Physics 8.322, Spring 2003
Homework #4

Due **Monday, March 10** by 4:00 PM in the 8.322 homework box in 4-339B.

1. This problem not included for copyright reasons.

2. Consider a spin s particle in a time-dependent magnetic field

$$\mathbf{B}(t) = B(\sin\theta\cos\phi(t), \sin\theta\sin\phi(t), \cos\theta).$$

Consider an initial state $|\psi, 0\rangle = |m, 0\rangle$ in an eigenstate of the Hamiltonian

$$H(t) = 2\mathbf{B}(t) \cdot \mathbf{J}/\hbar.$$

From the adiabatic theorem,
$$|\psi, t\rangle = e^{i\alpha_m(t)}|m, t\rangle.$$

Compute $\alpha_m(t)$.

3. Compute the geometrical phase γ_m generated by a complete rotation of the B field in the previous problem, using Berry's formulae (S.16, S.14) on page 467 of Sakurai. Compare to the answer from problem 3.

4. Prove that $\nabla \cdot \mathbf{V}_m(\mathbf{R}) = 0$, where $\mathbf{V}_m(\mathbf{R})$ is given by (S.14) in Sakurai.