1. **Line Shape Due to a Fluctuating Field** (6 Points)

   This problem provides an exercise in evaluating a correlation function and bears on related non-resonance phenomena such as relaxation in a fluctuating field.

   A two-level system, with states $|a\rangle$ and $|b\rangle$ that have energy separation $\hbar\omega_0$, is subjected to an oscillating perturbation, for instance the field of laser light, with a matrix element of the form

   $$\langle b|V(t)|a\rangle = \frac{x}{2}e^{-i\omega t}$$

   The two levels decay with a decay rate $\gamma/2$. Furthermore, the field amplitude x is not constant but flips between two values, $+A$ and $-A$. The flipping is random, occurring with a mean rate Γ. In another word, if at time t we have $x = A$, then on average, x will flip to the value $-A$ at time $t + 1/\Gamma$.

   The problem is to find the resonance line shape. This requires finding the correlation function $G_{ba}(\tau)$. As a guide, let's assume, without loss of generality, that $x = A$ at $t = 0$. Let us also consider the probabilities $p_+(\tau)$ and $p_-(\tau)$ which represent the probability that x will be $+A$ or $-A$ at $t = \tau$ respectively, given the initial condition $x(t = 0) = +A$. The correlation function can be expressed in terms of $p_+(\tau)$ and $p_-(\tau)$. From the coupled rate equations for $p_+(\tau)$ and $p_-(\tau)$, $G_{ab}(\tau)$ can be found, and from this the line shape.

   (a) Write the correlation function $G_{ba}(\tau)$ in terms of $p_+(\tau)$ and $p_-(\tau)$. Note that you will need to take the ensemble average since x flips randomly.

   (b) Derive a differential equation for $p_+(\tau)$ and $p_-(\tau)$. Just think about the definition of $p_+(\tau)$ and $p_-(\tau)$ and their relation to Γ. Solve for $p_+(\tau) - p_-(\tau)$.

   (c) Find the correlation function $G_{ba}(\tau)$ and the lineshape W_{ba}. Note: if you are unable to obtain the explicit form of $p_+(\tau) - p_-(\tau)$ from (b), assume the form $p_+(\tau) - p_-(\tau) = e^{-\beta\tau}$, where β is some constant.

2. **Bragg Scattering** (14 Points)
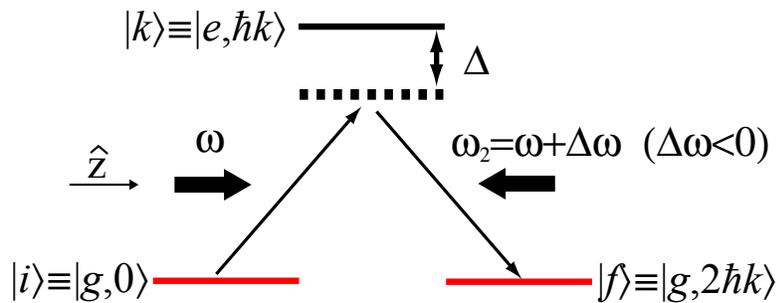
   Consider the above energy level diagram of an atom where the states are product states of an internal state and an external (momentum) state which can be written $|\text{internal}, \text{external}\rangle \equiv |\text{internal}\rangle \otimes |\text{external}\rangle$. In this problem the internal state is either $|g\rangle$ or $|e\rangle$, and the momentum state is $|\mathbf{p}\rangle$, where $\mathbf{p}$ is as indicated in the figure for each state. If two counterpropagating lasers are tuned as indicated, recoil momentum will be transferred to the atoms by redistributing photons between the beams. We want to look at this "Bragg scattering" in two ways:

   - by describing it as a two-photon stimulated Raman process, and
   - by considering the mechanical effect of the AC Stark shift potential seen by an atom.

Such an arrangement is a grating for matter waves and has been used in atom interferometers (See, for example, D. M. Giltner, R. W. McGowan, S. A. Lee, Phys. Rev. Lett. **75**, 2638 (1995)). The first observation of Bragg scattering of atoms by light was accomplished by a group at MIT: P. J. Martin, B. G. Oldaker, A. H. Miklich, D. E. Pritchard, Phys. Rev. Lett. **60**, 515 (1988). A comprehensive review of atom interferometry is given in the recent article by A. D. Cronin et al., Rev. Mod. Phys. **81**, 1051 (2009).

(a) Two-Photon Stimulated Raman Process

    i. In the above figure, what should $\Delta\omega$ be in order to realize resonance condition for the Raman process?

    ii. Assume the beams are counterpropagating along the z axis, have the same polarization, and can be expressed

$$\begin{aligned} E_1 &= E_0 \cos\left(kz - \omega_1 t\right) \\ E_2 &= E_0 \cos\left(-kz - (\omega_1 + \Delta\omega)t\right). \end{aligned}$$

Write down the interaction Hamiltonian. Don't forget to include the spatial dependence of the electric field. Although the dipole approximation essentially allows you to neglect the spatial dependence of the electric field in evaluating the matrix elements for the internal states, it is necessary for properly evaluating the matrix elements for the external states.

    iii. Write down the wavefunction of a particle with definite momentum $p = \hbar k$ in position space. Assume periodic boundary conditions in a 1D box of length L.

    iv. Calculate the two-photon Rabi frequency for the Raman process shown above in the figure. Assume that $|i\rangle$, $|k\rangle$, and $|f\rangle$ have external wavefunctions of the form you wrote down above with appropriate momenta. The dipole matrix element of the internal states is D_{eg}. Also, note that only the excited state $|k\rangle \equiv |e, \hbar k\rangle$ has nonvanishing matrix elements with the initial and final states. It is possible to consider this an effective two-level system involving only $|i\rangle$, $|f\rangle$, and the coupling between them.

    v. If H' is the perturbation due to E_1 and E_2, what is $\langle i|H'|f\rangle$ in terms of the given parameters? Hint: use the analogy between one-photon and two-photon transitions and the meaning of the corresponding Rabi frequencies.

(b) AC Stark Shift

    i. Calculate the AC Stark shift $U(z,t)$ of an atom in the ground state $|g\rangle$ due to the total electric field $E_1 + E_2$. For now, ignore the external state of the atom, but keep the spatial dependence of the electric field (while making the dipole approximation, of course). Assume the weak-field limit holds, and average over the oscillation at optical frequencies.

    ii. What is the coupling $\langle i|U(z,t)|f\rangle$ due to the mechanical potential presented by the AC Stark shift? Note that now we take into account the external states of the states $|i\rangle$ and $|f\rangle$, where these have the form as you wrote down in (a) iii. Compare this with the perturbation matrix element obtained in (a) v.

This problem illustrates that forces due to the AC Stark effect, *i.e.*, the stimulated light forces, correspond in the photon picture to a stimulated Raman process which redistributes photons between laser beams.

8.421 Atomic and Optical Physics I

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