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PROFESSOR: OK. Good morning, everybody. Let's get started. Let me just begin by asking if there are any questions, either about logistical issues or about physics issues?

OK. Today we'll be finishing our discussion of black-body radiation by talking about the actual spectrum of the cosmic microwave background that we find in our universe. And then move on to talk about the rather exciting discovery in 1998 of the fact that our universe today appears to have a nonzero cosmological constant.

So I want to begin by reviewing what we did last time. And one of the reasons why I do this is I think it's a good opportunity for you to ask questions that don't occur to you the first time we go through. And that, from my point of view, has been an extraordinary success. I think you've asked great questions. So we'll see what comes up today.

We began the last lecture by recalling, I think from the previous lecture actually, the basic formulas for black-body radiation, which is just the radiation of massless particles at a given temperature. And we have formulas for the energy density, the pressure, the number density, and the entropy density, all of which are given in terms of two constants, g and g^* , which is the only place where the actual nature of the matter comes in.

g and g^* are both equal to 2 for photons, but these formulas allow us to talk about other kinds of black-body radiation as well, black-body radiation of other kinds of particles. As neutrinos are also effectively massless, so they contribute. And in addition, e^+ plus e^- pairs, if the temperature gets hot enough so that the mass of the electrons becomes negligible compared to kt , also contribute to the cosmic background radiation. And if we want the higher temperatures, other particle will

start to contribute. And at the highest temperatures all particles act like black-body radiation.

The general formula for g and g_{star} is that there is a factor out front that depends on whether the particle is a boson or a fermion, a particle which does or does not obey the Pauli exclusion principle. Fermions do not, bosons-- excuse me, I said that backwards. Fermions obey the Pauli exclusion principle, bosons do not. G and g_{star} are both 1 for bosons. But for fermions there's a factor of $7/8$ for g and $3/4$ for g_{star} .

Yes?

AUDIENCE: Would you mind quickly restating why the positron-electron pairs act like radiation above that temperature?

PROFESSOR: OK. The question is, why do electron-positron pairs act like radiation at these high temperatures? And the answer is that radiation is just characterized by the fact that the particles are effectively massless. And the effective energy scale is kt , that's the average thermal energy in a thermal mix. So as long as $m_e c^2$ is small compared to kt , electrons and positrons think that they're massless and act like they're massless. And as I said, if you go to higher temperatures still, all particles will act like they're massless.

Coming back to the story of g and g_{star} , we have the factor out front which depends on whether they're bosons or fermions. And then that just multiplies the total number of particle types, whereby a particle type-- we made a complete specification of what kind of a thing it is. And that includes specifying what species of particle it is, whether it's a particle or an anti-particle if that distinction exists, and what the spin state is if the particle has spin.

So we can try this out now on some examples. First example, will be neutrinos which play a very important role in the early universe, and even in the particle number balance of today's universe. Neutrinos actually have a small mass, as we talked about last time and as I'll review again this time. But nonetheless, as far as

cosmology is concerned, they effectively act like massless particles although the story about why they act like massless particles is a little complicated. It's more than just saying that their mass is small, for reasons that we'll see.

But anyway, I'm nonetheless going to start by describing neutrinos as if they were massless, as was believed to be the case really until 15 years ago or so.

The massless model of the neutrino was a particle which was always left-handed. And by left-handed what I mean is that for neutrinos, if you took the angular momentum of the neutrino in the direction of the momentum, \hat{p} that means dotted with the unit vector in the direction of the spatial momentum, you'd always get minus $1/2$ in units of \hbar . And conversely, all neutrinos are right-handed which just means the same equation holds with the opposite sign. So neutrinos always have spins that oppose the direction of motion, and anti-neutrinos always have spins aligned with the direction of motion.

Now, it's not obvious but, if neutrinos were massless this would be a Lorentz invariant statement. If neutrinos have a mass, that statement is obviously not Lorentz invariant. As you can see by imagining a neutrino coming by, and you can get into a rocket ship, chase it, and pass it, and then see it going the other way out your window because you're going faster than it. You would see the momentum in the opposite direction from the way it looked to begin with. But the spin would look like it was the same direction as it did to begin with, and therefore the spin would now be aligned with the momentum instead of opposite the momentum.

So this could not possibly hold universally if the neutrino has a mass. But for the time being our neutrinos are massless. So we're going to take this as a given fact. And it certainly is a fact for all neutrinos that have ever been actually measured.

Given this model of the neutrino, the g for neutrinos is $7/8$ because they're fermions. Then there's a factor of 3, because there are three different species of neutrinos- electron neutrinos, muon neutrinos, tau neutrinos. Neutrinos come in particles or anti-particles which are distinct from each other, we think. So there's a factor of 2 associated with the particle anti-particle duality. And there's only one spin state. The

spin that's anti-aligned with the momentum, or aligned for the anti-neutrinos. But only one spin state in either case. So just a factor of 1 from spin states, and multiplying that through we get $2^{1/4}$ for g , and $9/2$ for g star.

Yes?

AUDIENCE: If we found out that they were Majorana, that they were their own anti-particles, would that change what we expect the temperature [INAUDIBLE] to be?

PROFESSOR: No, it would not. OK. The question was, if we find that they're Majorana particles-- which I'm going to be talking about in a minute-- where the particles would be their own anti-particles, which would mean that the right-handed anti-neutrino would really just be the anti-particle of the left-handed neutrino, it would not change these final numbers at all.

What it would do is, instead of having the 2 for particle anti-particle, we would have a 2 for spin states. So there would still be two kinds of neutrinos, but instead of calling them the neutrino and the anti-neutrino, the right words would be right-handed neutrino and left-handed neutrino. But the product would still be the same.

AUDIENCE: Wait, they have mass and they are Majorana?

PROFESSOR: If they have mass and Majorana, what I just said applies. The fact that they have a mass would mean at the lowest possible temperatures they would not act like black-body radiation. kT would have to be bigger than their mass times c squared. But that's only on the order of electron volts at most. So I'll talk later about why the true model neutrinos which have masses give the same result as this.

OK. Then we can also, just as an exercise, calculate g and g star. It's more than an exercise. We like to know the results. We can calculate g and g star for e plus e minus pairs, which is relevant for when kT is large compared to the rest energy of an electron. And again, they are fermions so we get a factor of $7/8$ appearing in the expression for g , and $3/4$ appearing in the expression for g star.

And then we just have to multiply that times the total number of types of electrons

that exist. There's only one species called an electron, so we only get a factor of one in the species slot of the product. There are both electrons and anti-electrons where the anti-electrons are usually called positrons. So we get a factor of 2 in particle anti-particle. Two spin states because an electron can be spin up or spin down, and that gives us $7/2$ and 3.

Given that, we can go ahead and calculate what the energy density and radiation should be for the present universe given the temperature of the photons, the temperature of the cosmic microwave background. And in doing that there's an important catch which is something which is the subject of a homework problem that you'll be doing on problem set seven.

When the electron-positron pairs disappear from the thermal equilibrium mix, if everything were still in thermal contact, its heat would be shared between the photons and the neutrinos in a way that would keep a common temperature. But in fact, when the e plus e minus pairs disappear, things are not in thermal contact anymore. And in particular, the neutrinos have decoupled. They're effectively not interacting with anything anymore. So the neutrinos keep their own entropy and do not absorb any entropy coming from the e plus e minus pairs. So all the entropy of the e plus e minus pairs gets transferred only to the photons. And that heats the photons relative to the neutrinos in a calculable amount, which you will calculate on the homework problem.

And the answer is that the temperature of the neutrinos ends up being only $4/11$ to the $1/3$ power, times the temperature of the photons. And that's important for understanding what's been happening in the universe since this time. That ratio is maintained forever from that time onward.

So if we want to write down the formula for energy density and radiation today it would have two terms. The 2 here is the g for the photons, and this, times that expression is the energy density in photons. The second term is the energy density in neutrinos. And it has the factor of $21/4$ which was the g factor for neutrinos. But then there's also a correction factor for the temperature, because on the right hand

side here I put t gamma to the fourth. So this factor corrects it to make it into t neutrino to the fourth, which is what we need there to give the right energy density for the neutrinos today. And this is just that ratio to the fourth power.

And once you plug in numbers there it's 7.01 times 10 to the minus 14th joules per meter cubed, which is, from the beginning, what we said was the energy density in radiation of the universe today.

OK. Finally I'd like to come back to this real story of neutrinos and their masses and why, even though they have small masses, the answers that we gave for the massless model of the neutrino are completely accurate for cosmology.

We've never actually measured the mass of a neutrino. But what we have seen is that neutrinos of one species can oscillate into neutrinos of the other species. And it turns out, theoretically, that that requires them to have a mass. And by seeing how fast they oscillate you can actually measure the difference in the mass squareds between the two species. So it's still possible actually, in principle, that one of the species could have zero mass. But they can't all have zero mass because we know the differences in the squares of their masses.

So in particular, Δm^2_{21} times c^4 , meaning the mass expressed as an energy, is 7.5 times 10 to the minus 5 eV squared. And larger values obtained for Δm^2_{32} , which is 2.3 times 10 to the minus 3 eV squared. We're still talking about fractions of an eV.

The other of the three possible combinations here are just not known yet.

Now, if neutrinos have a mass, that does actually change things rather dramatically because of what we said about-- the statement that the neutrinos always align their spins with their motion just cannot be true if neutrinos have a mass. And more generally, for any particle with a mass of arbitrary spin j , the statement is that, the component of j along any particular axis-- and we'll call it the z-axis-- always takes on the possible values in terms of \hbar going from minus j up to j with no emissions. It's different for massless particles. For massless particles every one of

these elements on the right hand side is independent and, by itself, a Lorentz invariant possibility.

But, coming back to neutrinos-- if the neutrinos have a mass, in addition to the left-handed neutrinos there has to be a right-handed partner. And the question then is, what's the story behind that? And it turns out we don't know the story. But we know two possible stories.

And one of the possible stories is that could be what's called a Dirac mass. And for Dirac mass what it means is that, the right-handed neutrino is simply a new type of particle which just happens to be a particle that we've never seen, but a particle which would have a perfectly real existence. It would however, to fit into theory and observation, be an extremely weakly interacting particle. The interactions of the right-handed one do not have to be the same as the interactions of the left-handed one. That is, the interactions can depend explicitly on $\hat{p} \cdot \hat{j}$. So depending its value, you could affect what the interactions are, again, in a Lorentz invariant way.

And in practice, the right-handed neutrinos would interact so weakly that we would not expect to see them in the laboratory. And we would not expect even in the early universe that they would have been produced in any significant number. So even though it would be a particle that, in principle, exists, we would not expect to see it. And we would not expect it to affect the early universe.

Alternatively-- and in some ways a more subtle idea-- is that the mass of the neutrino could be what's called a Majorana mass. Where Majorana, like Dirac, is the name of a person-- perhaps less well known than Dirac, but made important contributions in this context nonetheless.

In this case, it can only occur if lepton number is not conserved. And if lepton number is not conserved then there are really no quantum numbers that separate the particle that we call a neutrino from the particle that we call an anti-neutrino. And if that's the case, the particle that we call the anti-neutrino could, in fact, just be the right-handed partner of the neutrino.

So for the Majorana mass case we don't need to introduce any new things that we haven't already seen. We just have to rename the thing that we've been calling the anti-neutrino the neutrino with helicity plus 1 instead of minus 1-- it's with $\hat{j} \cdot \hat{p}$ with $\hat{j} \cdot \hat{p}$ equal to plus 1/2 instead of minus 1/2. So these would just be the two spin states of the neutrino instead of the neutrino and the anti-neutrino. And that's a possibility.

And this would also change nothing as far as the counting that we did. It would just change where the factors go instead of having a factor of 2 for particle anti-particle and this type counting. We'd have a factor of 2 in the spin state factor, and a factor of 1 in the particle anti-particle [INAUDIBLE]. The particle and the anti-particle would be the same thing.

OK. Any questions about that?

OK. Finally-- and I think this is my last slide of the summary. At the end of the lecture we just pointed out a number of tidbits of information.

We can calculate the temperature of the early universe at any time from the formulas that we already had on the slide. We know how to calculate the energy density at any time. And by knowing about black-body radiation we can convert that into a temperature.

And for an important interval of time, which is when kt is small enough so that you don't make muon anti-muon pairs, but large enough so that electron-positron pairs act like they are massless, and this very large range kt is equal to 0.860 m e v , divided by the square root of time where time is measured in seconds. So in particular, at 1 second kt is 0.86 m e v . And it does apply at 1 second. Because 0.86 m e v is in this range.

We also then talked about the implications of the conservation of entropy. If total entropy is conserved, the entropy density has to just fall off like 1 over the cube of the volume. Total entropy is conserved for almost all processes in the early universe. So the entropy falls off like 1 over a cubed. And that means that, as long

as we're talking about a period of time during which little g does not change-- and little g only changes when particles freeze out, like when the electron-positron pairs disappear-- but as long as little g doesn't change, s [INAUDIBLE] 1 over a^3 means simply that the temperature falls like 1 over a . And when little g changes you can even calculate corrections to this as, effectively you're doing when you calculate this relationship between the neutrino temperature and the photon temperature.

And finally, we talked about the behavior of the atoms in the universe as the universe cools. For temperatures above about 4,000 degrees the universe, which is mainly hydrogen, is mainly a hydrogen plasma. Isolated protons and electrons zipping through space independently. At about 4,000 Kelvin-- and this is a stat [? mac ?] calculation, which we're not doing-- but using the answer.

At about 4,000 Kelvin-- which is a number which depends on the density of hydrogen in the universe, it's not a universal property of hydrogen-- but for the density of hydrogen in the universe, at about 4,000 Kelvin hydrogen recombines. It becomes neutral atoms.

And slightly colder, at about 3,000, the degree of ionization becomes small enough so that the photons become effectively free. The photons decouple. In between 4,000 and 3,000 the hydrogen is mostly neutral, but they're still enough ionized so that the photons are still interacting.

So the most important temperatures-- the 3,000 Kelvin, when the photons are released, when the photons are no longer trapped with the matter of the universe. And last time we estimated the time at which that happens. That should be a small t , sorry. The time of decoupling is about 380,000 years. And that number is actually very accurate, even though we didn't calculate it very accurately.

And that's the end of my summary. Any questions about the summary?

OK. In that case, let's go on to talk first about the spectrum of the cosmic background radiation. And then we'll move on to talk about the cosmological constant.

CMB is cosmic microwave background. And that's a very, very standard abbreviation these days.

So when the cosmic microwave background was first discovered by Penzias and Wilson in 1965-- which, I might point out, is going to have its 50th anniversary in the coming year-- they only measured it at one frequency. It was a real tour de force to measure it at the one frequency and to convince themselves that the buzz that they were hearing in their detector was not just some kind of random electrical noise, but really was some signal coming from outer space.

And the main clue that it was some signal coming from outer space was that they were able to compare it with a cold load, a liquid helium-cooled source, and find that that comparison worked the way they expected. And the main reason for believing it was cosmological rather than local is that they got the same reading no matter what direction they pointed their antenna.

This just took a lot of radio technique skill to convince themselves that it wasn't just some radio tube that was malfunctioning or something. They even worried that it may have been caused by pigeon droppings in their antenna, I actually read about in Weinberg's book. But they finally convinced themselves that it was real. They were still not convinced really that it was a sign for the big bang and-- you may recall, again, from reading Weinberg that there were two papers published back-to-back. The experimental paper by Penzias and Wilson, which really just described the experiment, mentioning that a possible explanation was in this other paper by Dickie, Peebles, Roll, and Wilkinson which described the theory that this was radiation that originated with the big bang. But it's all based on one point at one frequency.

Shortly afterwards, I guess within the same year, Roll and Wilkinson were able to measure it at a slightly different frequency. And when I wrote my popular-level book I tabulated all of the data that was known in 1975. And this mess is the graph.

This shows sort of the full range of interesting frequencies. The solid line here is the expected theoretical curve corresponding to a modern measurement of the

temperature 2.726 degrees Kelvin. All of the interesting historical points are in this tiny little corner on the left, which is magnified above. The original Penzias and Wilson point is way down here at very low frequencies by the standards of radiation at 2.726 degrees Kelvin.

The Roll and Wilkinson point is there. These blobs indicate error bars. The [? cyanogen ?] points that you read about in Weinberg are shown there and there.

The first measurement that showed that, it didn't only go up but started to go down like black-body radiation should, was a balloon flight-- this 1971 balloon flight which produce that blob and that bound. This was an experiment by MIT's own Ray Weiss. And it was very important in the history because it was the first evidence that we weren't just seeing some straight line, but we were seeing something which did indeed rise and fall the way black-body radiation should.

A later balloon flight in 1974 produced error bars that are shown by this gray area. Incredibly broad. So the bottom line that this graph was intended by me to illustrate is that, in 1975 you could believe that this was black-body radiation if you so wished. But there was not really a lot of evidence that it was black-body radiation.

The situation did not get better quickly. The next significant measurement came in 1987 which was a rocket flight, which was a collaboration between a group at Berkeley and a group at Nagoya, Japan. I believe it was the Japanese group that supplied the rocket and the American group that supplied the instrumentation.

And they measured the radiation at three points. I can give you the number that goes with those graphical points. I guess what I have tabulated here is the effect of temperature that those points correspond to.

As you can see from the graph, those points are all well above the black-body curve. Significantly more radiation than what was expected by people who thought it should be black-body.

And 0.2 up there would correspond to a temperature of 2.955 plus or minus 0.017 K. The size of the vertical bars there are the error bars that the experimenters

found. And 0.3 was t equals 3.175 plus or minus 0.027 K.

So these were higher temperatures than the 2.7 that fit the lower part of the spectrum. And very, very small error bars.

So this data came out in 1987. And, in truth, nobody knew what to make out of it. The experimental group were well aware that this was not what people wanted them to find. And they certainly examined their data very carefully to figure out what could have conceivably gone wrong. And they were going around the country giving talks about this. And I heard one or two of them in which they described how surprised they were by the results, but emphasized that they analyzed the experiment very, very carefully and couldn't find anything wrong with it. And this was the situation for awhile.

I should point out that I think this point number three is something like 16 standard deviations off of the theory. And usually when somebody makes a measurement that's three or four standard deviations off of your theory, you really start to worry. 16 standard deviations is certainly a bit extreme.

Nonetheless, nobody had any good explanation for this. So, well, different people had different attitudes. There were some people who tried to construct theories that would account for this. And there were others who waited for it to go away. I'm pretty sure I was among those who waited for it to go away, and we were right.

So the next important piece of data came from the first satellite dedicated to measuring the cosmic background radiation. The famous COBE Satellite-- Cosmic Background Explorer-- I guess I didn't write down the name here. Oh, it's in the title. Preliminary measurement of the cosmic microwave background spectrum by the Cosmic Background Explorer, COBE Satellite.

So COBE was the first satellite dedicated to measuring the cosmic background radiation. It was launched in 1989, I guess, and released its first data in January of 1990.

Back in those days there was no internet or archive. So you may or may not know that the way physics results were first announced to the world were in the form of what were called pre-prints, which were essentially xeroxed copies of the paper that were sent out to a mailing list. Typically, I think, institutions had mailing lists of maybe 100 other institutions. And every physics department had a pre-print library that people can go to and find these pre-prints.

So this is the COBE pre-print. 90-01, the first pre-print from 1990. And this is the data. So it is kind of breathtaking, I think. It suddenly changed the entire field, and in some sense really change cosmology for the field. Where we only had approximate ideas of the way things worked, to suddenly having a really precise science in which precise measurements could be made, and cleared up the issue of the radiation. It wasn't just a mess like this, or a terrible fit like that, but a fantastically good fit. Really nailing the radiation as having a thermal spectrum.

So the history is that John Mather presented this data at the January 1990 American Physical Society meeting, and was given a standing ovation. And he later won the Nobel Prize for this work. He was the head of the team that brought this data. He won the Nobel Prize in 2006 along with George Smoot, who was responsible for one of the other experiments on the COBE satellite.

Yes?

AUDIENCE: So do we know what happened with the other measurements?

PROFESSOR: To tell you the truth, I don't think the other measurements ever-- the other people ever really published what they think happened. But the widespread rumor, which I imagine is true, is that they were seeing their own rocket exhaust. And there were, I think, some arguments going on between the Americans and the Japanese, with the Americans more or less accusing the Japanese of not really telling them how the rocket was set up.

Yes?

AUDIENCE: Are the error bars plugged on those points, or is it just that good?

PROFESSOR: Those are the error bars?

AUDIENCE: OK.

PROFESSOR: And even more spectacular, a couple years later, I guess it was-- this was actually just based on nine minutes of data or something like that. But a couple years later they published their full data set, where the size of the error bars were reduced by a factor of 10. And still a perfect fit. They didn't even know how to plot it, so I think they plotted the same graph, and said the error bars are a factor of 10 smaller than what's shown. It was gorgeous.

So I think I forgot to tell you what the spectrum is supposed to look like exactly. And this is just a formula that I want you to understand the meaning of, but not the derivation of. We-- as with the other stat mech results that we're relying on, we're going to relegate their derivation to the stat mech course that you either have taken or will take.

But the spectrum is completely determined because the principle of thermal equilibrium is sort of absolute in statistical mechanics. And in order for a black-body radiating object to be in thermal equilibrium with an environment at that temperature, it has to have not only the right emission rate but also the right spectrum. If the spectrum weren't right you could imagine putting in filters that would trap in some frequencies and let out others. And then you would move away from thermal equilibrium if the spectrum were right or wrong because you'd be trapping in more radiation-- you could arrange for the filters to trap in more radiation than they are letting out.

So the spectrum is calculable. And in terms of-- I guess this is energy density. I have to admit, I usually call energy density u and in these notes here it's called ρ . We'll figure out the units after I write it down and make sure that it is energy density. ρ_{ν} or $\rho_{\nu} d\nu$, means-- with this product it means the total energy density, energy per volume, per frequency interval, $d\nu$ -- well, it's times $d\nu$, so if you multiply by ν , this is the total energy for frequencies between ν and $\nu + d\nu$.

$d\nu$.

And the formula is $16\pi^2 \hbar^3 \nu^3$, divided by c^3 times 1 over e to the $2\pi \hbar \nu$ over kt , minus 1 $d\nu$.

OK. And actually, the unit's not that transparent. I believe this is energy density and not mass density. But maybe I'll make sure of that and let you know next time.

And this is what produces that curve that you saw on the slides. I've included the subscript ν here to indicate that it's the number which, when you multiply it by $d\nu$, gives you the energy density between ν and $\nu + d\nu$.

If instead you wanted to know the energy density between λ and $\lambda + d\lambda$, there'd be a kinematic factor that you'd have to put in here-- the factor that relates $d\lambda$ to $d\nu$. And you could imagine working that out.

I might add that, in Weinberg's book, he actually plots both λ of ν . So his curve looks somewhat different than the curves that I showed you. This is not exactly the same thing.

Now, what this extremely accurately black-body curve proves is that the early universe really was very accurately in thermal equilibrium. And that can only happen if the early universe was very dense. And of course, our model of the universe goes back to infinite density. So the model predicts that it should be in thermal equilibrium.

But in particular, the numbers that we have here, if you ask how much could you change the model and still expect these curves the answer is roughly that, all of the important energy-releasing processes have to have happened before about one year after the Big Bang. Anything that happened after one year would still show up as some glitch in the black-body spectrum. So the big bag model really is confirmed back to about one year on the basis of this precise measurement of the spectrum of the cosmic background radiation.

And the COBE measurement is still, by the way, the best measurement of the

spectrum. We've had other very important experiments, that we'll talk about later, which measure the non-uniformity of the black-body radiation. Which is very small, but nonetheless very, very important [? effect. ?]

So we've had WMAP and now Planck which have been dedicated to measuring the anisotropies of the radiation. COBE also made initial measurements of the anisotropies. And we'll be talking about anisotropies later in the course.

Yes?

AUDIENCE: [INAUDIBLE].

PROFESSOR: Sorry?

AUDIENCE: The units of the right-hand side are energy density.

PROFESSOR: Energy density. OK. Thanks. OK. Good. So my words were right. I should have called it u , I think, to be consistent with my usual notation. Thanks.

OK. Any other questions about the CMB? Because if not, we're going to change gears completely and start talking about one of the other crucially important observational discoveries in cosmology in the last 20 years.

OK. So what I want to talk about next is the very important discovery originally made in 1998-- also resulting in a Nobel Prize-- that the universe is accelerating.

And this was a discovery that involved two experimental groups, and a total of something like 52 astronomers between the two groups. Which actually meant that-- I'm exaggerating slightly, I suppose. But it really involved the majority of the astronomers of the world, and therefore there weren't a lot of astronomers to argue with them about whether or not the result was right. But there still was some argument.

The announcement was initially made at a AAS meeting in January of 1998 by-- which group was first? I think that was the High-Z Supernova-- where are they? Yeah. That was the High-Z Supernova Search Team. And then there was also a

group largely based at Berkeley. The High-Z Supernova Search Team was actually fairly diffused, although based to some extent at Harvard. And the Supernova Cosmology Project was based rather squarely in Berkeley, headed by Saul Perlmutter.

And they both agreed. And what they found was, by looking at distant supernovae of a particular type-- type 1a-- they were able to use these supernovae as standard candles. And because supernovae are brighter than other standard candles that had been studied earlier in history, they were able to go out to much greater distances. And that means to look much further back in time than previous studies.

And what they discovered was that the expansion rate of the universe today was actually faster, and not slower, than the expansion rate about five billion years ago. And that was a big shock because until then everybody expected that gravity would be slowing down the expansion of the universe. And when these guys started to make these measurements they were just simply trying to figure out how fast the universe was slowing down. And they were shocked to discover that it was not slowing down, but instead speeding up.

Initially there was some controversy about it. People did try to invent other explanations for this data. But the data has, in fact, held up for the period from 1998 to the present. And in fact, it has been strongly supported by evidence from these anisotropies in the cosmic microwave background radiation, which we'll be talking about later. But it turns out, you can get a lot of information from these anisotropies in the cosmic background radiation.

So the picture now is really quite secure, that the acceleration-- the expansion of the universe is actually accelerating, and not decelerating. And the simplest explanation for that, which is the one that-- well, certainly because it's the most plausible, and the one that most of us take seriously, and it's the only one that fits the data extraordinarily well. So we've not seen any reason not to use this explanation. The simplest explanation is that there's a nonzero energy density to the vacuum, which is also what Einstein called the cosmological constant.

So we should begin by writing down the equations that describe this issue. So we've learned how to write down the second order Friedmann equation, which describes how the scale factor of the universe accelerates. And on the right-hand side, once we included materials with nonzero pressures, we discover that we need on the right-hand side, $\rho + 3p$, over c^2 -- excuse me-- times a .

Now when the cosmological constant was born, was when Einstein first turned his theory of general relativity to cosmology. Einstein invented the theory of general relativity in 1916. And just one year later, in 1917, he was applying it to the universe as a whole to see if he could get a cosmological model consistent with general relativity.

Einstein at that point was under the misconception that the universe was static, as Newton had also thought, and as far as we know, as everybody between Newton and Einstein thought. If you look up at the stars, the universe looks pretty static. And people took this very seriously. In hindsight, it's a little hard to know why they took it so seriously, but they did.

So when Einstein discovered this equation he was assuming that the universe consisted of basically non-relativistic stuff. Stars are essentially non-relativistic hunks of matter. So he thought that ρ would be positive, the effective pressure would be zero. And he immediately noticed that this equation would imply that the scale factor would have a negative acceleration. So that if you tried to set up a static universe it would instantly collapse.

And as we talked about earlier, Newton had talked himself out of that conclusion. And I think the real difference, as I think we also talked about earlier, was that Newton was thinking of the law of gravity as an action at a distance, where you determine the total force on something by integrating the forces caused by all other masses.

And then things get complicated and divergent, actually, for an infinite, static universe. And Newton managed to convince himself that you could have a static universe of that type, a statement that we now consider to be incorrect even in the

context of Newtonian mechanics.

But this fact that it's incorrect even in the context of Newtonian mechanics was really not discovered until Einstein wrote down this equation. And then Einstein himself also gave a Newtonian argument showing that, at least with a modern interpretation of Newtonian mechanics. It doesn't work in Newtonian gravity either to have a static universe.

But Einstein was still convinced that the universe was static. And he realized that he could modify his field equations-- the equations that we have not written down in this course, the equations that describe how matter create gravitational fields-- by adding a new term with a new coefficient in front of it which he called lambda.

And this extra term, lambda, could produce a kind of a universal gravitational repulsion. And he realised he had to adjust the constant to be just right to balance the amount of matter in the universe. But he didn't let that bother him. And if he adjusted it to be just right, and the universe was perfectly homogeneous, he could arrange for it to balance the standard force of gravity.

We can understand what lambda does to the equations because it does, in fact, have a simple description in terms of things that we have discussed and do understand. That is, you could think of lambda as simply corresponding to a vacuum energy density. Einstein did not make that connection.

And not being an historian of science, I can speculate as much as I want. So my speculation is that, the reason this did not occur to Einstein is that Einstein was a fully classical physicist who was not at this time or maybe never accepting the notions of quantum theory. And in any case, quantum field theory was still far in the future.

So in classical physics the vacuum is just plain empty. And if the vacuum is just plain empty it shouldn't have any energy density. The quantum field theory picture of the vacuum, however, is vastly more complex. So to a modern quantum field theory-oriented theoretical physicist the vacuum has particle, anti-particle pairs appearing

and disappearing all the time. We are now convinced that there's also this Higgs field that has even a nonzero mean value in the vacuum.

So the vacuum is a very complicated state which, if anything characterizes it, it's simply the state of lowest possible energy density. But because of, basically, the uncertainty principles of quantum mechanics, the lowest possible energy density does not mean that all the fields are just zero and stay zero. They're constantly fluctuating as they must according to the uncertainty principle, which applies to fields as well particles.

So we have no reason anymore to expect the energy density of the vacuum to be zero. So from a modern perspective it's very natural to simply equate the idea of the cosmological constant to the idea of a nonzero vacuum energy density. And there are some unit differences-- just constants related to the historical way that Einstein added this term his equations. So the energy density of the vacuum-- which is also the mass density of the vacuum times c squared-- is equal to Einstein's λ times c to the fourth, over $8\pi G$.

And this is really just an historical accident that it's defined this way. But this is the way Einstein defined λ .

Now, if the vacuum has an energy density, as the universe expands the space is still filled with vacuum. At least, if it was filled with vacuum. If it was matter it would thin out. But we can imagine a region of space that was just vacuum, and as it expands it would have to just stay vacuum. What else could it become? And that means that we know that, for a vacuum, ρ dot should equal zero.

Now we've also learned earlier, by applying conservation of energy to the expanding universe, that ρ dot in an expanding universe, is equal to minus $3\dot{a}$ over a . Or we could write this as h times ρ plus p over c squared.

This is basically a rewriting of $d u$ equals minus $p d v$, applying it to the expanding universe. So I won't re-derive it. We already derived it. Actually, I think you derived it on the homework, was the way it actually worked.

But in any case, this immediately tells us that if ρ dot is going to be 0 for vacuum energy, this has to be 0. And therefore p vacuum has to be equal to minus ρ vacuum times c squared.

And if we know the energy density in the pressure of this stuff called vacuum, that's all we need to know to put it into the Friedmann equations and find out how things behave. Otherwise this vacuum energy behaves no differently from anything else. It just has a particular relationship between the pressure and the energy density, with a very peculiar feature- that the pressure is negative. And that's an important feature because we had commented earlier that a negative pressure can drive acceleration. And now we're in a good position to see exactly how that works.

To sort of keep things straight I'm going to divide the mass density of the universe into a vacuum piece and a normal piece, where normal represents matter, or radiation, or anything else, if we ever discover something else. But in fact it will just be matter or radiation for anything that we'll be doing in this course, or anything that's really done in current cosmology.

And similarly, I'm going to write pressure as p vac plus p normal. "N" is for normal. But p vac I don't really need to use, because p vac I can rewrite in terms of ρ vac. So in the end I can express everything just in terms of ρ vac.

And I can write down the second order Friedmann equation. And it's just a matter of substituting in that ρ and that p into the Friedmann equation that we've already written. And we get minus 4π over $3G$, times ρ normal plus $3p$ normal, over c squared.

And the vacuum pieces-- have two pieces because there's a vacuum piece there and a vacuum piece there. it can all be expressed in terms of ρ vac and collected. And what you get is minus 2ρ vac times a . Showing just what we were talking about. That because of that minus sign, multiplies that minus sign, vacuum energy drives acceleration, not deceleration.

And that's why vacuum energy can explain these famous results of 1998. And we'll

see later that, for the same reason vacuum energy or things like vacuum energy can actually drive the expansion of the universe in the first place in what we call inflation.

Yes?

AUDIENCE: So for the equation without the cosmological constant it's, let's say, ρ and p are about the constant, then wouldn't that be the equation for a simple harmonic function [INAUDIBLE] or the oscillation of a [INAUDIBLE] is some negative constant times a ?

PROFESSOR: That's right, except that you would probably not believe the equations with the bounds.

AUDIENCE: OK.

PROFESSOR: And when a went negative you wouldn't really have a cosmological interpretation anymore, I don't think. But it is, in fact, true that if ρ and p were constants-- I'm not sure of any model that actually does that-- this would give you sinusoidal behavior during the expanding and contracting phase.

Yes?

AUDIENCE: [INAUDIBLE] the vacuum energy is constant over time, is it also makes sense [INAUDIBLE]?

AUDIENCE: Are you asking, does it make sense for maybe the vacuum energy to change with time? I think, if it changed with time, you wouldn't call it vacuum energy. Because the vacuum is more or less defined as the lowest possible energy state allowed by the laws of physics. And the laws of physics, as far as we know, do not change with time.

It's certainly true that, in a completely different context, you might imagine the laws of physics might change with time. And then thing would get more complicated. But that would really take you somewhat outside the sphere of physics as we know it.

You could always explore things like that, and it may turn out to be right. But at least

within the context of physics as we currently envision it, vacuum energies are constant, pretty much by definition.

Now I should maybe qualify that within the context of what we understand, there may, in fact, be multiple vacua. For example, if you have a field theory one can have a potential energy function for one or more fields. And that potential energy function could have more than one local minimum. And then any one of those local minima is effectively a vacuum. And that could very likely be the situation that describes the real world. And then you could tunnel from one vacuum to another, changing the vacuum energy. But that would not be a smooth evolution. That would be a sudden tunneling.

OK. So this is what happens to the second order Friedmann equation. It is also very useful to look at the first order Friedmann equation, which is a dot over a squared, 8π over $3G$. And in its native way of being written we would just have 8π over $3G$ rho, minus k over-- k squared over a squared.

And all I want to do now is replace rho by rho vac plus rho n. And this is a first order Friedmann equation. And we can expand rho n if we want more details, as rho matter plus rho radiation. And rho matter, we know, varies with time proportional to 1 over a cubed. Rho radiation behaves with time as 1 over a to the fourth. So all of the terms here, except for rho vac, fall off as a grows.

And that implies that if you're not somehow turned around firsts, which you can be-- you could have a closed universe that collapses before vacuum energy can take over. But as the universe gets larger, if it doesn't turn around, eventually rho vac will win. It will become larger than anything else because everything else is just getting smaller and smaller. And once that starts to happen everything else will get smaller and smaller, faster and faster, because a will start to grow exponentially.

If rho vac dominates-- which it will, as I said, unless the universe re-collapses first-- so for a large class of solutions rho vac will dominate-- then you can solve that equation. And you have \dot{h} , which is a dot over a, approaches, as a goes to infinity, the square root of 8π over $3G$ rho vac.

So h will approach a fixed value for a universe which is ultimately dominated by ρ vacuum. And if \dot{a} is a constant, that means that a grows exponentially.

So we could maybe give this a name-- h_{vac} . The value h has when it's completely dominated by the vacuum energy. And then we can write that a of t is ultimately going to be proportional to e to the h_{vac} times t . Which is what you get when you solve the equation, \dot{a} equals this constant.

OK. Now one thing which you can see very quickly-- let's see how far I should plan to get today. OK. I'll probably make one qualitative argument and then start a calculation that won't get very far. I will continue next time.

One qualitative point which you can see from just glancing at these equations is that the cosmological constant, when added to the other ingredients that we've already put into our model universe, will have the effect of increasing the age of the universe for a given value of h . And that's something that we said earlier in the course, we're looking forward to.

Because the model of the universe that we've been constructing so far have always turned out to be too young for the measured value of h . That is, the oldest stars look like they're older than the universe. And that's not good. So we'd like to make universe look older. And one of the beauties of having this vacuum energy, as far as making things fit together, is that it does make the universe older.

And the easiest way to see that-- at least a way to see that-- is to imagine drawing a graph of h versus t . Hubble expansion rate versus t . And if we look at the formula for h here we see that the ρ_{vac} piece just puts in a floor as h evolves with time, instead of going to 0 as it would in most models-- at least, as it would in open-universe model. It stops at some floor. And certainly for the models that we've been dealing with, h just decreases to some-- this is supposed to represent the present time. So this is previous models.

Now as you might say, that what I'm trying to describe here is not quite a theorem if

you considered closed universes where this k piece could be causing a positive-- a negative contribution to h , which is then decreasing with time. Things can get complicated. But for the models that we've been considering which are nearly flat, that k piece is absent. And then we just have pieces that go like, 1 over a cubed, 1 over a to the fourth, and constant. All of which are positive.

Then in the absence of vacuum energy we would have h falling. And with the presence of vacuum energy it would not fall as fast because we have this constant piece that would not be decreasing. So this is previous models. This is with ρ_{vac} . And I'm always talking about positive ρ_{vac} because that is what our universe has.

So this would be the two different behaviors of h for the model without vacuum energy and the model with vacuum energy. And if we're trying to calculate the age of the universe we would basically be extrapolating this curve back to the point where h was infinite, at the big bang. And we could see that, since this curve is always below this curve, it will take longer before it turns up and becomes infinite. So the age will always increase by adding vacuum energy.

With ρ_{vac} h equals infinity is further to the left. And notice that I'm comparing two different theories, both of which are the same age today. Because that's what we're interested in. We've measured the value of h . We're trying to infer the age of the universe.

OK. Maybe I'll just say a couple words about the calculation that we'll be starting with next time. We want to be able to precisely calculate things like the age of the universe, including the effect of this vacuum energy. And we'll be able to do that in a very straightforward way by using this first order Friedmann equation.

We know how each term in this Friedmann equation varies with a . And we can measure the amount of matter, and the amount of radiation, and in principal the amount of curvature-- it's negligibly small-- in our current universe. And once you have those parameters you can use that equation to extrapolate, to know what h was at any time in the past. And that tells you how the derivative-- it tells you the value of \dot{a} at any time in the past. And if you know the value of \dot{a} at any time

in the past, it's a principle just a matter of integration to figure out when a was 0.

And that's the calculation that we'll begin by doing next time. And we'll be able to get an integral expression for the age of the universe for an arbitrary value. We'll, at the end, express the matter density and the radiation density as fractions of ω , fractions of the critical density. And for any value of ω matter, ω radiation, and we'll even express the curvature as an ω curvature. The effective fraction of the critical density that this term represents.

And in terms of those different ω s, we'll be able to write down an integral for the total age of the universe. And that really is going to be state of the art. That is what the Planck team uses when they're analyzing their data to try to understand what the age of universe is according to the measurements that they're making. So we will finally come up to the present as far as the actual understanding of cosmology by the experts.

So that's all for today. see you all next Tuesday.