8.513 Problem Set 2

(Dated: September 21, 2004)

Due Sept 28

1. Consider the imaginary time propagator $K(x, y; \beta) = <x|e^{-\beta H}|y>$ for a quantum system with Hamiltonian H.

   (a) Show that for large β, $K(x, y; \beta) \sim \psi_0^*(y)\psi_0(x)e^{-\beta E_0}$ where $\psi_0(x)$ is the ground state wavefunction and E_0 is the ground state energy (assuming a unique ground state).

   (b) Use the expression for $K(x_0, x_0; \beta)$ derived in class for the simple harmonic oscillator to calculate the ground state energy and the ground state wavefunction.

   (c) From the result for the propagator, calculate the partition function for the simple harmonic oscillator and show that it agrees with the result obtained by more usual means directly by summing over the various energy eigenvalues.

   (d) Consider the calculation of $<x^2>$ at any finite temperature T. Show that this may be expressed as

   $$<x^2> = \frac{1}{Z}\int dx x^2 K(x, x; \beta) \tag{1}$$

   Hence calculate $<x^2>$ for the simple harmonic oscillator.

2. Carefully derive a path-integral representation for the propagator of a particle moving in a vactor potential $\vec{A}$ in N spatial dimension. The Hamiltonian is

   $$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + V(\vec{r}) \tag{2}$$

   Here e is the electric charge.