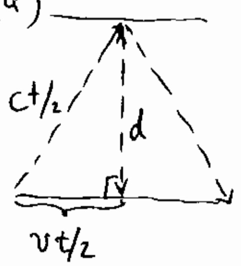
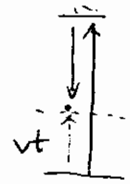
8.311 Electromagnetic Theory Problem Set #8 Solutions

1. (a)



$$\left(\frac{ct}{2}\right)^2 = d^2 + \left(\frac{vt}{2}\right)^2 \Rightarrow \boxed{t = \frac{2d}{c}\frac{1}{\sqrt{1-v^2/c^2}} = \frac{2d}{c}\gamma}$$

(b)



$$t = \frac{d'}{c-v} + \frac{d'}{c+v} \qquad d' \to \text{contracted length } d' = \frac{d}{\gamma}$$

$$\boxed{= \frac{2d'}{c}\gamma^2 = \frac{2d}{c}\gamma}$$

2. Use the identity $(1-\beta_1^2)(1-\beta_2^2) = (1+\beta_1\beta_2)^2\left[1 - \left(\frac{\beta_1+\beta_2}{1+\beta_1\beta_2}\right)^2\right]$

3. In instantaneous rest frame, the rocket's velocity changes from 0 to $g\,d\tau$ ($d\tau$ – proper time)

In the earth's reference frame, this velocity change is from v to $\frac{v+g\,d\tau}{1+\frac{v g\,d\tau}{c^2}} \simeq v + g\,d\tau\left(1-\frac{v^2}{c^2}\right)$

during time interval $dt = \frac{d\tau}{\sqrt{1-v^2/c^2}}$

Therefore in the earth's reference frame: $\frac{dv}{dt} = g\left(1-\frac{v^2}{c^2}\right)^{3/2}, \quad v\big|_{t=0} = 0$

This produces $v = \frac{gt/c}{\sqrt{\left(\frac{gt}{c}\right)^2+1}}\cdot c \Rightarrow \boxed{t = \frac{c}{g}\sinh\frac{g\tau}{c}}$

$$x = \int_0^t v\,dt = \boxed{\frac{c^2}{g}\left(\sqrt{1+\frac{g^2t^2}{c^2}}-1\right)}$$ these are results for the first 5-yrs in the rocket's frame

By the time the rocket returns, the earth's year will be year 2355, and the rocket will travel to a distance as far as 173 light years away from the earth.

4. See the discussion on light cone in Jackson/other texts.

(a) if $T < d/c$, handicap can be eliminated by switching to a reference frame moving long y direction at a speed $v = c^2T/d$. If $T \geq d/c$ this is impossible.

(b) the y-coordinate of the runner in the new frame will be Lorentz-contracted by $\sqrt{1-\frac{v^2}{c^2}}$ while the starting time will be the same.

5. The equation of motion is

$$m \frac{d^2\vec{r}}{dt^2} = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

This should be derivable from both Lagrangian and Hamiltonian approaches

6. $$\frac{d}{dt}\left(\frac{m\dot{\vec{r}}}{\sqrt{1-\frac{\dot{\vec{r}}^2}{c^2}}}\right) = e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

7. b). from $\frac{d\vec{p}}{dt} = e\vec{E} \Rightarrow \vec{p} = e\vec{E}t = \frac{m_0\dot{\vec{r}}}{\sqrt{1-\frac{\dot{\vec{r}}^2}{c^2}}}$

$$\Rightarrow \begin{cases} \vec{r} = \dfrac{c\vec{E}}{eE^2}\left(\sqrt{m_0^2c^2 + e^2E^2t^2} - m_0c\right) \\ t = \dfrac{m_0c}{eE} \sinh\left(\dfrac{eE}{m_0c}\tau\right) \end{cases}$$