8.511 Theory of Solids I **Problem Set 1** **Due September 21, 2004**

1. Prove that the product of the volume of the first Brillouin zone and the volume of the unit cell of the Bravais lattice equals $(2\pi)^3$.

2. Show that rotations about any axis that takes a Bravais lattice into itself must be either 1, 2, 3, 4 or 6 fold.

3. The common building blocks for most high temperature (high T_c) superconductors are copper oxide layers, as depicted in Figure I.1.6. Assume the distance between copper atoms (filled circles) is a. For simplicity let us also assume that in the third dimension these CuO_2 layers are simply stacked with spacing c, and there are no other atoms in the crystal. In first approximation the layers have a four-fold symmetry; the crystal is tetragonal.

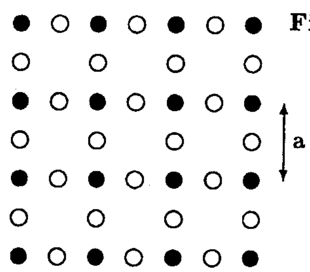

Fig. I.1.6. CuO_2 lattice

   (a) Sketch the Bravais lattice and indicate a possible set of primitive vectors for this crystal. What is the unit cell, and what is the basis?

   (b) In $LaCuO_4$ one discovers, at closer inspection, that the CuO_2 lattice is actually not flat, but that the oxygen atoms are moved a small amount out of the plane ("up" or "down") in an alternating fashion (in Figure I.1.7, a + means up and a - means down).[1] What is the primitive cell and lattice spacing for this crystal? What is the reciprocal lattice? Describe (qualitatively) what happens in the X-ray diffraction pattern as the distortion is decreased gradually to zero.

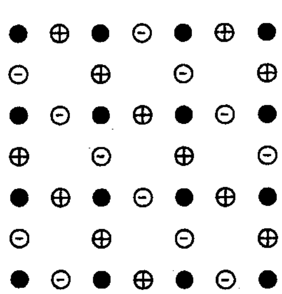

Fig. I.1.7. Distorted CuO_2 lattice

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[1] $LaCuO_4$ is an antiferromagnetic insulator. High temperature superconductivity was discovered in a closely related compound, $La_{1-x}Ba_xCuO_4$. See J.G. Bednorz and K.A. Müller, Z. Physik B **64**, 189 (1986).