PROFESSOR: So the subject here is about if you want to understand interactions between neutral objects. So let's start in a very basic way by saying we have an atome A, we have an atom B, they are separated by a distance R, and we want to know what is the force between them.

In a semiclassical picture, we would say that everything is-- the force which is responsible for that must be the Coulomb force. So the atom A and B consists of charges, so we should find the charge density in our system A, the charge density of B, we take the coulomb energy and we integrate over the volume. And this is how in e and m, in the most general way, you would to write down the electrostatic energy of a system. You take all charges, each charge has a coulomb potential with the other charge [INAUDIBLE] indicate over everything.

But for objects which are localized and we can do multiple expansion, our objects are neutral, neutral atom. So therefore there is no coulomb term, and the next term is the dipole. So therefore, starting with classical physics, we would find that the interaction of those charge density distributions one for atom A, one for atom B, should actually interact with this Hamiltonian h prime.

OK, but now, if you look at it classically, we find that classically, the expectation value for the dipole operator of each atom is zero. But you will also find-- if you said, well, OK, that's what I expect, because an atom is in isotopic space. How can it have a dipole moment?

But maybe, what could happen is, if there is a fluctuation of a dipole moment here and a fluctuation there and the fluctuations are correlated, then the two systems may repel or attract each other. So therefore, when we are talking about

fluctuations, we are talking about expectation values for d squared.

But also in classical physics, this is exactly zero. So what classical physics tells us, it says, in classical physics, it costs more energy to create a dipole moment, or even a fluctuating dipole moment, than one gets back from the dipole interaction. So the system will never spontaneously form some form of dipole moment. And in classical physics, two neutral atoms will have no attraction at all. So it costs more energy.

So now, we have to do quantum mechanics. And every so often, I have to go through a classical argument to make you wonder about the quantum mechanical result. Because the quantum mechanical result, you're used to it. You've heard about the van der Waals interaction. But now you know that there is really something quantum at work.

In quantum mechanics, the expectation value of the square of the dipole operator is not zero. I mean, you can see that if you have an electron, which is in an harmonic oscillator, and if it is at the bottom of the oscillator-- well, the dipole moment-- what I want you to say is the dipole moment for a charged system is related to-- between electron and proton-- to the displacement operator, R.

And even if you have an equilibrium position-- like in an harmonic oscillator, where R is equal to zero, R squared-- the expectation value-- is not zero, because whatever your stable equilibrium is, you will have zero point fluctuations around it. And so therefore, now in quantum physics, it's not that we have to provide energy to create a d squared. We have it. We have it by necessity, because of non-commuting operators, because of the nature of the quantum physics.

So now, the fact that we have fluctuating dipole moments already, that means if now with the fluctuating dipole moments between the two atoms are synchronized, this will then lead to a non-vanishing average force between the atoms. So that's the idea.

But the good thing is, you don't even have to know about it-- about fluctuations and all that. You just take-- and this is your homework-- you just take the operator H

prime. And look what happens when you have an atom in the ground and excited state and what happens when you have both atoms in the ground state.

In the first case, you get the leading result in first order perturbation theory. For two ground state atoms, you have to go to second order. In first order, the potential is 1 over R cubed. And you will calculate the C3 coefficient.

In second order, you get a van der Waals potential, which is 1 over R to the sixth. And it's fairly straightforward to calculate the C6 coefficient. Any questions?

So let me reemphasize, the beauty of this perturbation result that it's so simple. But the downside is, you don't really understand what you're doing. I mean, you are solving the Schrodinger equation. You are finding an expectation value. But the nature of the effect-- how it is related to fluctuation, what is really behind it-- is sometimes obscured.

So let me give you one interpretation, which I really like. And I learned that actually from Dan Kleppner. When he taught the atomic physics course at MIT 20 years ago, that was one element of it.

And I told you at the beginning that I'm actually proud that our atomic physics course is really kind of-- has hopefully kept-- the best of the decade long tradition of teachers like Norman Ramsey, Dan Kleppner, and Dave Pritchard. So this is now really due to Dan Kleppner. And I will later today actually post Dan Kleppner's original lecture notes about this effect.

So we want to understand the van der Waals force in a completely different system. And I think that helps. We have two LC circuits. There's L, C, C, L.

And what happens is, the two capacitors are closed. So there is this stray field of one capacitor reaching the other capacitor. So I won't go through the detailed math here, because it's not necessary.

But what we have here is, we have two coupled systems-- two coupled capacitors. And the only thing I want to use is that the stray field of a capacitor has a diopolic

characteristic. And it decays with 1 over R cubed.

So what we have right now is, each LC circuit, when it's isolated, has a resonant frequency of omega naught. But now, when we have two pendulums-- two oscillators-- which are coupled, we get two modes, which are omega plus and minus.

And if we simply solve for two modes, and they have some coupling, well, we get an upshift and downshift by the coupling. But if you look more carefully at the solution for two coupled modes, it has also a quadratic effect, which is sometimes neglected, but it's there. So this is, in general, what you will find, if you couple two modes.

Now in quantum mechanics, when we ask for the oscillator in the ground state, we can find the zero point energy. And so the energy in the ground state is 1/2 the zero point energy in the mode plus, in the mode minus. The first order coupling term, the 1 over R cubed cancels out. But what remains is a contribution, which is 1 over R to the sixth.

So we need quantum physics. Because in classical physics, the ground state of the LC circuit is nothing happens. No charge, no current, no nothing. I mean, this is a classical ground state, and you would not get any effect. So realize that this van der Waals potential requires quantum mechanics, and it is due to the zero point energy of the atomic oscillators.

Just hold the thought, when we talk about atoms in the electromagnetic field, we have two oscillators. We have an harmonic oscillator, which is a fluctuating dipole moment on each side of the atom. But we have many, many harmonic oscillators in between, which is the electromagnetic field.

And during the course of this lecture, I will refer to both the zero point energy of the atomic oscillator and the zero point energy of the oscillator, which is the electromagnetic field. And to give you the result at the end, I will sort of explain different effects by the zero point oscillation of the atoms, other effects by the zero point oscillation of the electromagnetic field.

But at the end of this section, which is probably after spring break on Monday, I will actually tell you that you have to be careful. If you have quantum physics, you have to use a consistent description. And you always need that both the field and the atoms are quantized and give zero point fluctuations.

And when I'm telling you now that the van der Waals potential is due only to zero point fluctuations of the atomic oscillator, there may be another way of getting the same result by looking at the zero point fluctuations of the electromagnetic field. So I just want you to take what I'm saying to you-- I'm not saying to you anything wrong. But it may not be the only way of expressing it. And I want you just to pay attention to it.

But certain effects can be simply described by just simply looking at the atomic oscillators. Why do we need the quantized electromagnetic field when all we want to do is have a stray field coupling two capacitors. So we want to always learn about the simplest way to understanding an effect. But then in the end, there are maybe certain subtleties.

OK. But let's keep it simple. So it's a zero point energy of the two oscillators. And maybe let me emphasize this by translating this description-- what we learned from the capacitors. Let me give you the Coulomb description of the atom.

So an atom really never have in free space a spontaneous dipole moment. But because of zero point oscillation, it will have a dipole moment squared. So what happens now is, let's say at some moment, we have a fluctuation. One atom happens to have a fluctuation of the dipole moment. This creates an electric field at the position of the atom, b.

So this is a fluctuating field. The fluctuating field, because there's a finite polarizability, induces now a dipole moment of the atom, db. I will later introduced polarizabilities. But here, it's just proportional to Eb. So that's proportional to da over R cubed.

So now, we have no longer randomly fluctuating dipole moments with da db, which

are out of phase. Positive and minus cancels out. We have a dipole moment, da, which creates-- through polarizability-- an aligned dipole moment, db.

And for those, we have now the dipole-dipole interaction, which is db da over R cubed. And when db changes sign, da changes signs. So the product of them will always stay the same. And now, we find that we have an interaction.

If you look, db was 1 over R cubed. And if you multiply that with the R cubed of the dipole-dipole interaction, we obtain, in a very different way, the van der Waals potential, which is 1 over R to the sixth. So therefore, what we can take away from that is that the 1 over R to the sixth potential is caused by the zero point fluctuations of the atomic dipole moments.

OK. So now you have already three different ways to look at the van der Waals potential. One is just doing perturbation theory and not understanding anything. The second one is the ground state of two coupled oscillators, a la two capacitors. And finally, the fact that spontaneous fluctuations create stimulated fluctuations, and then there is a quadratic term, which doesn't vanish.

OK. Well, let's now consider that the electromagnetic field-- that there is an electromagnetic field. Until now, we have not really used the fact that we have an electromagnetic field with photons. And the electromagnetic fields, each mode, is an harmonic oscillator. And each mode of the electromagnetic field has zero point fluctuation of itself.

So let us now discuss what can the vacuum fluctuations of the electromagnetic field do for us. Well, if we have fluctuations-- momentary fluctuations-- of the electric field of the vacuum, the fluctuation at position, a, and position, b, will induce now dipole moments of the atoms, a and b, by multiplying with the atomic polarizability.

And if you assume-- I'll say a little bit more about it-- that we're talking about the long wavelengths fluctuation of the electric field. Then the two dipole moments, which are induced by those vacuum fluctuations, are actually in phase. So the key word is now that the vacuum fluctuations create correlated dipole moments. So the

dipole moments are no longer independently fluctuating by themselves.

So if you write down now the dipole-dipole interaction, we will find a result which is proportional to the product of the two polarizabilities. The dipole interaction has an intrinsic spatial dependence, which is 1 over R cubed. But now, the important term is that we have to calculate the correlation function of the electric field at position, a, and position, b. And these are the vacuum fluctuations.

I'm not doing the calculation, because it would be somewhat messy. I really post on the web a fairly easy-to-read paper by Larry Spruch in Physics Today. And he gives you a little bit more details. But you have already the physics in this equation.

What you should use now is, you have to use the density of modes. And for each mode, the zero point energy, 1/2 h-bar omega. And then, what you get is, you get an interaction, which is 1 over R to the seventh. Yes.

AUDIENCE: When you're accounting here only the dipole moment, which are [INAUDIBLE]. So they're coming from the fluctuation of long wavelengths in your EM. And once they [INAUDIBLE] those but they are EM field fluctuation from vacuum. So what are the effect of those shorter waves? Are they just higher energies, or just--

PROFESSOR: Excellent question. Actually, I need that-- in 10 minutes-- I need exactly the answer to your question. So let me give you the answer right now.

If you look at this expression, there are two things which come into place. One is we have now two atoms separated by a distance, R, which is ra minus rb . If you take, now, high frequency modes, which are very short wavelengths, and you integrate and sum over all of them, it's pretty clear that you will get plus and minus, which will completely average out.

So it seems very clear that the modes with wavelengths, lambda, shorter-- definitely much shorter-- than R average out. So therefore, the bulk of the contribution will actually come from the modes with wavelengths smaller than R.

But now, there is something else. The density of states for the electromagnetic field

dramatically increases with frequency, omega. So we have many more modes at short wavelengths.

So therefore, the argument says, well, the long wavelength modes, up to wavelengths, R, are the dominant ones. They don't average out. But because the density of modes increases, the shorter the wavelength is, by far the dominant contribution, will come from the modes, which are centered around lambda equals R.

So the density of modes implies now that those modes will dominate. So that's the answer. But ultimately, you have to put everything into an equation, have an integral, solve the integral, do some approximation. And you will exactly see from the density of modes that the very long wavelengths don't contribute. And you will also see that because of rapid averaging of E to the ikr phase factors, the high frequency modes will not contribute.

OK. So we have now two different power laws. One is 1 over R to the sixth. And one is 1 over R to the seventh.

So let's explain that now. So we obtain the 1 over R sixth by using the uncertainty principle for atoms. And here, we have used the uncertainty principle, the zero point fluctuations for electromagnetic waves.

When we derive the 1 over R sixth potential, we actually used simply the electrostatic instantaneous Coulomb field dipole field. And this is only valid for short range when-- you know, I said we have a fluctuation and the fluctuation created another one, which was in phase. But if one fluctuation has to send out an electromagnetic wave, the second fluctuation may not be in. When propagation effects come into play, there's a time lag between the two oscillators and may actually be out of phase.

So we really assumed in the derivation, without ever saying it explicitly, that there are no propagation effects. And this is called the short range potential. Whereas, for the second argument that we said, we have two distant atoms, which get

synchronized by being driven by fluctuations of the electromagnetic wave at wavelengths, lambda equals R. This is what happens at long range.

So this is a famous result, that we have a van der Waals force, which is the instantaneous force. And when propagation effects come into play, this goes by the name Casimir-Polder potential. The moment when propagation effects come into play-- when radiation plays a role-- we get a different power law.

So again, I'm just playing with ideas. I'm taking a train of thought, atoms fluctuate; another train of thought, electromagnetic fields propagate; and just see, what are the ramifications? What are the consequences? So now, I want to eventually give you a treatment, which has both aspects in one. And this is our diagrammatic treatment.

First, before I do with that, do you have any idea what distinguishes the short range from the long range potential?

So you have two atoms. You pull them apart. And what distance do we have the 1 over R to the sixth physics? At what distance do you have the 1 over R to the seventh physics?

AUDIENCE: When [INAUDIBLE] transitions?

PROFESSOR: When the distance is the wavelengths of the atomic transition. That's the only thing which matters. You don't get it immediately from here. Because-- I mean, I used an instantaneous potential, which didn't have any scale.

And here, I said we have vacuum fluctuations of all wavelengths. So here, I had a picture, which clearly works at long separation. And propagation effects are built in, because it's a vacuum fluctuation, which sort of act at position, a and b. And the wavelengths-- their propagation-- is part of the formulation.

But yes, Bohr's conjecture that it is the wavelengths of the atom-- the wavelengths of the resonant radiation-- is correct. But I want to show you now how we can obtain this result in a, I think, really elegant and beautiful way by looking at the diagrams

we have just learned.

OK. What did we just have? Time to do that, yes. Any questions here?

The reason why I want to show it diagrammatically is actually two fold. One is, it's really beautiful how it comes out. But the second thing is, when I introduced the Feynman diagrams and all of these formulation in terms of propagators, I gave you a wonderful picture. But in the end, I reduced it to first and second order perturbation theory.

So I want to show you at least one nontrivial example where you would not get the result easily without this formalism. So enjoy one nontrivial application of diagrams.

So if you follow the systematic way, how we have set up our formulation of quantum electrodynamic with a photon field-- the quantized photon field-- then the two neutral atoms have no direct Coulomb interaction. I just want to point that out. Because a few minutes ago, when I said a classical system is just Coulomb field-- each charge element has a Coulomb potential with the other charge element-- this was just the opposite approach.

But in our QED formulation, we have sold our soul. We have said we only really got the near field, the longitudinal field, as belonging to the atoms. Everything which happens at longer range is included in our atom-photon Hamiltonian.

So therefore, there is no direct Coulomb interaction for the longitudinal field. Everything, everything now, has to come from the interaction-- the perturbation operator-- for the quantized radiation field. And therefore, we must get it out.

We have one atom; it's d1 at d prime. I think that's how it's-- I'm using now the nomenclature, which is used in atom-photon interaction, pages 121 to 126. And so, there is one atom at the unprimed location, one atom at the time primed location. And everything has to come out now of the formulation of the quantized radiation field.

And we have discussed that lengths at the transverse electric field become

operators, a plus a dagger-- the symbol for the dipole operators are acting on the atoms. We've done all that.

So then the only thing which happens is-- and this is described by a and a dagger-- is the exchange of photons. So the only way how those two neutral objects can interact is through the radiation field. And through the radiation field means they have to emit photons. So I want to show you now that the van der Waals interaction can be thought of as one atom emitting a photon. But this virtual photon is absorbed by the other atom and vice versa.

And you may have heard quite often-- but I'm not sure if you've seen it explicitly-- that a lot of interactions in physics are actually mediated by the exchange of virtual particles. The famous example is that the nuclear force comes from a virtual exchange of pions. But then, people say, the Coulomb force comes from a virtual exchange of photons. But usually, we don't really show you how this virtual exchange of photons work.

So in that sense, I'm proud that I can show you, at least the basic outline, how virtual exchange of photons between two neutral atoms leads to the van der Waals potential. So that's our agenda.

OK. Now the fact that this takes six pages in the book-- and the book is not solving all equations-- means if you really want to do it quantitatively, it has a certain complexity. But what I've done is, I've sort of distilled out the grand idea and which also shows you what happens when you go from short range to long range. And this is what I want to present you.

And therefore, I take [? Benji's ?] question and will not discuss all the modes of the electromagnetic field. I will immediately use the fact that the dominant contribution comes from modes. And since we don't have real photons, we don't have energy to create real photons when everything is in the current state. We call them virtual photons.

Virtual photons are photons which are immediate. But a short time later, because of

Heisenberg uncertainty, have to disappear. So the dominant contribution comes from virtual photons at the wavelengths, which is D, the distance between atoms.

So the frequency, which will play an important role, is simply the frequency of those virtual photons. And I gave you already the reason-- the density of states which favor higher frequencies. But higher frequencies cancel, because of the E to the ikr terms. So we only focus on those.

The second thing I need is that when we do perturbation theory, the energy shift in perturbation theory-- well, if you do second order, you go from the initial state to an intermediate state. And then, you go from the intermediate state, one, to the final state. And what you have to do is, you have to divide by the energy denominator.

If you do higher and higher order perturbation theory, in our diagrammatic presentation, that means we have propagators at intermediate energies. And I told you that the time an atom can spend in an intermediate state is 1 over the energy defect. And if you integrate the E to the i energy defect factor over time, you get something which is proportional to 1 over delta E.

So therefore, if you perform the time integration of the perturbation analysis, you find by integrating over the short time particles spend in the intermediate state, you actually generate those energy denominators. That's actually the relationship between the time propagation I showed you earlier. And when you integrate over the time, you get for this intermediate state, a contribution, which is 1 over the energy defect over the intermediate state.

So therefore, if you go now to higher order perturbation theory, we simply have a product of matrix elements in the numerator. And in the denominator, we have a product of energy defects. You are using that all the time for second order perturbation theory, but the structure is that you can just sort of daisy chain the expression. And you get higher order perturbation theory.

So therefore, all we will have to understand to figure out what happens when virtual photons are exchanged, I will just show you two relevant diagrams. And we just

look-- we stare at the diagrams-- and figure out what are these denominators? What are those energy defects? And we will find that there's a difference at long range and short range. So that's the agenda.

So we will need two energies. One energy is the energy of the virtual photon, which is exchanged. And the second energy, or frequency, is the resonant frequency of the atom.

So let's look at two relevant diagrams. So one is atom, a; one is atom, b. We are interested in interactions between atoms in the ground state. And after a sufficiently long time, both atoms have to be back in the ground state.

Let me just get this organized, one, two, three, four. And the result is obtained in fourth order perturbation theory. It has to be fourth order, because we need photon exchange. To emit a photon here, absorb it here, is second order.

But then, one atom is in the excited state and this can't be. So a second photon is required to undo the effect. So we need exchange of photon pairs.

Each exchange means absorption and emission. That means two vertices. And photon pairs means another two. So we need fourth order perturbation theory.

OK. But it's much, much easier than you think. So we have a ground state atom. And now the action starts. This is the first vertex. It emits a photon.

The photon is absorbed by the other atom, which puts the atom into the excited state. This atom is also in the excited state, of course. And then, we know the atom cannot stay in the excited state for a long time. It can now emit a second photon, which brings it back to the ground state. And the second photon is absorbed by atom, b, and puts the atom back into the ground state.

So this is one relevant diagram. And let me analyze it in the energy defects. Here, in the first intermediate step, the energy defect is-- well, we have ground-ground. This is sort our reference.

But now, we have put one atom into the excited state. That causes energy, omega

naught. And we have created one photon, omega. Here in that time step, there are no photons, but there are two excited atoms. And in the third time step, we have still one atom in the excited state and one photon.

Now, this is one relevant class. I mean, you can now use permutations at atom, b, excites photons first and such. But the structure of many of these diagrams will be the same that we have those three energy defects.

But now, we have another possibility. And this is, one atom emits a photon, but it feels it doesn't want to be excited, because the excitation energy is so precious. It wants to immediately return to the ground state. So we could have a situation that the atom fires a second photon before the first photon is absorbed.

So the diagram-- the relevant diagram-- is now this. So the atom started in the ground state, is in the excited state, but as soon as possible, goes back to the ground state. And the other atom starts out in ground state, is in the excited state, and is back in the ground state.

So what we have here is, we have again three intermediate steps. In the first step, of course, we have an excited atom and a photon. And we have the same here. But now the difference is, that in the intermediate step, our energy defect is not two atomic excitations, it is two photonic excitations.

And now we want to compare those two diagrams for short and for long range. And it becomes clear that short range is where the frequency of the photon-- its wavelength is lambda. Short range means high frequency. The frequency of the photon is larger than the atomic excitation energy. Or that means, the distance is smaller than the resonant wavelength of the photon. Whereas, long range is that the atomic excitation is more precious than the photonic excitation.

OK. So I need three more minutes. So it just fits into the class time.

So we are here in the limit that omega is larger than omega naught. So that means, if we take now the product of the three energy denominators, omega dominates

over omega naught. So we'll have this structure omega, omega naught, omega. Omega, omega naught, omega.

Whereas, at long range, omega naught dominates. We have omega naught, omega naught, omega naught. So the structure of the product of energy denominators is 1 over omega naught.

If you look on the right hand side, we have replaced-- in the intermediate part in the propagator-- the energy defect by omega naught by omega. So therefore, in this case, we have 1 over omega cubed and 1 over omega naught squared, omega.

So we know that when we sum up over diagrams, we have to sum up over all possibly diagrams. And the ones which dominate at short range are those ones and at long range are the other ones. So in other words, at short range, one photon is exchanged and disappears. And then, the next photon comes.

But at long range, we have an exchange that one atom sends out two photons almost simultaneously. And the other atom absorbs them. Because it sends them out almost simultaneously. Because here, we are talking about long wavelength photons, and photons are cheaper. Rather send out two photons before you spend too much time in the excited state.

OK. Let's wrap it up. Since omega is proportional to 1 over the wavelengths, which was-- and I said the photons we are concentrating, is the photons which have a wavelength of the distance, D .

So what we are realizing now is that, if you just compare those diagrams at long range, we go from 1 over omega squared to 1 over omega. So at long range-- omega is-- we have an additional factor of omega, which is 1 over D.

And so now, if you assume-- I can't show you that without solving the real problem-- but if you assume that at short range, we have 1 over R to the sixth-- van der Waals potential-- then at least, I've proven to you that at long range, you get one more power in the distance. And this is a transition from an instantaneous potential, where propagation effects do not matter, to what is called retarded potential.

So that's all I wanted to show you about interactions between two neutral atoms. Are there any questions? Timor?

AUDIENCE: Does it matter the direction of both the photons that we drew? For example, can atom, a, emit to b, b absorbs, and b emits to a, rather than--

PROFESSOR: In practice, it matters. When you solve it, you have to sum over all possibilities.

Remember, quantum mechanics in the system does everything. It doesn't care whether it's allowed or not. It tries everything out, whether it violates energy or not. And you have to sum over everything.

But if you then analyze all the possible diagrams, you will actually figure out that they can-- no matter what you do, what permutation of photons and atoms you have-- the fourth order diagrams will be such that they are always distinguished by in this intermediate zone, whether you have two atomic excitations or photonic excitations.

And so what I do for you were the two diagrams, which are representative for a whole class of diagrams here. But absolutely, yes, you are right.

OK. So that's all about neutral atoms. I know you have spring break next week-- MIT spring break. Maybe that's good news for the Harvard people, who-- when is Harvard's spring break?

AUDIENCE: We just finished.

PROFESSOR: OK. So you have-- at least for this class, you have a second spring break. So we are not yet finished with van der Waals and passing interactions, but we finished the neutral atoms. And on Monday after spring break, we talk about interactions between metal plate and atoms in two metal plates.