8.821/8.871 Holographic duality

MIT OpenCourseWare Lecture Notes

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Lecture 1

1: HINTS FOR HOLOGRAPHY

In this chapter, we will get a favor of the holographic duality. We first study gravity system and derive black hole thermodynamics where holography principle emerges. Then we investigate gauge theory in the large N (t' Hooft) limit. At last, we compare such a theory and the string theory and give hints on holographic duality.

1.1: PRELUDE: GRAVITY V.S. OTHER INTERACTIONS

Let us first do a simple exercise: which of the following interactions does not belong to the group, and why?

a) electromagnetism b) weak interaction c) strong interactions d) gravity

The answer would be d). a)-c) are all interactions that can be described by gauge theories in fixed spacetime (Minkowski spacetime):

Quantum electrodynamics (QED): $U(1)$ gauge field + Dirac fields,
Electroweak interaction: $SU(2) \times U(1)$,
Strong interaction: $SU(3)$.

And the basic theoretical structure is well understood by Path-integral formalism plus Wilsonian Renormalization Group. Then any calculation can be reduced to this algorithm, although it does not mean we can necessarily perform it. On the other hand, gravity is quite different. From theory of general gravity (GR):

$$\text{Classical gravity} = \text{Spacetime} \tag{1}$$

However, how do we understand quantum gravity? Spacetime here should become dynamical. There are many puzzling questions. Is spacetime fundamental or emergent? Is it continuous or discrete? What is the quantum nature of black holes? How did the universe begin? One intriguing feature about gravity is that it is the weakest interaction, which may be a fundamental aspect.

In 1997, Juan Maldacena discovered the famous duality:

$$\text{Quantum gravity in Anti de Sitter spacetime} = \text{Field theories (many-body system) in a fixed spacetime} \tag{2}$$

The two sides should be be considered as different descriptions of the same quantum system. This duality provides a "unification", which has far-reaching implications for both sides of the equation. Maldacena's original paper has been cited by more than 10,000 times in SLAC database. But the subject is still in its infancy, and many elementary issues are not yet understood. In a sense it is still like magic. Eq. (2), when totally understood, will be comparable to other milestones of physics, *e.g.* Newton's universal gravity, Maxwell's electromagnetism, Boltzman's statistical mechanics , Einstein's relativity, *etc.*

The goal of the course:

- Motivate and "derive" the duality.
- Work out the dictionary and develop tools for the duality.
- Understand physical implications for both sides of the duality.
- Learn important features.
- Examine open questions.

Emergence of gravity

Duality (2) implies that quantum gravity plus spacetime can emerge from a non-gravitational system. The idea itself is not new: In 1967, A. Sakharov observed that certain condensed matter systems have mathematical descriptions similar to those in GR. This led to a natural question: could GR arise as an effective description of some condensed matter systems? In 1950's, people already speculated that GR is a macroscopic description just like hydrodynamics.

From field theory perspective, it is natural to ask whether massless spin-2 particles (gravitons) can arise as bound states in a theory of massless spin-1 (photons, gluons) and spin-$\frac{1}{2}$ particles (protons, electrons). If the answer is yes, we can conclude that gravity can be emergent. For example, in Quantum Chromodyanmics (QCD) , there are indeed massive spin-2 excitations. Could one tweak such a theory that massless spin-2 particles emerge? Such hopes were however dashed by a powerful theorem of Weinberg and Witten [1].

Theorem 1 : A theory that allows the construction of a Lorentz-covariant conserved 4-vector current J^μ cannot contain massless particles of spin $> \frac{1}{2}$ with non-vanishing values of the conceived charge $\int J^0 d^3x$.

Theorem 2 : A theory that allows a conserved Lorentz-covariant stress tensor $T^{\mu\nu}$ cannot contain massless particles of spin > 1.

Remarks:

1. The theorems apply to both "elementary" and "composite" particles.
2. The theorems are consistent with the fact that massless photons in QED = Maxwell + Dirac fields, as photons do not carry any charge.
3. The theorems are consistent with Yang-Mills (YM) theory. Consider $SU(2)$ YM: A^a_μ, $a = 1, 2, 3$ as gauge fields. For example, $A^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu)$ are massless spin-1 fields charged under $U(1)$ subgroup generated by $\frac{\sigma^3}{2}$. But there does not exist a conserved, Lorentz-covariant, gauge invariant current for this $U(1)$ symmetry. (see pset 1)
4. It does not forbid gravitons from GR. In GR, there is no conserved, Lorentz-covariant stress tensor.
5. Theorem 2 indicates that none of renormalizable quantum field theories (QFTs) in Minkowski spacetime can have an emergent graviton. That is why no matter how we tweak QCD, this can never happen.
6. A hidden assumption of the theorems is that those particles live in the same spacetime as the original theory. This loophole was utilized by holographic duality.

Proof.

Suppose we have such a theory that allows Lorentz-covariant conserved current and stress tensor, and there exist massless particles of spin-J. One-particle state are denoted as

$$|k, \sigma\rangle, \qquad k^\mu = (k^0, \mathbf{k}), \quad \sigma = \pm j(\text{helicity}) \tag{3}$$

We have

$$\hat{R}(\theta, \hat{k})|k, \sigma\rangle = e^{i\sigma\theta}|k, \sigma\rangle \tag{4}$$

where $\hat{R}(\theta, \hat{k})$ is the rotational operator by an angle θ around $\hat{k} = \frac{\mathbf{k}}{|\mathbf{k}|}$. More about representations of Poincare group can be found in Ref. [2]. The conserved, Lorentz-covariant current is J^μ, with the conserved charge $\hat{Q} = \int J^0 d^3x$; the conserved, Lorentz-covariant stress tensor is $T^{\mu\nu}$, with the conserved momentum $\hat{P}^\mu = \int T^{0\mu} d^3x$. Then

$$\hat{P}^\mu|k, \sigma\rangle = k^\mu|k, \sigma\rangle \tag{5}$$

If $|k, \sigma\rangle$ is charged under the symmetry generated by J^μ with charge q:

$$\hat{Q}|k, \sigma\rangle = q|k, \sigma\rangle \tag{6}$$

We want to show that:

1. if $q \neq 0$, $j \leqslant \frac{1}{2}$
2. $j \leqslant 1$

First we claim that Lorentz invariance implies:

$$\langle k,\sigma|J^\mu|k',\sigma\rangle \xrightarrow{k\to k'} \frac{qk^\mu}{k^0}\frac{1}{(2\pi)^3} \tag{7}$$

$$\langle k,\sigma|T^{\mu\nu}|k',\sigma\rangle \xrightarrow{k\to k'} \frac{k^\mu k^\nu}{k^0}\frac{1}{(2\pi)^3} \tag{8}$$

where $\langle k,\sigma|k',\sigma\rangle = \delta_{\sigma,\sigma'}\delta^{(3)}(\mathbf{k}-\mathbf{k}')$. You need to prove the claim in pest 1, one self-consistency check would be when looking at 0-component of Eq. (7), we have $\langle k,\sigma|J^0|k',\sigma\rangle \xrightarrow{k\to k'} \frac{q}{(2\pi)^3}$.

For massless particles, $k^2 = k'^2 = 0$. This implies that $k^\mu k'_\mu < 0$, *i.e.* $k + k'$ is timelike. We can choose a frame, such that $\mathbf{k}+\mathbf{k}' = 0$ and $k^\mu = (E,0,0,E), k'^\mu = (E,0,0,-E)$. In this frame, a rotation by θ around the z-axis has the effect:

$$\hat{R}(\theta)|k,j\rangle = e^{ij\theta}|k,j\rangle, \qquad \hat{R}(\theta)|k',j\rangle = e^{-ij\theta}|k',j\rangle \tag{9}$$

Now consider

$$\langle k',j|\hat{R}^{-1}(\theta)J^\mu\hat{R}(\theta)|k,j\rangle \tag{10}$$

Then we have

$$e^{2ij\theta}\langle k',j|J^\mu|k,j\rangle = \Lambda^\mu_\nu(\theta)\langle k',j|J^\nu|k,j\rangle \tag{11}$$

here $\Lambda^\mu_\nu(\theta)$ is defined by the rotational transformation acting on a vector by a angle θ around z-axis, *i.e.*

$$\Lambda^\mu_\nu(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{12}$$

Similarly:

$$e^{2ij\theta}\langle k',j|T^{\mu\nu}|k,j\rangle = \Lambda^\mu_\rho(\theta)\Lambda^\nu_\lambda(\theta)\langle k',j|T^{\rho\lambda}|k,j\rangle \tag{13}$$

Note that Λ^μ_ν only has eigenvalues $e^{\pm i\theta}, 1$, thus $\langle k',j|\hat{R}^{-1}(\theta)J^\mu\hat{R}(\theta)|k,j\rangle$ can only be nonzero if $j \leqslant \frac{1}{2}$. Otherwise, Eq. (7) is contradicted. $\langle k',j|\hat{R}^{-1}(\theta)T^{\mu\nu}\hat{R}(\theta)|k,j\rangle$ can only be nonzero if $j \leqslant 1$. Otherwise, Eq. (8) is contradicted. Thus we have proved the theorem.

$\square$

Weinberg-Witten Theorem forbids the existence of massless spin-2 particles, which is a hallmark of gravity, in the same spacetime a QFT lives. But there is a loophole: emergent gravity can live in a different spacetime, as in a holographic duality. We are not yet ready to go there without some preparations:

1. Black hole thermodynamics $\Rightarrow$ holographic principle.
2. Large N gauge theories $\Rightarrow$ gauge/string duality.
3. A bit of string theory which would be useful for building intuitions and perspectives.

References

[1] S. Weinberg and E. Witten, Physics Letter B. **96** (1-2): 59-62 (1980).

[2] S. Weinberg, *The Theory of Quantum Fields*, Cambridge University Press (2005).

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