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PROFESSOR:

OK, in that case, let's go on. At the end of last class, we were beginning to talk about the Doppler shift. And we defined our terms. And I guess I'll repeat the definitions on the blackboard here. We are talking initially about the case where the observer is stationary and the source is moving with velocity v . We are talking initially about sound waves, waves which have a fixed speed relative to some medium.

And the speed relative to the medium will be called u . v will be the velocity of recession, as shown on the diagram. Δt_s , s for source, will be the time interval between wave crests as measured at the source-- so period of wave at source. And Δt_o will be the period of the wave at the observer. And what we're trying to calculate is the relationship between Δt_o and Δt_s .

OK, at this point, I'd like to go to the screen and go through the different stages of what happens as this process takes place. We start in frame one with an observer in some location, a source at some different location. Source is moving to the right. Source is emitting the first wave crest in this slide number one. Nothing too interesting so far.

Next picture, the source emits the second wave crest. But meanwhile, the source has moved. The time between wave crests as seen by the source is Δt_s . So the distance that the source will move during that time interval is v times Δt_s . And we'll call that Δl . And this really is the important slide. I think I have it highlighted here. That picture is what really counts for the whole Doppler shift. It says that the second wave crest has to travel a little further than the first wave crest by this amount Δl . So Δl will be the crucial quantity that will control the answer. Third slide, the waves have traveled. Now the first crest in this third frame

has just hit the observer. Next frame, the last crest, the second crest, has hit the observer.

And to figure out the Doppler shift from those images, all we have to do is realize that if both objects were stationary, there'd be no time difference between observation and source. Each could occur later by some fixed amount, the amount of time it would take the sound wave to travel. But they will occur later by the same amount if there was no motion. So if there's no motion, Δt_o would be equal to Δt_s , if there was no motion.

But because there is motion, we said that the wave, the second crest, is going to have to travel further by the amount Δl . So it'll be delayed by the amount of time it takes for the wave to travel Δl . And that's just Δl divided by v -- so plus Δl divided by v . And we know what Δl is. Δl is just v times Δt_s . I'm sorry, this is divided by u . u is the speed of the wave. v is the speed of the source.

So here we have in the numerator v times Δt_s , and the denominator is u . This is just what we said Δl is. So this really is our result. It tells us what Δt_o is in terms of Δt_s . And we can solve for the ratio. It tells us that Δt_o over Δt_s is equal to $1 + v$ over u . Solve that equation.

Now there's a standard definition that's used to describe redshifts, which is that this ratio, which is also the ratio of wavelengths, which is how I usually think of it, the wavelength at the observer divided by the wave length at the source-- the wave length being disproportional to the Δt . The wave ones are just the wave speed times Δt .

This is defined to be $1 + z$ where z is what's called the redshift. And the astronomers take out the one so that if both objects are stationary, z is equal to 0. That corresponds to know redshift. That means the wave length is the same at the source and at the observer. So the ratio of the wavelength as its observed to the wavelength of the source is what's called $1 + z$.

So in this case, it follows immediately that the redshift for this case is just v over u . So maybe I'll just write that again in a box and label it. z is equal to v over u corresponds to the nonrelativistic case, or the sound wave case, with the source moving. OK? Everybody on board? Any questions? OK, straightforward enough, I think.

OK, now we will go on to do the alternative simple case, where the observer is moving and the source is stationary. So keep the source on the right and the observer on the left. But this time it's the observer that's moving again with speed v . So v is always for these two cases in my notation the relative velocity between the source and the observer.

So now we have a new sequence of pictures. So first picture again is fairly trivial. The first wave crest is being emitted by the source. Picture number two, the second wave crest is emitted by the source. Picture number three, the first wave crest arrives at the source. And picture number four now, the second wave crest arrives at the source. And for this case, it is this last frame where all the action is.

The point is that between the time when the first crest hits the source and the time when the second crest hits the source, that is the time between the third and fourth frame, the source has moved. And it's moved by distance which is v times the time between those images. And the time between these images is just the time that the source experiences between the receipt of the two waves. And it's therefore what we've been calling $\Delta t_{\text{sub } 0}$, the time interval as measured by the observer. So the distance traveled is just v times $t_{\text{sub } 0}$. And it's that last frame inside that box where all the action happens for determining the answer for this problem.

So we can put that into equations also. This time it's slightly more complicated. It starts basically with the same idea. $\Delta t_{\text{sub } 0}$ is equal to $\Delta t_{\text{sub } s}$, which is what it would be if there was no motion. But it's a little bit longer because of the extra distance the second pulse travels. And that extra distance is again called Δl . So the time delay is again Δl divided by u , the wave speed. But this time we have a different formula for Δl . This time Δl is v times $\Delta t_{\text{sub } 0}$, instead

of last time it was v times Δt_s .

So this time the equation is ever so slightly more complicated, because Δt_0 appears on both sides of the equation. Last time we immediately wrote down an equation for Δt_0 . But nonetheless, this is one equation with one unknown, so of course it's trivial to solve it for Δt_0 . And when we do that, we get Δt_0 . I'll maybe divide by Δt_s . By just doing a little bit of algebra on that, you discover it's $1 - v/u$ inverse.

And then z , the redshift, is $\Delta t_0 / \Delta t_s - 1$ by definition.

AUDIENCE: Is that $1 - v/u$?

PROFESSOR: Oh, do I have it wrong here? Yes, it's $1 - v/u$. Thank you very much. Right, coming from the v/u here, obviously, as you noticed. OK, so to write down a final equation for z , it's $\Delta t_0 / \Delta t_s - 1$. So it's $1 - v/u$ inverse minus 1. And then just putting things over our common denominator, it ends up being v/u divided by $1 - v/u$. And that then is our final answer. So this now is the answer for the nonrelativistic case-- again, we haven't done relativity yet-- and the observer moving.

OK, now it's worth pointing out that when velocity is small compared to the wave speed, as is often the case if we're going to be talking about light which we'll be talking about in a minute, but could be the case with respect to sound as well. Then these two formulas are almost the same. They're both proportional to v/u , when I'm talking about the case where v/u is a small quantity, where the motion is slow compared to the sound speed or wave speed. And the only difference is that denominator.

Here we have a denominator of $1 - v/u$. And here the v/u is the whole show. There's no denominator. And if v/u is small, the denominator is close to 1. So the two answers are going to be almost the same. And one can describe that a little bit more succinctly perhaps, by saying that z with the observer moving minus z with the source moving is equal to v/u quantity squared times

$1 - \frac{v}{u}$ in the denominator. Just a little bit of algebra to get that result.

And what that shows explicitly is that second order in $\frac{v}{u}$. It's proportional to $\frac{v}{u}$ squared, not to $\frac{v}{u}$. So if $\frac{v}{u}$ were a part in 1,000, the difference would be a part in a million. So for slow velocities, it doesn't matter whether it's the source that's moving or the observer that's moving. But these answers can, of course, be very different if the velocity v is comparable to u . OK, any questions about that? Yes?

AUDIENCE: Does this violate Galilean relativity?

PROFESSOR: Does it violate Galilean relativity? Not really. Although, one has to maybe be careful about how one defines Galilean relativity. What makes it legitimate as a classical mechanical calculation is that unshown, but crucially important on the blackboard, is the air that the sound wave is moving through. So in one of these pictures, the air was at rest. The other picture, the air was moving. Or rather, well, I said that wrong. The air was at rest in both pictures. But if you want to make a Galilean transformation to relate one picture to the other, then after you've made the Galilean transformation, the air would be moving and it would not really be the same picture.

So it is all consistent with Galilean relativity. And one has to remember that the air is playing a crucial role here. When we say that the observer or the source is at rest, the full sentence really is that it's at rest relative to the medium that's transporting the wave. Any other questions? Yes?

AUDIENCE: This is not really a question, just kind of an observation. I just find it interesting that in the case if v is greater than u that this one is always positive, no problem. But if v is greater than u in the other case, then you have a negative number, which, I don't know if that-- I just find that weird.

PROFESSOR: Right, let me think. If v is greater than u , then the observer moving case becomes negative. And it presumably means the wave never reaches it. If the observer is moving faster than the wave speed, the wave never reaches them. And that's why

the answer is peculiar. If the source is moving faster than the wave speed, the wave still reaches the observer. So there is a nontrivial and believable answer in that case.

OK. What we're going to move on to now is the relativistic case. And as I think I described in the course handouts, relativity is has sort of an odd role in this class. We need a little bit of relativity. But at the same time, there are the courses in relativity. So I don't want this to be a course in relativity. In the old days, when there weren't so many other courses in relativity, we did in fact spend two weeks of this course doing special relativity. But I don't think that's worthwhile anymore.

Well, I'll maybe ask for a show of hands. How many of you have had relativity in some other course? That's what I thought, most of you, perhaps not all. Well, maybe I should ask the other question. How many of you have not had relativity? OK, a number.

So I do want to make the course completely intelligible to the people in that second group. Special relativity is not a prerequisite for this class. So what my goal will be to tell you enough special relativity so that you'll be able to follow what comes next. But I will not be deriving those results. I'll just leave for other courses for people who want to take them. And if you don't want to take them, that's fine too. But I want to make this course coherent.

So what we're going to do is discuss the consequences of special relativity without trying to relate those consequences directly to the underlying ideas of special relativity. I will, however, mention where special relativity comes from. It arose in the mind of Albert Einstein, because he realized that the physics that we knew well, basically Newtonian physics at that time, possess this property, Galilean relativity, which came up in a question just a minute ago. Galilean relativity says that if you look at any given physical process in a frame that's moving at a uniform velocity relative to the first frame that you use to describe it, it should also be consistent with the laws of physics in the second frame.

Incidentally, I-- maybe I'm more ignorant about history than most-- only learned a

few years ago what Galileo had to do with this. It did actually play a very important role in the history of Galileo in the physics that he was debating about. A key issue in the time of Galileo was whether the earth moved around the sun or the sun moved around the earth. And that was something Galileo was intensely involved in. And one of the arguments that said that it must be the sun that moves-- it can't be the earth that moves-- was that if it's the earth that moves, it means we move at very high velocity. The velocity of the earth around the sun is high by ordinary standards. And obviously, we'd feel that motion, people thought. So it was a proof that it must be that the earth is stationary and the sun moves. Because otherwise, we'd feel the effect of this high velocity motion.

So it was crucial to Galileo's point of view that it was really the earth that was moving that you don't detect motion. If you're in uniform motion, the laws of physics are exactly the same as they would be if you were at rest. And that's basically what Galilean relativity is all about. And it was in fact very clearly enunciated by Galileo in his writings.

So that was the case for mechanics. But at the same time, in the 1860s, Maxwell invented Maxwell's equations, or completed them is maybe a more accurate description for what Maxwell really did. Most of those equations already existed. And a prediction of Maxwell's equations is that light travels at some fixed speed, which could be calculated in terms of epsilon naught and mu naught that appeared in those equations, a speed that we call c . Now if light travels at speed c , it would mean that if you got into a spaceship and chased a light beam, it'd say half the speed of light. The implication of the physics that was known at the time would have been that, from the point of view of that spaceship traveling at half the speed of light, the light pulse would only be receding at half the speed of light. You would have half caught up with it. But that would mean that from the frame of this rapidly moving spaceship, the laws of physics must somehow be different. Maxwell's equations must not hold in their standard form.

So there was this tension between Maxwell and Newton, if you like. The tension was not a contradiction. It would be perfectly possible for there to be a fixed frame in

which Maxwell's equations had their simple form. But Newton's equations could perhaps have the same form in all frames. And that in fact was what people thought at the time. To account for this situation, physicists invented the idea of an ether which was a medium through which light waves travelled, similar to the air in which sound waves traveled. And the frame in which Maxwell's equations had their simple form was the rest frame of the ether. And if you moved relative ether, the equations would be different. And that's what people thought in 1904. And it was a consistent point of view, but it meant that there was this dichotomy between electromagnetism and mechanics.

So Einstein thought maybe physics is not so sloppy. Maybe there's a more elegant way which all this plays out. And he realized that if you modified the equations that are used to transform between one frame and another, you could make Maxwell's equations frame invariant. You could make it so that Maxwell's equations are valid in all frames. And if we go back to our example of the spaceship chasing the light beam, with these new transformation equations that Einstein suggested, it would turn out, even though it's very contrary to intuition, that when the spaceship measures the speed of the light pulse, it would still measure that the light pulse was moving away at speed c , even though it had moved at half c trying to catch up with the light pulse.

So it's not obvious how that can happen. But it turns out it can happen. We'll be talking a little bit more about how it happens. And that was basically Einstein's proposal. It was a proposal that there is no ether, that the laws of physics both electromagnetism and mechanics are the same in all frames. And in order to do that, he had to say that the equations of transformation between one frame and another are different from what Galileo believed.

So these are what we call the Lorentz transformations. We might write them down later in the course, but we're not going to write them down today. But what goes into them are three physical effects, which we will talk about. If we're talking about the time dilation, we only need one of those three effects. So I'm going to start by just discussing that for a minute, and then we'll come back at the end of the class, or

perhaps next class, depending on how timing works out, to discuss the other two primary effects that are needed to make up the theory of special relativity and explain how it could be that the speed of light could look the same to all observers, even if we're talking about observers that might be moving.

OK, so time dilation is the simple statement that if I were to watch a moving clock, the moving clock would appear to me to be running slower. I'll just mention for a moment now that I put the word appear in quotation marks. That means we're going to come back and discuss in detail what is meant by the word appear. But to finish the sentence first, the moving clock would appear to me in my reference frame to always be running slower. And so by a very predictable amount, a famous expression in special relativity, gamma, where gamma is 1 over the square root of 1 minus beta squared, where beta is just an abbreviation for v over c , the velocity of the clock divided by the speed of light.

So as long as v over c is small, this is a small effect. Gamma is near 1 . And running slowly by a factor of 1 means not running slowly at all. So the fact it was near 1 means it's a very small effect. But moving clocks will always appear to be running slower. That's one of these three effects of special relativity that we'll be discussing in the course of lecture notes one.

Now let me come back now to talk about this word "appear," because that's a little bit subtle. Might just add that there was a series-- this is just an aside-- but I guess broadcast last year there was a four-part series of Brian Greene's *Fabric of the Cosmos* that was broadcast on PBS. And the interesting thing about that, which is relevant here, is that he tried to illustrate time dilation. And he did it by having sort of a parable of a person sitting in a chair and somebody else carrying a clock over his head, walking towards the person sitting in the chair. And the camera showed what the person sitting in the chair would see and showed the clock running slowly. That's wrong. It's not what he would actually see. And that's the crucial issue of the word "appear" here.

When we say that the clock appears to run slowly, we're not talking about what an

observer would actually see. The complication of literally seeing is that when you see something, what you're doing is you're measuring the light pulses as they arrive at your eyes at a given time. And since light has a finite travel time, it means that you're seeing different things at different times. In particular, if there's a large object coming towards you, say this laser pointer coming towards me, I would be seeing the front of it at an earlier time than I would be seeing the back of it. The other way around, actually. I'd be seeing the back at an earlier time, because the light that leaves here at an earlier time would take longer to reach my eye, reach my eye the same time as light which leaves the front of the object later.

So the point is that as this is coming toward me, I'm seeing different pieces of it at different times in terms of the actual existence of this laser pointer. And that makes things complicated. So what you actually see when you take into account special relativity is fairly complicated. You can calculate it, but there's no simple expression for it. You really just have to calculate point by point what you'll be seeing for every part of the object at a given time, nothing very simple.

So the simple expression, which just says, clocks run slowly by a factor of gamma, and we'll learn later expressions about how things contract and how simultaneity changes, those simple expressions are not based on what any observer would actually see. But rather, they're based on what ends up giving a simpler picture, a picture in which you imagine that what we're discussing is not what an individual sees but rather what a frame of reference sees. So we're talking not about what the observer sees but rather what is seen in the observer's frame of reference. And a frame of reference, I think you could think of it pretty concretely, as kind of a structure of rulers connected together to each other to form a grid of rulers and with clocks located everywhere along this grid.

So all the observations are made locally. That is, if you want to measure a time in a given reference frame, you don't use a central clock, waiting for the light pulse to reach that central clock. Rather, you measure the reference frame is filled with clocks, all of which have been synchronized to start with. And if you want to know what time an event happens, you look at the clock that's next to it. And that clock

tells you what time that event happened.

So that's typically what we draw when we draw a coordinate systems and so on. It's really the way we normally think. The point is, though, if you really want to think what one observer would see, it's more complicated. Then you have to take into account the light travel time. So it's only after you take out the way travel time and calculate how local clocks would compare that you see this time dilation in the simple form, that the clock always runs slower.

So in particular, for this example of the person sitting in a chair and the clock coming towards them, that's exactly what we're going to be talking about. That is the Doppler shift. And what we'll find is when the clock comes towards them, he will see the blueshift not a redshift. He'll see the clock running faster not slower, the opposite of what was shown in the TV program. But the difference is that what causes it to look like it's going faster is the fact that each pulse travels a shorter distance if the clock is coming towards the observer. And that becomes a bigger effect than the fact the clock itself, if it were measured relative to clocks that it passes, would be running slowly. Yes?

AUDIENCE: So in that case, if the clock, let's say, was moving toward you really fast, could you measure it when it was directly perpendicular to you? And then that would be the special relativity time dilation?

PROFESSOR: Well, if it was moving alongside you. If it was coming right at you, it would just hit you. And that was the case, Sonya, more or less. But yes, you're exactly right. If the clock were moving at right angles to the observer so that the observer saw it, what you really want for this to be the pure effect is you want to have the velocity of the clock to be perpendicular to the velocity of the photon that the observer is seeing, as measured in the observer's reference frame. Actually that matters. Then you see would see pure time dilation effect. That's exactly right.

OK, so might just add that Brian Green and a bunch of people here at MIT, actually - I was involved and so were some others. There's a number of MIT people on this program. So we ended up having a long conversation with Brian Green about it by

email. And everybody at MIT thought it was just plain wrong. Brian Green actually took the position that it was very intentional on his part, and he was just trying to illustrate the time dilation effect, and he didn't want to talk about the Doppler shift. And since he didn't want to talk about it, he could ignore the fact that it was there. We all thought that was bad pedagogy. But we never convinced Brian, I should say.

OK, now what we want to do is go through these Doppler shift calculations again, this time recognizing that moving clocks run slowly by a factor of γ . So we're now going to be doing the relativistic case where the wave is sound waves-- excuse me, where the wave is light waves. I'll get this straight. And the velocities might be comparable to the speed of light. So this time dilation effect is large enough so that we want to take it into account.

Now in this case, what we are hoping, and everything would be wrong, inconsistent, if we can find this, we're hoping that in this case, the two answers should be the same. It shouldn't matter whether the source is moving or the observer is moving. Earlier it did matter, and we said that was explicable, because we knew that air was involved. And if we made a velocity transformation to go from one picture to the other, from the picture where the source is moving to the picture where the observer was moving, the air would have a different velocity in the two pictures. It would be stationary in one and moving in the other. So we would not expect to get the same answer as we would have gotten assuming the air was stationary.

But in this case, if anything has a different velocity when you go from the picture where the source is moving to the picture where the observer is moving, it would have to be the ether that has a different velocity. But the basic axiom of special relativity is that there is no ether, at least there's no physical effects coming from the ether. So you might as well pretend it does not exist. You can't really prove that it does not exist if it has no properties. It still exists. But the basic axiom of special relativity is that there are no physical effects coming from this ether. So for special relativity, we should get the same answer, whether it's the source that's moving or the observer that's moving. It's really the same situation, just viewed in different reference frames. And special relativity says it cannot matter what reference frame

we're doing the calculations in. So it's the same pictures, but this time we want to take into account the fact that moving clocks run slowly by a factor of gamma.

So looking at these pictures, we can first start just glancing at them and saying, where's there a clock that's moving? That might be something we need to think about. And maybe I'll ask you-- which of these four frames shows a moving clock?

AUDIENCE: All of them?

PROFESSOR: Sorry?

AUDIENCE: All of them?

PROFESSOR: All of them, I guess so. But the time interval measured on the clock is only actually relevant to one of them.

AUDIENCE: Two.

PROFESSOR: Two, exactly, two. Here we said that the source is moving, and we said the time measured on the source's clock between the emission of these two wave crests. Incidentally, we're usually talking about continuous waves like light waves. And then we're talking about the time between successive wave crests. Well, if we just as well imagine that the source is emitting a series of pulses where each pulse represents a wave crest, and somehow to me that sounds a little simpler to describe, because you don't have to think about the sine wave associated with the signal that the source is actually creating.

So in any case, the time between these pulses, I'll call them, as measured by the source clock, is what we call Δt_s , is the time that the source would actually measure. And the source is moving in this picture. So relative to our frame, we want to think of this entire series of pictures as all being consistent in our frame. It's very important, since transformations between frames are a little complicated in special relativity. It's very important when you're doing any problem to pick what frame you're going to use for your description and be sure to stick to it. If anything is initially described in another frame, you have to figure out what it looks like in your

frame in order to fit it together with the other events that you're describing in your own particular reference frame.

So for this problem, our frame will be the frame of the slide, the frame, which is at rest relative to the observer. So we could also call it the observer's frame. And relative to that frame, the source is moving. And therefore, the clock-- a source is emitting a series of pulses. That's a clock. Anything that does anything at regular intervals is a clock. So the source represents a moving clock. And we need to take into account the fact that the source will be running slowly by a factor of gamma.

And otherwise, nothing changes. The observer has a clock also, which the observer is going to use to measure the time between crests. But the observer's clock is at rest in our frame. So there's no time dilation associated with the observer's clock, only a time dilation associated with the clock on the source. So again, the important issue is all inside that yellow box. And what do I do now is look at the equations and see how the equations are modified.

I guess I should start back at the beginning of the blackboards. Maybe I should turn the blackboard lights on when I work on the blackboard. So the time integral as seen by the observer will be equal to-- last time we just had Δt_s as our first term, which would be what it would be if there was no velocity. And that's still true if there was no velocity. But if the clock is going to be running slowly by a factor of gamma, that would mean that the time interval that we would measure, even if there was no change in path length, which will be the next term, if there was no change in path length, the time that we would measure as the observer would be different from the time interval as measured by the source by a factor of gamma. But one has to figure out whether the gamma goes in the numerator or the denominator. And sometimes it's a little tricky. It helps a lot, I think, to just sort of imagine an example. Any example is clear, it turns out. But when you try to write down the answer in general, you sometimes get it wrong. That's my experience with myself and with other people.

So this clock is wanting slower. And say we're talking about a second time interval. If

this clock is running slower, it means it takes longer for it to take off a second. We would see it as maybe running slower by a factor of two. It would mean that it would only take off a second every two seconds. So that means that the time interval that we would see-- and that's what we're trying to calculate, the time interval as the observer-- is going to be longer than Δt_s by a factor of gamma. So the first term changes from what it was before. And what we're doing is we're rewriting this equation. So the first term changes from what it was before by putting in a factor of gamma in front of it. Because the source clock is running slowly.

And then the second term is still Δl divided by u . But the equation for Δl changes also, because Δl is the time interval that it takes for the light to travel the extra distance, where the extra distance was because of the time between the clicks. And that is now changed because of the time dilation of the source clock. So the second term also changes by a factor of gamma. So it's gamma times Δt_s plus v times gamma times Δt_s divided by u .

So the whole answer just changes by a factor of gamma. So it's gamma times $1 + v/c$ times Δt_s . And now if you do a little bit of arithmetic here-- gamma, remember, is $1/\sqrt{1 - v^2/c^2}$. And a squared minus b squared-- I'm now thinking of the denominator $1 - v^2/c^2$. Maybe I'll write this on the side. $1 - v^2/c^2$ -- it's worth remembering here-- can be written as $(1 + v/c)(1 - v/c)$.

And we have the square root of that appearing here in the denominator. Gamma is $1/\sqrt{1 - v^2/c^2}$. So this becomes the square root in the denominator, resulting the square root of $1 + v/c$ appearing in the answer. And this remains in the denominator. And what we get simplifies to simply $(1 + \beta)/\sqrt{1 - \beta^2}$ times Δt_s .

So this is the special relativity answer, the relativistic answer, and this is for the source moving. Now we expect that the answer won't depend on whether the source is moving or not, but certainly the calculations do. So this is what we got

from the calculation, which we assumed that it was a source that was moving, and the observer was stationary. It's just corrected from the previous answer by this factor of gamma. Any questions about that?

OK, so to summarize on the slide what's changed is that this time interval, this extra distance Δl for the relativistic problem, is not $v \Delta t_s$ as it was. But rather what we said is that it's $\gamma v \Delta t_s$, because the clock on the source is running slowly by a factor of gamma. And it was this difference that we just used on the calculation on the blackboard to get the new answer.

OK now the next calculation, now we're going to do the same calculation again, which we already did for the nonrelativistic case. But this time it will be the observer moving. And we're going to try to do the relativistic case. So this time it's the clock carried by the observer that's running slowly. And remember, this is running slowly relative to us, relative to our frame of reference, where our frame of reference is by definition the frame of reference of the slide that we're looking at.

Source is stationary, so Δt_s is just an honest time as we would measure it. But times as measured by the observer, Δt_o , are going to be different. So to correct the calculation for relativity, that's the crucial box as it was even in the nonrelativistic case. And we're going to have to change that equation by replacing Δl . Instead of $v \Delta t_o$ is $v \Delta t'$.

Now $\Delta t'$ is not exactly Δt_o or $\gamma \Delta t_o$. It's a little trickier. Let's see. Hold on. The definition I want to use for $\Delta t'$ is that it's the time as measured between the third frame and the fourth frame. But it's the time as we measure it. I still want to describe everything in terms of the way we would measure it. So $\Delta t'$ is not measured either by the source or the observer. It's measured by us. And it's related to the time as the observer would measure it by a factor of gamma, because relative to us, the observer's clock is running slowly by a factor of gamma.

AUDIENCE: So we're the source?

PROFESSOR: Well, we're stationary relative to the source, because the source is stationary. But we're the frame of reference of the slide, is the way I like to think about it. But it is the frame of reference, same as the frame of reference as the source. That's right.

So Δt_0 is related to Δt_{prime} by a factor of γ . And again, one has to think a little bit to make sure one gets the γ in the right place, the numerator or the denominator. We're saying that the observer clock appears to be running slowly relative to us. And that means that during the amount of time it would take for it to say take off one second, since it's running slowly, should take more than a second relative to us. And that's what this formula would give, if we multiply by γ . It would say that Δt_{prime} is equal to γ times Δt_0 . So we would say that the time that the observer clock ticks off one second we might measure two seconds. That's the direction of the time shift, all given by that formula.

And those are all the pictures. Now we just have to write down the equations that go with those pictures. And the hope is that we'll get the same answer as we got last time. I think I'll leave that on the board. Let's see. What do I do? No, I won't leave it on the board.

OK, this time the calculation we're mimicking is the calculation up here. This was the calculation we had for the nonrelativistic case where the observer was moving. Now we want to put in the time dilation that would correct that calculation. And the key equation is that the time interval between the receipt of the two wave crests as we would measure it, which is why I call it Δt_{prime} , so prime is not the source or the observer. It's us. It's the same frame of reference as the source, but the source isn't located there. So we think of it as the time interval as measured in the frame of reference. That's the frame of the slide, which is our frame. And it is equal to γ times Δt_0 , where Δt_0 is the time as it would actually be measured on the observer's clock. So, we're keeping their labels s and 0 to mean what would actually be measured on the clocks at the source and at the observer. That's what we're trying to establish a relationship between.

So the basic formula that we have up top there would first become Δt_0 is

equal to-- hold on. Now actually, we would not start by writing down an equation for Δt_0 . Rather what we want to do is start by writing down the relationship as we would see it in our frame, which is going to involve $\Delta t'$. We'll worry about Δt_0 later. So our equation is going to become $\Delta t'$ is equal to Δt_s plus v times $\Delta t'$ divided by the wave speed, which now we'll call c , since we're doing a relativistic problem.

So this is the basic equation for the time delay as we would see it in our frame, where $\Delta t'$ is the time between the receipt of the first and second crests, the time between frames three and four as we would measure it. And now we can do the same thing that we did over there, first for $\Delta t'$. And we discover that $\Delta t'$ can be written as $1 - v/c$ inverse, just doing algebra on this equation, times Δt_s . So that equation relates $\Delta t'$ to Δt_s .

And now we use the equation up top here to see what the observer himself would actually measure. And that becomes just $1/\gamma$ times $\Delta t'$. So $\Delta t_{\text{observed}}$ is equal to $1/\gamma$ times $1 - v/c$ inverse times Δt_{source} . And now since it's important we get the same answer, I'm going to write in some intermediate algebra here just so we can all really see that it works out.

The $1/\gamma$, I am going to write as-- remember γ is $1/\sqrt{1 - \beta^2}$ -- so $1/\gamma$ is the square root of $1 - \beta^2$. And I'm going to write that as the square root of $1 + \beta$ times $1 - \beta$. So this is the factor $1/\gamma$. Now we have explicitly here a factor of $1/(1 - \beta)$. The inverse makes it $1/(1 - \beta)$. Did I write something wrong?

AUDIENCE: From the second until the third time?

PROFESSOR: Second to third, you're there?

AUDIENCE: I thought it wasn't--

PROFESSOR: Yeah, this goes to the other side. It is minus. OK, finally, the $1 - \beta$, you see

occurs to the first power in the denominator. And the $1/2$ power to the numerator. So we do indeed get exactly what we wanted. Δt_0 is equal to the square root of $1 + \beta$ divided by $1 - \beta$ times Δt_s . And now we know that this is the relativistic answer for either source or observer moving.

And then if we want to write down what z is, this is $\Delta t_{\text{observer}}$ divided by $\Delta t_{\text{source}} - 1$. So that becomes just the square root of $1 + \beta$ over $1 - \beta$.

So we found what we expected, what we wanted to find to be consistent with the basic ideas of relativity. It's the same answer no matter which one is moving, because it doesn't matter what frame of reference we do the calculation in. OK, questions about either the calculations or the ideas behind them?

OK in that case, let's move on. I do want to come back and talk about the other two kinematic effects of special relativity, which are Lorentz contraction and a change in the notion of simultaneity. But before I get to that, there's one other issue I want to discuss first, mainly because it's something you need to understand to do the problem set that's due tomorrow. And that is the situation that's needed to describe clocks which might be accelerating.

Special relativity really only describes inertial reference frames and how things change from one inertial frame to another. So if you know how a clock behaves if it's at rest in one reference frame, special relativity completely dictates without any ambiguity what it would look like at a frame that was moving at a uniform velocity, relative to the original frame, which means it completely dictates how that clock is going to behave if it is moving at a constant uniform velocity.

But nonetheless, in the real world, we have very few clocks around that are completely inertial. Every clock that we see around us from the clock on the wall, which is moving with the earth or my wrist watch which moves more, is constantly undergoing accelerations. So we want to be able to talk about clocks which are accelerating. So we need to say a little bit about how we would do that and how we do it if the clocks were moving at relativistic speeds, which also happens with

satellites, for example. The GPS system as you've probably been told wouldn't actually work unless the calculations were done in such a sophisticated way that they even take into account the effects of general relativity as well special relativity. So moving clocks and how they behave is a crucially important topic technologically.

So what do we say that a clock that's accelerating? There's I think a common myth that to describe acceleration you need general relativity, and therefore we have to put off talking about accelerating clocks until we take a course in general relativity. That's actually totally false. What general relativity does is provide a theory of gravity which basically says the gravity and acceleration are intimately linked. And that's where acceleration gets pushed into general relativity.

But special relativity alone is enough for us to describe any system that could be described by equations that are consistent with special relativity. Special relativity does not describe gravity. So in any situation where gravity is important, special relativity is at a loss to make real predictions about what should happen. But as long as gravity is absent, as long as we're only dealing with electromagnetic forces that we think we understand, there's nothing that prevents us from using the equations of special relativity to describe what happens. We have to use the dynamical equations of special relativity that talk about how things respond to forces.

And whenever there's a force, there's an acceleration. But there really are such equations. We can combine, for example, electromagnetism with relativistic mechanics to describe a system of particles that are interacting electromagnetically, completely consistent with special relativity. And even those particles are accelerating, we could say everything we want to say about them.

So in particular, if there's a physical clock, to the extent that we could describe that clock as made out of particles whose physics we understand, special relativity will still tell us what that clock will do even when that clock accelerates. The answer, however, from that calculation, you might imagine, is very, very complicated. Because the physics of any actual clock, my wrist watch as an example, is pretty damn complicated. And we're not really going to write down the equations that

describe my wrist watch to figure out how it's going to behave when it accelerates.

So what are we going to do? Let me point out that you already have pretty much experience with accelerating clocks, because all of you-- well, many of you-- wear wrist watches like I do that are accelerating all the time. And they tend to work. You basically assume that even though they're accelerating they've been designed well enough so they can withstand the accelerations that your wrist gives them and still read the right time.

On the other hand, one could imagine contrary situations. Probably my wrist watch would survive this, but if you take a mechanical clock, a windup clock, and heave it against the wall and let it smash against the wall and come to a stop, as it smashes against the wall, it wanted to go a very large acceleration. And if the acceleration is large enough, we can predict the effect it will have the clock, even though it's a complicated interaction. If the acceleration is large enough, it'll simply break the clock and it will stop. And that's one possible effect that acceleration can have a clock.

And other effects are similar in nature. If there's any effect that the motion of my hand has on the wrist watch, it would be a mechanical effect that you'd calculate by understanding the mechanics of how the watch works, not by understanding any principles of general relativity. What's at stake underlying this-- you might wonder what the real difference is-- special relativity can make precise predictions about how a clock will behave if it moves at a uniform velocity, even without knowing anything about the details of that clock. Special relativity could make that prediction because there's a symmetry, Lorentz symmetry, which relates those two situations. And that's an exact symmetry of nature. So no matter what the clock is made out of, if it's moving at a uniform velocity, special relativity tells you there's no doubt it would run slowly by a factor of gamma.

On the other hand, there's no such principle of any kind, either in special relativity or general relativity, about acceleration. So if you want to know the effect of acceleration on a clock, it really depends in detail on how large the acceleration is of

course and the detailed physics of the clock and how much acceleration it takes to effect it and in what way it affects, precisely. So what's the bottom line? The bottom line is that when we want to talk about an accelerated clock, which we really do all the time, what we always do is simply assume that the clock is built well enough so that the acceleration does not affect its speed. And that really can be said very precisely. The assumption that we're going to be making is that these are ideal clocks, meaning that they're built well. And when we say the acceleration does not affect the speed of the clock, what we're saying is that the clock will run at precisely the same speed as another clock that's, say, moving instantaneously alongside it with the same velocity but with no acceleration.

So at any point in the motion of my arm here, my wrist watch will have some specific velocity. The velocity will affect in some tiny way the speed of the clock by this factor gamma, which will be very close to 1 for that case. But we're going to assume, if we call my watch an ideal clock, that in the even time, it will be running at exactly the same speed as a clock which is not accelerating, but which is moving with the same velocity as the wrist watch. Therefore, the factor of gamma will be there, but there'll be no effect of acceleration. The speed of the clock will be determined only by its velocity relative to our reference frame. OK, is that clear enough? And that's what you need to assume about some accelerating clocks that show up on the problem set. That's why I wanted to get it in today.

OK, if there are no other questions about that, I'd like to come back and talk a little bit more about special relativity and its consequences. Sometime later in the course, we'll talk a little bit more about what I would call the dynamical consequences of special relativity, which include well known equations like $E = mc^2$, for example. But before one talks about energy and momentum, which are quantities which I will dub dynamical, there kinematic effects of special relativity of which this time dilation is one. And by kinematic I really just mean the consequences of special relativity for the measurements of times and distances.

And if one limits oneself to discussing consequences for times and distances, kinematic consequences, there really are precisely three and no more

consequences of special relativity. And really all special relativity in some sense is embodied by these three statements that we're going to be talking about, the first of which was time dilation, which we've already seen. I'll just remind you. Time dilation says that any clock which is moving at speed v relative to a given reference frame will appear in quotation marks to an observer using that reference frame to run slower than normal by a factor denoted by the Greek letter gamma.

Turn out the board lights in case they're distracting. And again, "appear" refers not to how it would actually look to any particular observer, because any particular observer in a particular location is going to be waiting for light rays to reach that observer. And they'll take different amounts of time depending on where they start. "Appear" refers to measurements made in the reference frame of the observer, where we assume that all the actual measurements are made by on the spot clocks and rulers who measure where the object actually were at the time these events happened and not what it looks like at sometime later when light rays reach some observer.

OK, the second consequence, and again all these will involve the word "appear." And I'll always write it in quotation marks to remind you that it's not exactly what a person would see. The second one is another famous effect of special relativity, Lorentz contraction, or sometimes called Lorentz-Fitzgerald contraction. Any rod which is moving at a speed v along its length relative to a given reference frame will appear-- and again appear to an observer using that reference frame-- to be shorter than its normal length by the same factor, gamma. A rod which is moving perpendicular to its length does not undo that change in apparent length. So these pictures kind of show it all. A rod, that bar is the rod, a rod which is moving at speed v will look contracted, will appear to be contracted, by a factor of gamma. And a rod which is moving perpendicular to its length, a rod which is like this moving this way, has no such effect, we'll appear to have its natural length.

And this is a very famous consequence of special relativity. It means that moving rocket ships shorter and shorter and faster they go, and so on. And again, you should remember, it's not what you'd actually see. It's what you'd measure if you

had on the spot local observers making these measurements, which you then later compile. OK, that's actually all I want to say about the contraction. Any questions?

OK, next and last is an effect we just talked about less because it's a little bit more intricate to describe. But the other crucially important effect-- these would not be consistent if you didn't have all three. The other crucially important effect is the changing simultaneity, the relativity of simultaneity. And it takes more words describe it, so there's more words on the slide than on the other slides. And the pictures are a little bit more complicated too. But the pictures do say it all, really.

The point is that if you have a system consisting of two clocks which have been synchronized in their own rest frame-- they're both at rest with respect to each other and they've been synchronized in their own rest frame-- and they're connected by say a rod, which has some length which we'll call l_0 in the rest frame of the two clocks. If that whole system moves relative to us by speed v along the length, those clocks, even though they were synchronized in the rest frame, would not look synchronized to us but rather would look like they're out of synchronization.

And in particular, it will look like the trailing clock, the one on the back of this combination of clocks, will look a little bit like it reads a little bit later in the day by a factor which is β times l_0 times c . β remember is v over c . l_0 is the distance between the clocks as measured in the rest frame of the clocks. And c is of course the speed of light. On the other hand, if these two clocks were moving in a direction which is perpendicular to the line that joins them, then there's no change in the synchronization.

Now I should mention that this really is crucially important to the consistency of the whole picture. And actually showing that the picture is consistent is more than we're going to do. It's not impossible to do at this level by any means. But it's more than we're going to do in this class, since we're not focusing on special relativity. But when you hear about special relativity and look at these postulates, you might realize that there seems to be a pretty obvious tension between the idea that moving clocks run slowly and that all observers experience the same laws of

physics. Because it means that if you and I are moving with respect to each other, I would claim that your clock was running slowly. But at the same time, you would claim that my clock was running slowly. Because from your point of view, you're at some fixed velocity and therefore an inertial frame. And I'm moving relative to you. So as you would describe it, I would be a moving clock. And my clock would be running slowly.

So I think your clock is running slowly. You think my clock is running slowly. That seems to contradict. What happens if we hold the clocks next to each other and really just watch how they compare? Which one gets ahead? How can we disagree on that? Well of course, we can't hold the clock next to each other and also have them moving relative to each other. That's part of how you get out of this conflict. But let's think in a little bit more detail about what we're really saying when I say that your clock is running slowly. Remember, I want to make all my observations not by watching you, because then there's this time delay effect which complicates things. I make all of my observations by having a family of local observers surrounding me all at rest relative to me. And they report back to me. And only when I receive those reports and piece them together do I get the simple picture of what happened where when, which is the simple picture that is described by these "appear" relationships.

So when I say your clock is running slowly, what I mean is that if I have a network of clocks all at rest relative to me, and your clock comes shooting by, I would measure what time it reads as it goes past all my local clocks. Rather, they would measure what time your clock reads as it goes past each local clock. And then they would all report back to me. So if your clock is running slowly, for example, let's say by a factor of two, it would mean that when your clock passes my clock and my clock reads one second, your clock would only read half a second, because it's running slowly. It hasn't ticked off as much time. When it passes some later clock of my sequence of clocks, where my clock reads two seconds, your clock will read one second, and so on. So in that sense, I would say your clock was running slowly.

Now that has to be consistent with you thinking that my clocks are all running slowly

as well. So if you agree that my clocks were all synchronized, then you would conclude that my clocks must be running fast. Because when your clock reads a half second, my clock reads one second. When your clock reads one second, my clock reads two seconds. You would say that my clocks were reading fast if we just made that direct comparison.

But at the same time, we know that's not the right answer. You should see the same physics and I see. If you and I are moving with respect to each other, you should see my clocks running slowly. So the way out of that is this question of simultaneity. From the point of view of your clocks going past all my clocks, if you just looked at the time on my clocks as you passed them, you would actually think that they were running fast relative to your clock. But you would also, however, not think of those clocks were synchronized with each other. So you don't determine what speed they're running by looking at two different clocks.

If you want to figure out whether my clocks running fast or slow, you want to look at one of my clocks and see how it changes with time, not comparing different clocks. Because the different clocks would just be out of synchronism with respect to each other as you would see it. And we're not going to go through the details. But if you do look at my clocks consistently using a family of your clocks that are stationary relative to you, just as I thought about a network of clocks when I was trying to measure the speed of your clock, then everything's consistent. You would see all my clocks running slowly. I would see all of your clocks running slowly. And because we disagree on what's simultaneous, there are no contradictions. So simultaneity is absolutely crucial to get out of what would otherwise be a glaring contradiction in the whole system.

OK, that's about all I really wanted to say today. But let me just give a preview of things that we'll talk about later in the course concerning relativity. So far I think we've said all we are going to say, unless the questions. I think we've said all we're going to say about the kinematic consequences of special relativity. And we're not going to be trying to derive them, as I said. If you're interested, by the way, the notes recommend several references, including the lecture notes from eight to 86

from earlier years when special relativity was included as a real topic. So certainly if you're interested in learning about this and you haven't already seen it, I'm happy to help you. But otherwise, it will not be part of this course to discuss how these three consequences of special relativity arise from the basic postulates of special relativity.

But later we will be saying things that follow further along the line by pursuing the consequences of special relativity for momentum and energy, which will be important to us. The connection is the important connection and simple connection that energy and momentum are only interesting to us if they're defined in a way which makes them conserved quantities. That's why energy and momentum are important in physics. Because for a closed system, the total energy and the total momentum do not change. Energy and momentum can be transferred from one part of a system to another. But energy and momentum cannot be either made nor destroyed.

Now if we took Newton's definitions of energy and momentum, and used relativistic kinematics, what we would find is that if we looked at, say, a collision of two particles, the Newtonian definitions of energy and momentum, if we took those seriously, would tell us what might happen in a collision. Usually there's an angle that's undetermined. But given an angle, it determines everything else. If we used special relativity to then describe what that same collision would look like in a different frame, we would find that these Newtonian definitions of energy and momentum would not be conserved in the other frame, if they were conserved in the first frame. The conservation laws are dependent on what reference frame you're using.

So what Einstein amended was slightly modified definitions of energy and momentum, which are determined by the criterion that these slightly modified definitions of energy and momentum should, if they are conserved in one frame be also conserved in any other frame, which are related by the relationships to special relativity. So that's why it was essential once one changed the kinematics of going from one frame to another to also change the definitions of energy and momentum

so that the conservation laws would hold in all frames using the new transformation equations to get from frame to frame. And that's why later in the course we will be introducing a slightly modified, slightly non-Newtonian, definitions of energy and momentum of moving particles. OK, that's all for today. I will see folks next Tuesday.