8.321 Quantum Theory-I Fall 2017

Prob Set 11

1. **Sakurai 4.11**

2. **Adapted from Sakurai 5.1**

   A simple harmonic oscillator (in one dimension) is subjected to a perturbation

   $$H_1 = bx \tag{2}$$

   where b is a real constant.

   (a) Calculate the energy shift of the ground state to *lowest non-vanishing order*.

   (b) Solve this problem *exactly* and compare with the result obtained in (a).

   (c) Next set $b = 0$ but consider anharmonic perturbations (i) $H_1 = \lambda x^3$, (ii) $H_1 = \lambda x^4$. In each of these two cases find the energy level shifts to lowest non-vanishing order.

   Hint: Write $x = \sqrt{\frac{\hbar}{2m\omega}}\left(a + a^\dagger\right)$, and work with the matrix elements of the ladder operators.

3. **Sakurai 5.2**

4. **Degenerate perturbation theory**

   Consider two spin-1/2 particles described by a Hamiltonian

   $$H_0 = -J\vec{S}_1 \cdot \vec{S}_2 \tag{3}$$

   with $J > 0$. Find the spectrum of H_0 and analyze how it will change in the presence of a small perturbation

   $$H_1 = -BS_1^z \tag{4}$$

with $|B| \ll J$ (the magnetic field acts on one of the two spins). Analyse the problem using perturbation theory in H_1 at leading order in small B.

5. **Sakurai 5.9**

6. **Sakurai 5.20**

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