Assignment #6   **PHYSICS   8.284**

Due 11:04am 2006 March 24 Friday

- Reading: Clayton §4.3 through equation (4-62). Hansen & Kawaler §§6.3-5.

1. Our midterm exam (24 March) will be closed book *except* for a single sheet of 8.5×11 inch paper (two sides) on which you may write anything you think will help you on the exam. Please prepare such a sheet and submit it as your first problem for this set *and bring a copy of the sheet to the exam.*

2. Using homology arguments, we found the following relation for the dependence of the luminosity of a star on its mass M, radius R, Kramer opacity coefficient κ_0, and mean molecular weight μ: $L = CM^{11/2}\mu^{15/2}\kappa_0^{-1}R^{-1/2}$, where C is a constant.

   a) Use the Sun to estimate the value of C/κ_0 taking the Sun's composition (mass fractions) to be X = 0.70, Y = 0.28, Z = 0.02.

   b) Suppose one had a star with X=0.98 and Z=0.02, i.e. no helium, but otherwise identical to the Sun. By what factor would this star be brighter or fainter than the Sun? *Just for fun:* Can you use the narrowness of the main sequence to turn this into an argument that most of the helium in the Sun must have been formed in the big bang?

   c) A red giant has a pure helium core that we can approximate as an n=1.5 polytrope with a central temperature of 10^8 K. Use the value of C/κ_0 obtained above to estimate the luminosity of a Y = 0.98, Z = 0.02 helium core with mass $0.5M_\odot$ in units of $L_\odot$.

   Assume that the dominant source of opacity in cases a), b) and c) above comes from atomic species heavier than helium.

3. The scaling (homology) relations derived in class were obtained by assuming an equation of state and by reducing the equations of hydrostatic equilibrium and radiative equilibrium to dimensional relations. We can do likewise with the last of the equations of stellar stucture, the energy generation equation, letting $L = C'M \left(\frac{M}{R^3}\right)T_c^n$. The assumptions embodied in this are that some constant fraction of the mass is involved in energy generation, that the rate is proportional to the central density, and that it is also (as we shall see) a strong function of the central temperature.

   a) We derived the relation $L = CM^{11/2}\mu^{15/2}\kappa_0^{-1}R^{-1/2}$ without recourse to an energy generation relation. Following the development in class, derive a *new* scaling relation for L which depends on M, κ_0, and μ but not R.

   b) Assume that $n = 4$ for a star like the Sun. Find the power law dependence of radius on mass.

   c) The "effective" temperature T_{eff} at the surface of the star is defined so that $L = 4\pi R^2 T_{eff}^4$ where R is the radius of the star. Use our scaling relations to eliminate R and determine the dependence of luminosity on effective temperature. Use the data in the following table to convert this into an approximate slope for the observed main sequence, $dM_V/d(B - V)$, evaluated at the Sun's position on the main sequence.

Main Sequence Properties

type	$B - V$	$T_{eff}(K)$
O5	-0.45	35,000
B0	-0.31	21,000
B5	-0.17	13,500
A0	0.00	9,700
A5	0.16	8,100
F0	0.30	7,200
F5	0.45	6,500
G0	0.57	6,000
G5	0.70	5,400
K0	0.84	4,700
K5	1.11	4,000
M0	1.24	3,300
M5	1.61	2,900

4. The radiative transfer equation has a term which is equal to the gradient (the radial derivative) of the radiation pressure.

   a) Use the fact that the total pressure must be greater than the radiation pressure and that the total opacity κ must be greater than the Thompson opacity κ_T to derive the inequality,

 $$L(r) < L_{edd} \equiv \frac{4\pi GM(r)c}{\kappa_T} \quad ,$$

   where L_{edd} is called the "Eddington" luminosity after Arthur Eddington. A star of mass M cannot be brighter than this.

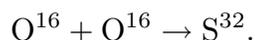   b) Compute the Eddington luminosity (both in erg/s and $L_\odot$) for a star of one solar mass.

5. Consider three stars. In the first star, oxygen is being produced via the reaction

   $$\mathrm{C}^{12} + \mathrm{He}^4 \rightarrow \mathrm{O}^{16} \quad .$$
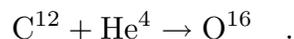
   In the second, magnesium is being produced via the reaction

   $$\mathrm{C}^{12} + \mathrm{C}^{12} \rightarrow \mathrm{Mg}^{24}.$$
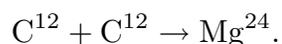
   In the third sulfur is being produced viat the reaction

   $$\mathrm{O}^{16} + \mathrm{O}^{16} \rightarrow \mathrm{S}^{32}.$$

   Suppose that all three reactions are proceeding at the same rate. What is the ratio of the temperatures at the centers of the three stars? In the spirit of ruthless approximating, *ignore* factors which multiply exponentials, (e.g. the densities of the reactants and the cross section factor $S(E)$).