8.512 Theory of Solids II
Spring 2009

8.512 Theory of Solids

Problem Set 6

Due April 6, 2004

1. (a) We can include the effects of Coulomb repulsion by the following effective potential:

$$V(\omega) = V_p(\omega) + V_c(\omega)$$

where $V_p = -V_0$ for $|\omega| < \omega_D$ is the phonon mediated attraction and $N(0)V_c = \mu > 0$ for $|\omega| < E_F$ represents the Coulomb repulsion. Write down the self-consistent gap equation at finite temperature. Show that $\Delta(\xi)$ is frequency dependent even near T$_c$ so that the T$_c$ equation becomes

$$\Delta(\xi) = -N(0) \int d\xi' V(\xi - \xi')\Delta(\xi') \frac{1 - 2f(\xi')}{2\xi'} \tag{1}$$

This integral equation is difficult to solve analytically, but we may try the following approximate solution:

$$\begin{aligned} \Delta(\omega) &= \Delta_1, \;\; |\omega| < \omega_D \\ &= \Delta_2, \;\; |\omega| > \omega_D \end{aligned}$$

Now rewrite Eq.(1) as

$$\Delta(\xi) = -N(0) \int d\xi' V_p(\xi' - \xi)\Delta(\xi') \frac{1 - 2f(\xi')}{2\xi'} + A \tag{2}$$

where

$$A(\xi) = -N(0) \int d\xi' V_c(\xi' - \xi)\Delta(\xi') \frac{1 - 2f(\xi')}{2\xi'} \tag{3}$$

Convince yourself that $A(\xi)$ is a slowly varying function of ξ for $\xi << E_F$, so that we may approximate $A(\xi)$ by $A(0)$ in Eq.(2). Produce an argument to show that in the region $\xi > \omega_D$ the first term in the R.H.S. of Eq.(2) is small compared with A so that in fact $\Delta_2 \approx A(0)$. In the same spirit show that

$$\Delta_1 \sim N(0)V_0\Delta_1 \ln \frac{\omega_D}{kT_c} + \Delta_2$$

Combining this with an equation for Δ_2 using Eq.(3), show that the T_c equation becomes

$$1 = \ln\left(\frac{\omega_D}{kT_c}\right)(N(0)V_0 - \mu^*) \tag{4}$$

where $\mu^* = \frac{\mu}{1+\mu\ln(E_F/\omega_D)}$. $\mu^* < \mu$ is called the renormalized Coulomb repulsion. It can be thought of as an effective repulsion with a cutoff at ω_D instead of E_F. Equation (4) shows that the condition for superconductivity is $N(0)V_0 > \mu^*$ and not $N(0)V_0 > \mu$. For screened Coulomb repulsion, estimate μ and μ^* for a typical metal.

(b) Upon isotope substituting $M \to M + \delta M$, how is the Debye frequency affected to leading order? Assuming that this is the only effect, how is $\delta T_c/T_c$ related to $\delta M/M$, (i) in the absence of Coulomb repulsion, and (ii) including Coulomb repulsion.