HONG LIU: So last time, we talked about-- we introduced the concept of D-branes. And then we quantized open strings on the D-branes. And we see the massive spectrum on the D-branes includes, say, massless gauge field, and also some massless scalar fields.

And then I described that one can interpolate the dynamics of the scalar fields actually as the motion of the D-branes. So in other words, at the beginning, even when we quantize the open string, we started with a rigid boundary condition, so we started with a rigid ring. But now, after you quantize it, then you get the fluctuations. And because of those fluctuations, the D-branes become dynamical. Those fluctuations of the D-branes make it into a dynamic object in principle to make it move, or et cetera.

So now let's say a little bit regarding the math of a D-brane. In the gravitational theory, anything gravitates. D-brane will have energy. It will have a mass, et cetera. so let's talk about what should be the mass of a D-brane if it's a dynamic object.

So here, there's a very simple and intuitive answer. So on the D-brane, there are many open strings. In principle, there are an infinite number of open string excitations live on the D-brane. And each of them can be considered, say, as a space time field living on the D-brane, et cetera.

So actual definition for the mass of a D-brane is that this should be the energy of a D-brane, which essentially should be the ground state, the energy of the ground state of the D-brane. The energy of the ground state of a D-brane could be corresponding to the energy of the D-brane none of those strings are excited. That should correspond to the vacuum energy of open strings living on it. So this is very intuitive definition and obviously makes sense.

So we can write the mass of a D-brane, DP-brane, as the tension, which is the mass per unit volume times the total volume. And this should be equal to the vacuum energy of all the open strings. So say each open string excitation corresponding to a field. You have a tachyon. You have gauge field. You have massive scalar field, and also infinite number of massive fields. All those fields, they have vacuum energy. So you need to sum all of them together. The sum of those vacuum energies would be the mass of the D-brane.

So this can be achieved just by doing the vacuum diagram of open strings. So we describe in the closed stream case, if you want to find the vacuum energy in the closed string, you just sum of all possible. So this will be just the vacuum diagram.

In other words, so the difference between the open string is that open strings have boundary and closed strings are closed. So that means the sum of all two dimensional surfaces with this one boundaries but no external open strings. This is the natural definition of the vacuum diagram, as we would do.

And we will do in the Euclidean path integral. So you can do this in the Euclidean path integral. You sum over all surfaces. In the case when you need the sum of all surfaces, in some sense, the only way we know how to define such a sum is to do the Euclidean path integral.

And this sum-- so previously, we talked about the vacuum energy of the closed stream, the sum of all possible closed surfaces, say of different topology. So here, again, you sum of all possible surfaces with this one boundaries. So the simplest surface with one boundary is a disk. The difference with closed string case, now you have to sum over surfaces with boundaries. So the simplest one would be a disk. And the next would be annulus. Now you have two boundaries rather than one boundary.

And exactly you can consider more and more complicated diagrams. You can consider more and more complicated diagrams. And the way to weight those surfaces exactly the same as before is that you have this g string than to the power minus chi, and the chi is the Euler number, just exactly as we described before.

And Euler number now to apply surfaces with boundaries would be-- so previously, Euler number is 2 minus 2h. h is number of genuses, or number of holes. But now you also need to include the number of boundaries, which are called b. So when you include the boundaries, then that will change your Euler number, and also change the weight for each diagram.

You can also add handles here. You can also add handles here. You can also add genuses to the disk. You can also add h here.

So according to this counting, then this would count as gs minus 1. So this one has zero holes and one boundaries. So this is 2 minus 1. So this is 1. So this is gs to the power negative 1.

And this one has low hole but two boundaries. 2 minus 2 is 0. This is 1 to the power gs to the 0. And then you have higher diagrams. You have higher surfaces with all positive powers, gs. Yes?

AUDIENCE: What about like a Mobius strip that has one boundary?

HONG LIU: Right. So the Mobius strip is a very good question. A Mobius strip is unoriented surface. So here, we can see the oriented string. You can consider unoriented string. But most of what we said applies to that case. It's just we have to worry a little bit about orientation, so we don't go there.

AUDIENCE: Will that really contribute to the vector [INAUDIBLE]?

HONG LIU: Hm?

AUDIENCE: Will the Mobius strip contribute to the--

HONG LIU: Yeah, yeah, it will. In the case when you have unoriented string.

AUDIENCE: But you make a restriction and say, on this D-brane, we have or not have unoriented--

HONG LIU: Here, we only consider oriented surfaces. We only consider oriented strings. We have not talked about unoriented strings.

AUDIENCE: But no restriction in principle.

HONG LIU: You can. It's actually a technical complication I don't want to go into right now. Yes?

AUDIENCE: So we think of vertical axis as time. So the disk would be a string kind of nucleating and propagating--

HONG LIU: No. This is a Euclidean and you can think of time as whatever you want.

AUDIENCE: Right. But still, so the disk would be like a nucleating open string that propagates and then disappears, right?

HONG LIU: Yeah. For example, this open string you can consider. Heuristically, you may be able to think of some kind of a single string, just rotate, for example.

AUDIENCE: Like forever?

HONG LIU: Yeah. For example, I just said a single. I'm just saying it's hard to interpret as a time now. But the time in this direction would be periodic time if you think from that point of view.

But the good thing is that when you go to Euclidean, what you call time and the spatial direction then becomes obscured. It depends on your convenience.

AUDIENCE: In the center, is that a genus?

HONG LIU: No. The center is completely smooth.

AUDIENCE: It's not like a torus?

HONG LIU: No. A disk is a disk. A disk is not a torus.

AUDIENCE: But there's a--

HONG LIU: You're talking about this one?

AUDIENCE: Yeah.

HONG LIU: Oh. This one is annulus. This guy is annulus. This is a flat surface.

AUDIENCE: So the inside and outside are the--

HONG LIU: They are different. If you identify this and that, then they become a torus. When you identify this one and that one, then they become a torus. Then you get rid of the boundaries. When you identify them, then there's no boundary anymore because they become a circle. Good?

So that means if I have weak coupling, that means when gs is much smaller than 1, which is the cases we can only consider because if you have gs more than 1, then you have some infinite number of diagrams. And we don't know how to deal with this. So weak coupling, when gs more than 1, then the brane tension, then the D-brane will be always scaled with string coupling at 1 over gs because of that, because this term will dominate. This term will dominate. And then the energy should be 1 over gs.

This is a very important result. It's a very important result. The mass of the D-brane is actually 1 over gs.

So on dimensional ground, you can just essentially write down what's the tension of the D-brane because the only dimensional parameter is alpha prime. So the dimension of the D-brane should be-- so this is mass per unit volume. So you have a p dimension of volume, then that would be p plus 1. So the mass dimension of the tension would be p plus 1.

So just on dimension ground, I can write gs because it's 1 over s, alpha prime 1/2 p plus 1. So that gives you the right dimension. And then you can have some numerical constant which you need to determine. You have some numerical constant which you can determine string theory by doing that path integral. Any questions about this? Yes?

AUDIENCE: Do these open string vacuum diagrams have any interpretations like half of a closed string diagram? So if you put two disks together, you have a sphere.

HONG LIU: Right. We will talk about this in a minute. Yes?

AUDIENCE: With the logical decomposition of the powers of g, why is the disk g minus 1.

HONG LIU: This comes from this formula. As we discussed before, the weight of different topology is always weighted by some constant to the power of Euler number. Now, the Euler number, if you have surfaces with boundaries, then the Euler number depends on the number of boundaries. And then you can just work it out. So this is based on simple topology. Any other questions? Yes?

AUDIENCE: I'm sorry. I'm just a little bit confused about the vacuum energy here as the one-- remember when you calculate the mass of the string. You know, we have a naught term there. There is no excitation.

HONG LIU: Sorry. Say it again.

AUDIENCE: So when we calculate the mass of the open string and there is a0 term, which is completely different.

HONG LIU: That's completely different. So that a0, we considered before, it's the zero point energy for the oscillation modes on the screen. So that a0 is that we are considering this string, and the zero point energy for the oscillation mode on this string.

But here, we are considering the zero point energy not of the string. We are considering the zero point energy of the D-brane. And the zero point energy of the D-brane would be to write down the vacuum energy of all the fields living on the D-brane. And all the fields living on the D-brane corresponding to all the-- now, each string excitation becomes a field on the D-brane.

And so that's corresponding to sum of that. And that, then, in turn corresponding to sum of these kind of surfaces. Any other questions?

So there's an alternative way to think about how to compute the D-brane mass or energy as follows, which is actually extremely instructive. There is an alternative way of doing this.

So let's consider just D-brane. So consider the interaction between the two D-branes. So let's consider two D-branes separated by some distance. And then they have a mass. Then they will interact gravitationally. In particular, in a weak coupling limit they're pretty massive. They're very massive because it's 1 over the g string. So when g string is small, which is the only regime we're working with, so the D-brane is very heavy.

And so you can ask, what is the gravitational attraction between the two? What is the interaction between the two? And we know that at low energies, say if the two D-branes are not excited, if their distance are very far apart, then the leading interaction between them just comes from the massless mode because only massless mode mediates normal interactions. And so interaction between them just comes from graviton or this [INAUDIBLE], essentially just corresponding to small number of massless closed string modes. Only those massless modes will contribute because the massive mode only contributes short range interactions.

AUDIENCE: Why not the vector mode in open string?

HONG LIU: No. Vector mode of open string only lives on each brane.

AUDIENCE: But we can have an open string like--

HONG LIU: I will talk about that separately. Just wait a little bit. So if I think purely from the alternative gravity point of view, not from string theory point of view, I have two massive objects. I want to look at the interaction between them. And then interaction will be proportional to gn, say their mass. So if I factor out the volume factor, it would be just GN TP squared. So this essentially is the gravitational interaction between the two.

And from the string theory point of view, such a diagram corresponding to you exchange your closed string. So this diagram corresponding says, suppose you have brane one, brane two. So this picture that brane one will emit graviton absorbed by the other brane. And then that's how we measure, say, the newtons force between them.

And it translates to the string theory picture. This corresponding to one D-brane emits a closed string, and then absorbed by the other D-brane. And when you go to [INAUDIBLE], which only massless mode matters, and then becomes this picture. So this is the string theory version of that diagram.

So now essentially, what you need to do to calculate this thing in the string theory is to calculate this thing in the diagram. So form the string theory point of view, now what you need to consider is to do path integral on the topology of a cylinder with one boundary on the brane one and the other boundary on brane two. So this corresponds to exchange of a closed string in this direction.

AUDIENCE: Question.

HONG LIU: Yes?

AUDIENCE: What's the mechanism for the D-brane emitting a closed string? Or equivalently, on the other picture, why can it emit a graviton?

HONG LIU: It's coupled to graviton.

AUDIENCE: So how did we introduce the coupling?

HONG LIU: Hm?

AUDIENCE: How did we introduce the coupling? I mean, we introduced them as boundary conditions for open strings.

HONG LIU: Yeah.

AUDIENCE: So does that naturally introduce coupling?

HONG LIU: No, no, no. This is what I'm writing here. And this diagram, you emit from a closed string corresponding to look at the cylinder. One boundary of the cylinder on the location with one D-brane, and the other boundary of the cylinder on the location of the other D-brane. And then you just integrate over this surface. And then that will give you the graviton exchange. That will give you the closed string exchange between the two.

AUDIENCE: So the coupling of the closed string to the brane kind of naturally arises?

HONG LIU: In the boundary condition imposed here. So you impose this closed string to initiate it from-- so you impose the boundary condition here so that this closed string starts from brane one and then ends on brane two. And then you integrate over all surfaces this cylinder topology.

AUDIENCE: We know the interaction constant for closed string is g closed, but is it the same here?

HONG LIU: No. That's what I'm going to talk about. Is this clear?

OK. So now I'm going to mention two things. First, as I said before, whenever we do some calculations, we often do analytic integration to the Euclidean signature. So now it will be the same. When we do this calculation, we have a closed string start at location of the brane one, and move forward in time to end on the brane two.

So this is the simplest diagram. You can also add some holes here. You can also add some holes here. And then that corresponds to higher order diagram structure. So now don't worry about that. Just the simplest diagram. Yes?

AUDIENCE: Was it time dimension inside brane to define time dimension as one of the dimensions that lived inside brane?

HONG LIU: Yeah. But this is not a space time [INAUDIBLE]. This is virtual time. This is virtual time associated with this graviton. So essentially, I do create the closed string here on this brane and then propagate then absorbed by the other D-brane.

AUDIENCE: Can you also think about this as like an open string--

HONG LIU: Yeah. One second. I'm going to explain. So now there are two remarkable things about this diagram. The two remarkable things about this diagram. First, this is a cylinder diagram. And this is a diagram with two boundaries because we have to emit a closed string from here. And so you have one boundary. You have an initial closed string, then you have a final closed string. So this is a surface with two boundaries with no holes. And if you calculate the chi, so this would be zero. Then that means this diagram is gs to power of 0.

And then from here, we know that then this from string theory point of view will be gs to the power of 0. And so this is another way to see that the TP should be 1 over g string. Because we said before that the G Newton-- so we explained before G Newton would be order of gs squared. G Newton is the g string squared. So do you remember G Newton is g string squared? Good.

But something remarkable about this diagram is that you can also view this diagram. So right now, we see it from this direction. Now we can also view it from the other direction. Viewed from this direction, so now try to think about this direction as the time and this direction as the sigma. So right now, we are seeing this as virtual time and this as sigma. This is a closed string.

So now think about this direction as sigma and this direction as time. And then this is an open string with one end ending on brane two and the other end ending on brane one, and then going in the loop. So this is the one loop open string. So even though this is tree-level exchange in closed string. So here is the tree-level exchange in closed string. So here is one loop in open string.

So this tells you the same process. You can really view it from two perspectives. From one perspective, it's the standard point of view, is that we exchange some closed strings. So we exchange some gravitons, some massless particles, some particles between these two.

But there's another way to think about it is we say, because we have two D-branes here and because D-branes correspond in two places, open string can end, then I have open string connect between them. And this one loop open string is essentially corresponding to the vacuum diagram of those open strings connect between them. So when you add the vacuum energy of all those open strings ending between them, then you're effectively calculating the interaction between the two D-branes.

So we have two completely different perspectives to look at the same process. And this is very, very deep and profound. Deep is profound.

Because that means the process that you can think from closed string perspective can be fully understood in a different way from the open string perspective, in a completely equivalent way. And this is normally called the channel duality. So it's a very simple geometric fact about two dimensional surfaces. But physical significance is very profound.

AUDIENCE: I have a question.

HONG LIU: One second. Let me finish. And this is precisely the string theory origin of the holographic duality or the idea, say, of t we are going to see in a couple of lectures. Just because of this simple geometric picture. This side is gravity, and the D-brane is about gauge theory. And then we see gravity to be equivalent to gauge theory. That's something we are going to see later. Yeah?

AUDIENCE: So you said the dynamics of closed string can be fully understood by open string. Is that why you say it's a closed surface formed by a closed string? How can you interpret?

HONG LIU: Yeah. I'm talking about this particular diagram. I'm just saying this gives you a hint of something very profound.

AUDIENCE: And one more thing. Why is the gs 0?

HONG LIU: No. This is the surface of two boundaries.

AUDIENCE: But that is for open string.

HONG LIU: No. Chi is for everything. Chi is everything. Doesn't matter open or closed string. This is the universal formula. This is open. The open string just means we imposed the boundary condition on the open string. The topology is the same. We understand the topology is the same. Yes?

AUDIENCE: Why do you call it one loop string? Where is the loop?

HONG LIU: Because this is the open string. So this is the open. Think from this point of view. This is the open string on brane one and brane two. And then you go around once, go around in circle. So this is one loop.

AUDIENCE: What's the free momentum?

HONG LIU: What is this? What is this? This is one loop if you have a particle. And so you have a string. Then you go around the circle. This is one loop. And indeed, when you sum over such surfaces, you will lead to sum of all possible momentums, et cetera. So the field theory momentum is one of the modes you have to sum over when you do past integral over surface of such topology. Yes?

AUDIENCE: Once you go to strings connecting different branes, your quantization conditions change.

HONG LIU: We will talk about that. Quantization condition almost does not change. We will talk about that. We'll talk about that in a little bit. But right now, it's just intuitively clear you have open string connect between them and they just go around. You have open string connect between them. You can just go around the circle.

AUDIENCE: But this correspondence doesn't count the tree-level around the open string.

HONG LIU: Count what?

AUDIENCE: If we generalize the tree-level open string.

HONG LIU: No, no, no. This doesn't have to account for the tree-level open string.

AUDIENCE: Tree-level open string contributes to the interaction?

HONG LIU: No. Here, I'm only talking about this diagram and just say this hints that there are certain closed string processes can be completely described by the open string. So what want to extrapolate is that open string is a more fundamental description because the tree-level diagram in closed string can be described by the open string. And now if you can generalize that maybe everything closed string can be described by open string.

But you don't want to do in the opposite way. Open string is open string. Good? Any other questions?

AUDIENCE: Once you go higher dimensions, when you leave tree-level closed string can be distinct?

HONG LIU: Yes. Things become more complicated. Things become more complicated. But the similar picture will exist.

But nobody has made it work. Nobody has made it work at full string theory level to construct the whole closed string theory from the open string theory. Nobody has made it work. But there are many such kind of indications from the geometry of the surface point of view.

So now let's talk about relaxing the strength of open string interactions. Actually, before I do that, now is a good place to go back to examine what we discussed at the end of last lecture. Now is a good time to go back to talk about what we did at the end of the last lecture.

So in the last lecture, at the end, we described that one can work out the low energy effective action of the massless modes on the D-branes. So the massless modes on the D-branes on the gauge fields along the D-brane, so A alpha from 0, 1, to p. And then the scalar field and a label all the perpendicular directions.

So you can write down effective action for them. I mentioned if you work out things carefully, then you find the prefactor is actually just the brane tension. And if last thing is excited, p plus 1. If last thing is excited, then you just have the vacuum energy, so you will have a one. So this is just the brane mass, the total brane mass. This is et. So if last thing is excited, then you just have the zero point energy, which is just tp times the volume.

But now, if you also have gauge field, then based on general argument, you must have the Maxwell. And if the scalar field is excited, then you also have the action for massless scalar field.

And then we mentioned that for example, you can consider special case. Suppose A alpha is not excited but the brane, rather than a scalar field that moves in a coherent way the same at all points on the D-brane, just phi a, is a function of t rather than x. So phi, in general. Suppose the brane coordinates our x0 and the p. So in general, A alpha is the function of x0, xp. And phi a is x0 and xp. So they describe you can have arbitrary profile on the world-volume.

But suppose, say, let me consider the uniform situation which I only consider every point has the same behavior for phi. And then this s just becomes 1/2, dt just becomes dt. And D-brane plus 1/2 and D-brane phi dot squared.

So this is precisely the motion of just a massive object. OK And m is just this guy. So if last thing depends on the spatial coordinate, then the integration of the spatial coordinate becomes the volume. Combine the volume with that, becomes the mass, and then just becomes that. I think this is minus sign. This is plus sign.

So as I mentioned, this is another way to see that the D-brane becomes dynamical, and that in particular, this phi describes the motion of the D-brane. So in fact, this result can be much, much strengthened. But I will only quote the result. I will only quote the result.

It turns out for D-brane with constant. So as opposed to the D-brane move with constant velocity, so now you can also have a motion in the spatial direction. You have a constant of this and F alpha beta. You can also excite the gauge field, but the field strength is constant.

Or this quantity is small. They don't have to be strictly constant, but at least their derivatives are small.

In such a situation, one can actually sum all the higher order terms from string theory corrections. So this is just a low energy, just like field theory. And in such a situation, you can actually sum over infinite number of higher order terms.

And what do you find? You find so-called wave sum of all infinite number higher terms. You find so-called Dirac-Born-Infeld action. You find that this effective action becomes like this. This is very [INAUDIBLE] result. So I just want to mention it.

You can actually sum into this form. So you can sum into this form. And this g alpha beta is the [INAUDIBLE]. So let me just explain a little bit the physics of this formula.

So let's consider the case of the phi is not excited at all, just phi for the constant, say, for example. Then this term vanishes. Then this g alpha beta just becomes eta alpha beta. And now you just have a square root, say, your Minkowski metric plus F alpha beta. Forget about this 2 pi alpha prime. This is just some dimensional factor.

And then you just have eta alpha beta plus F alpha beta. And then you write the determinant. And suppose when F alpha beta is small, when alpha prime times F alpha beta is small, then you can expand it in powers of F alpha beta. It's a simple exercise but instructive exercise. You see that precisely reproduces the Maxwell term. But this will give rise to higher nonlinear terms. There will be higher order nonlinear terms. So this can be considered as a nonlinear generalization of the Maxwell theory.

It turns out actually, this theory was considered in the '30s by this guy, Born and Infeld. Actually, maybe '30s or '40s. Anyway, prehistory. They invented as a way to avoid-- they want to avoid the similarity of the Maxwell theory. So in the Maxwell theory, if you have a charged particle, and then the location of the charged particle, then the field due to that charged particle is singular and the location of that particle. And so they want to avoid that singular behavior, so they invented this Born-Infeld action.

And for many years, this action does not have any applications. But if you invent something nice, it will find its use. Just like in this movie, *Jurassic Park,* life finds a way. Life always finds a way.

So that's Born-Infeld. Now let's set F equal to 0. Let's just look at g alpha beta. Now let's look at g alpha beta. So g alpha beta, we can write it in a slightly more transparent form. We can write a form which makes it a bit more transparent.

I can write it as the following. So even mu, remember, is the Minkowski metric of the full space time. And I can write this as following. x mu beta x mu x mu with x alpha equal to x alpha, which is along the brane direction, and xa to be phi a.

So if you look at this formula, you can see this is an induced metric for some brane embedded in the full Minkowski space time. And the x describes such embedding. This generalization of this induces the metric formula we encountered before for the string. But right now, the only difference is that now alpha beta, they run all in the world-volume direction of the D-brane. And then this becomes an induced metric on the D-brane when it's embedded in the space time.

And this x alpha equal to x alpha, it just means that when we embed it, and we choose the world-volume direction to be the same as the space time direction along the brane direction. And in the perpendicular direction, this is just phi a. And if you look at this, this is exactly that. It's exactly that because x alpha equal to x alpha. And then you just get the eta alpha beta. And then for the other direction, you get this one. Is this clear?

So when f equal to 0, so this S just becomes eta g alpha beta. So this is precisely the volume element of DB-brane. Because this is the induced metric, and then this is just the total volume element of the DP-brane.

And we see this is precisely is the relativistic generalization. So this is just a generalization of the [INAUDIBLE] action we wrote earlier, which is for a string. Then this would be a two dimensional area. And here, you just integrate over the volume element of the whole D-brane.

So we see that this Born-Infeld corresponding to really describes the relativistic motion of a p dimensional object. Describes the relativistic motion of a p dimensional object. And this Dirac-Born-Infeld, when you combine these two together, it magically combines these two things into a single thing. Yes?

AUDIENCE: So I recall you saying earlier that-- you said that people have played with this idea of thinking about branes instead of just generically higher dimensional objects and strings but no one really understood the theory of these things because the topology and geometry were too complicated. So it seems to me that wouldn't you run into that same problem right here if it's indeed some generalization of the [INAUDIBLE] action?

HONG LIU: Yeah, but we don't try to quantize it. At least we don't try to quantize this action. And we know how to quantize this action. And this is just our ordinary field theory.

AUDIENCE: I have a question. Here, we must impose the big x as the coordinates in the target space.

HONG LIU: That's right.

AUDIENCE: So phi a must be kind of a constant? I mean, why there should be a constant part on--

HONG LIU: No. If the derivative of those things are not small, then there will be many other terms. This will not be the only action.

AUDIENCE: Given a constant.

HONG LIU: Sorry?

AUDIENCE: You said with a constant.

HONG LIU: The partial alpha phi a equal to constant.

AUDIENCE: Oh, equals a constant. So that means--

HONG LIU: No. What I'm saying is that if these are constants, then this is our exact string theory action. And when these are not constants, and then this is a leading approximation, there will be higher order terms which depend on their derivatives. Yes?

AUDIENCE: Sorry. One thing I just don't understand-- why is it that we don't want to try to quantize anything? Shouldn't it be quantized in principle? These are the sort of classical analogs of things you want to quantize.

HONG LIU: Yeah.

AUDIENCE: So this is to say that this object that we don't really know how to quantize is-- we just don't do it because we don't know how.

HONG LIU: Yeah.

AUDIENCE: I see. OK. It's not because-- fair enough.

HONG LIU: Yeah. You should try anything. And only those people who have succeeded in the history books. Only a few have won the battle in the history books. And if you just fail the battle, you're not in the history book. So people have tried this but failed it, but that won't be written in the books.

AUDIENCE: Sure.

AUDIENCE: Sorry. So you did the square root by summing over all the massive terms in the--

HONG LIU: Sorry?

AUDIENCE: You get the square root term by summing the series, including the massive fields?

HONG LIU: No. This is still the gauge field and the phi.

AUDIENCE: Right. So how do you get it? What's the series that you're summing?

HONG LIU: Hm?

AUDIENCE: What's the series that you're summing?

HONG LIU: Oh. I'm just saying in the string theory, typically you don't start by f squared. You have f cubed, f four, et cetera. You can sum all of them together. Even for the massless mode, these are just leading terms. And these terms would be the smallest number of derivatives, and so they dominate at low energies. But in general, even just for the effective serial massless mode, you can have many, many other terms.

AUDIENCE: I have a question. If we assume that partial alpha phi a is a constant, then we can solve out the phi a is proportional to x alpha. But how can you assume they're just the coordinate in target space?

HONG LIU: No, no. This is a function of alpha.

AUDIENCE: Yes. But since it's a function of x alpha, why it can be regarded as the coordinate in target space?

HONG LIU: Sorry. I don't understand.

AUDIENCE: All the coordinates should be independent in target space.

HONG LIU: Sorry. I don't understand. They are independent. These are the virtual coordinates. These are the volume coordinates of D-brane. And these are the target space coordinates. I'm just choosing the function of the target space coordinates. I'm choosing here just to be identical to the world-volume coordinate. And this one I choose to be some function of the world-volume coordinate. Of course I can do that.

Any other questions? So again, this highlights that D-brane is really a dynamical object. In fact, at low energies, they move like [INAUDIBLE] motion. And they actually move relativistically if you give enough velocity, et cetera.

Because of the fluctuations of the D-brane, they become a really full dynamical object. They have a mass. They can move around. And now you can deform their shape. If you have enough energy, you can bend them, et cetera. You can do whatever you want.

So let me mention one last thing. So you may ask, why somehow those fields which describe the motion of the D-brane, they're corresponding to the massless modes on the world-volume of the D-branes? Whether this is a coincidence, or why it somehow happens to be the massless mode on the D-brane which describes the motion of the D-brane.

So this is not an accident. This is not an accident. The reason is that-- so why modes describing motions of D-branes appear as massless modes? So this is not accident.

So underlying reason, it's because the underlying Minkowski space is translation invariant. So that means that no matter where you put the D-brane, it should be a well-defined configuration. Should be a well-defined configuration no matter where you put on the D-brane.

Then that means that the [INAUDIBLE] action for the phi cannot contain a potential term. They cannot be potential term. They should not be, say, somewhere is the minimum, somewhere is the maximum, cannot happen. Everywhere must be the same. So it means the dependence on y can only be derivative. Can only depend on derivatives. And of course, at low energies, if you have derivatives, then can only be the massless particle.

So translation invariant. So this means that any phi a equal to constant should be allowed configurations. That means cannot have potential. So max term is like a potential for phi.

To say in the fancy words of Quantum Field Theory II or Quantum Field Theory III, that the phi a, in other words, phi a are the Goldstone bosons for breaking translation symmetries.

So previously, Minkowski space is translation invariant. And now if you put a D-brane there, then you break that translation symmetry. The location of the D-brane breaks that translation symmetry. If I even put the D-brane anywhere, then that means the modes, the dynamics control the location of the D-brane must not have any potential, can only have derivative terms. So in other words, when you put the D-brane in, you spontaneously break the translation symmetry on the line in Minkowski space.

So let me mention one thing. Then we can have a break. I'll mention one quick thing. So let me say a few words on the strength of the open string interactions. So previously, we described that the closed strings, they interact by such joining and splitting procedure. And the strength here is capped by gs. So the closed string coupling is essentially the gs, which describes such a process.

So if you have an open string, of course you have a similar process, just string ends joined together. You can join string ends together. So here, you really just have open string. So now these lines are the boundaries of open string, or the endpoints of open string. So you have two open strings joined together from another one. So let's call this interaction go describing the interaction of the open string.

So the question is, how is this go related to gs? And there's a single way we can figure it out. So let's consider the simplest situation. Just have open string propagate in time. Again, this is two boundaries open string. We just propagate in time.

Now let's consider a more complicated process. So the open string propagates in time. So this is just a simple surface with one initial open string and one final open string. And you can see the complicated process because we have a hole in the middle. So now the string worksheet is like this, just this part. We have a hole in the middle. And this is another configuration to have some initial open string propagate to some final open string.

So now we know, by counting we did before described here. So here, we're adding one boundary. We are adding one boundary. So this adds a boundary. So that means we must add a factor of gs. Because from this formula, this is g minus chi. And chi is minus b. So if we increase one boundary, then you increase a factor of gs.

But we can also view this diagram as a single open string comes in. The opposite of this splits into two open strings and then they close together. So one split operation, and the one join operation. So that should correspond to go squared. So then this means that we conclude that gs must be proportional to go squared. The open string coupling strings is the square root of the closed string interaction. Yes?

AUDIENCE: What's the strength of the process when closed string becomes open string and vice versa?

HONG LIU: Sorry?

AUDIENCE: What's the strength of the process when closed string becomes and open string?

HONG LIU: Right. Yeah, you can consider such process. Again, you can just do it by counting the topology of the surface. Such process can exist.

OK. Then let's have a short break. So what time is it? It's 38. When should we start again? 41? OK, 41. Let's start at 41.

So we have talked about D-branes, et cetera. And we have already seen some remarkable aspects of the D-brane, including this channel duality between the closed string exchange can be considered open string loop. And now we are going to see a lot of magic of the D-brane.

And this comes when you put several D-branes together. So normally, our conventional intuition says if you find some particle, say in this case, you find the D-brane. So if you put two particles together, nothing much really changes. It's two particles. Put three particles together. Not much changes. It's three particles.

But when you put multiple D-branes together, things change a lot in a very profound way. So now let's consider just two D-branes. So let's consider two D-branes. Let's consider example.

So let me first just tell you the naive intuition. Suppose you have D-brane one, D-brane two. So for this one, we have a u1 gauge field. For this one, you have a u1 gauge field. Because each one, we have a gauge field, a Maxwell. When you put together, from conventional wisdom, you say, maybe I just have two Maxwell. From conventional wisdom, you two Maxwell. Naively, if I put them together, I just have two Maxwells. 1 plus 1 equal to 2.

But in string theory, 1 plus 1 equal to 4. It's actually equal to 2. It's also equal to 2, depending on how you think about it. Anyway, one way to think about it is 1 plus 1 becomes 4.

So to see 1 plus 1 become 4 is very easy. So let's consider these two branes on top of each other. But in order to distinguish these two branes, I just separate them a little bit. But you should really think of them on top of each other.

And so now you have four types of strings. You can have string going to 1, 1, going to 2, 2, then going to 1, 2, going to 2, 1. So 2, 1 and 1, 2 are different because the oriented string. So I put arrow there. So this is from 1 to 2. This is from 2 to 1. So we have four types of strings-- 1, 1, 1, 2, 2, 1, 2, 2.

AUDIENCE: Why is oriented--

HONG LIU: Hm?

AUDIENCE: Why the 1 to 2, 2 to 1 are different?

HONG LIU: It's because for this string, sigma 0 here. For this one, sigma pi there. For this string, sigma 0 there. And for this one, sigma pi there. Let me just elaborate on this point. So suppose I have a string like this. Then this string is sigma 0 point ending on 1, sigma equal to pi ending on 2. But if I have a string like that, then there's a sigma 0 ending here and sigma pi ending there.

So you have four types of open strings. And now if you think about how we quantize those strings, and the four types of open strings, they actually have identical spectrum. Because for all of them, the boundary conditions are exactly the same. Because the boundary conditions only know the location of the D-branes. So all four types of open strings have identical spectrum.

So in other words, each string excitation-- say this is the state on the worksheet-- each state becomes four states because I can label IJ. Now suppose I use I and J to label 1 and the 2. I and J can be 1 and 2. So depending on whether this is 1, 1 string, or 1, 2 string, or 2, 1 string, or 2, 2 string. So this is what I said 1 plus 1 equal to 4. Because naively, you would say I have two massless modes. But now I have four.

For example, the massless modes become four copies of them. In other words, you can think each open string excitation a 2 by 2 matrix. I can use 2 index to label them.

In particular, for example, the corresponding fields-- so each string excitation corresponding to some field. For example, the gauge field associated with this now has two index, I J. And similar with phi a I J. Of course, this generalizes immediately to if you have n branes, then just becomes n times n matrices. So 1 plus 1 plus 1 to n becomes n squared.

So now let me give some remarks. So this basic structure turns out to be, again, very, very profound. Now let me give some remarks.

So there's a reason I call-- so this is something with the 2 index. So of course, you naturally call it a matrix. But there's another reason to think about this really as a matrix. It's because the strings, as we were doing there, the open strings, they interact by joining their ends.

So this naturally leads to-- when those strings interact with each other, and those parts naturally just emerges as a matrix product, I, J indices. So it's easy to see. So let me just draw that. Let me just do it here to save some time.

Suppose this is I. This is J. So this is sigma equal to 0, sigma equal to pi. And the sigma equal to pi joins with stigma 0 to end of the other one. But of course, if you want to join them together, their J's have to be the same. This is K. Then you go to I, K. And when they join together, then you sum of all possible J's. Then this is like a matrix product.

So if I draw it in the diagram not very well. Now let me separate I, J, K to be three things. But they don't have to be separated. I, J, K can also be the same. But in order to emphasize this picture is that you have a string to go from I to J. Suppose this is I, this is J, this is K. Go from I to J, then from J to K. So sigma 0, sigma pi. And the pi end joins with the sigma 0 end. And then here, you get the string.

So that diagram roughly can also be think of a diagram like this. Two strings join into one string with index I and K. And the I, K, K can all be the same. I just make them different to make it clear. And of course, when you join J together, you have the sum of them because they can be in principle all possible J's. So it naturally appears as a matrix product. Just follows by the nature of string interaction.

And now there's another remarkable thing is that if you can see the phi a, so the same thing applies for a alpha applies to any field. It's that 1q-- so this corresponds to a string with sigma equal to 0 at 1 and sigma pi. So this corresponds to a 1, 2 string. And then we can also think about the 2, 1 string.

So it turns out that these two can be considered as complex conjugates of each other for the following reason. Again, now let me just again separate this 1 and 2 to make it clear. So this is a 1, 2 string. So this is a 2, 1 string.

So I claim string interactions defined by this way have the following symmetry. So string interactions described by joining the ends or splitting the ends have the following symmetry. It's that I can associate each brane by a phase factor. So I explained to you i theta I. So I labeled to the brane and the theta can be some different-- can be a phase factor.

And now the rule is that if the sigma is equal to 0 and on that brane, then I multiply it by the exponential of i theta I. And the sigma equal to pi ends on that brane, ends on I. Let me write it more explicit. So if sigma 0 ending on I, then I multiply by a phase factor, exponential i theta I. And if the sigma equal to pi factor ending on I, then I multiply by exponential minus i theta I.

So let's consider this operation. And I claim this operation is the symmetry of the string interaction. So let's first consider if you just have a single brane. So if you have a single brane like this, then just nothing changes because you multiply one end by i theta and the other by exponential minus i theta, does not change.

But now, if you have such kind of interactions, because the sigma 0 and sigma pi ends join together, and they can only join if they are ending on the same brane, then those factors always cancel each other. And so this would be a symmetry. Is this clear?

So under this operation, phi a ending on the same brane is invariant. And phi a I J then transforms by a phase factor theta I minus theta J of phi a I J. And the phi a J I transforms as a factor minus I theta I minus theta J. So we can actually think of them as complex conjugates. So they're transforming opposite way under this phase change. Yes?

AUDIENCE: And this is because we're considering them to be u1 branes?

HONG LIU: Sorry?

AUDIENCE: Is this because we're considering them to be u1?

HONG LIU: No. This is a good point. I will talk about this more. Right now, let's think about each brane separately.

AUDIENCE: I have a question. In principle, can we write the interaction of open string with the same note, J, J by random number?

HONG LIU: Sorry. What do you mean by random number?

AUDIENCE: Say it's I, J, K, L.

HONG LIU: No, no, no. The strings can only join together if they're ending on the same brane.

AUDIENCE: Oh. OK.

HONG LIU: So the J's have to be the same. So this guarantees that this will symmetry. Good? So in this sense, they're complex conjugates. And now we can build on this a little bit further. So this actually works. Doesn't matter whether the branes are coincident or not coincident. This is a generally true.

Now let's consider all the branes are coincident with each other. So for coincidental branes, since branes are indistinguishable from each other, so they are higher dimensional generalization of what we call identical particles. So if they're indistinguishable from each, we can shuffle their indices. We should have the symmetry to reshuffle their symmetries. So whether we call this 1, 1 or this 1, 2 or this 1, 1 should not matter.

So if I combine these two facts, two observations together, then when we have n coincidental branes, then there's in fact u(n) symmetry. If that whole string interaction is invariant, say if I have psi, I, J goes to say U, I, K, U, J, L, star, psi, K, L. So back here on U just corresponding to I reshuffle all the indices. So I have to do the same to U. So I reshuffle the two indices in the same way.

But I have a star here. It's because of this reason. Because in some sense, the sigma 0 and sigma pi, they're only symmetries if I multiply opposite phase factor.

I can rewrite this as a matrix notation. If you think about each side as a matrix, then this is the symmetry corresponding to psi U psi dagger. And the U can be arbitrary unitary matrices. So in fact, when you have n coincidental branes together, there's u(n) symmetries for the string interactions.

So to say it in a more fancy mathematical language is that each open string excitation transforms under the adjunct representation of this u(n). So this is like a join representation. If you have u(n) symmetry, then this is like a join representation.

So on the string worksheet, this is really a global symmetry. So it's just a phase factor associated with each n. There's nothing. It's a global symmetry.

But the remarkable thing is that in the space time, this becomes a gauge symmetry. So this tells you, because of the presence of this u(n) gauge symmetry and because of each mode transforms under a join representation of some u(n), that as a space-time field, they also must transform under a join representation of some u(n). And interpreting the space-time, then this u(n) must be a gauge symmetry.

On the worksheet, it's a global symmetry. But in space-time-- so let me just write it down. In space-time-- or in other words, in the world volume of D-branes, this u(n) must be a gauge symmetry for the following reason. Because the only way we know-- for example, this gauge field becomes transformed under a join representation of some u(n) symmetry. And all the excitations will have this symmetry.

And then the only way we can make sense the gauge field under such symmetry is that this is a gauge symmetry and this is the gauge boson for that symmetry. Is it clear? So let me just say it in words. It must be a gauge symmetry. And in particular, a alpha I J must be the corresponding gauge bosons.

Because this is the only way we know how to make sense, because this is the only way we know can happen at low energies. Some gauge fields transformed as a matrix interact with each other. And this can only be Yang-Mills theory. And if this is Yang-Mills theory, then this must be a gauge symmetry. So this is the basic argument. And at low energies, we must have Yang-Mills theory.

So something remarkable happens. So each D-brane, when we separate them. is a Maxwell theory. When you put them together, they become Yang-Mills theory. And somehow, they become non-Rubinian. And everything comes from, in the very trivial way in string theory, you just count the indicies. But the physical implication is profound.

And this can be confirmed, again, by just starting the explicit string theory scattering amplitude. You can calculate the scattering of 3a in string theory. Then you find it's precisely-- at low energy, precisely given, but the same vertex as the Yang-Mills theory, et cetera.

So you can do that. Then you find the low energy effective action can be written as the following. Some Yang-Mills coupling trace. Let me just write down the Yang-Mills theory. You find the low energy effective action can be written in this form. So this is standard Yang-Mills field strings because now everything is a matrix. Everything is a matrix. And both A alpha and phi a now are embedded in matrices.

So this g Yang-Mills is the Yang-Mills coupling which describes how the gauge fields interact with each other. And obviously, this should be related to the open string interaction because-- I think I have already erased it. This kind of interaction, joining the string. This corresponding to two strings joined into one string, and this proportional to g0. And this, from the field theory point of view, it just controls the interaction of the three A's. So that must be the Yang-Mills coupling.

So this must be the Yang-Mills coupling. And then this should be related to gs to the power 1/2, which we just derived slightly earlier. So now, on dimensional ground, the g Yang-Mills square can be written as-- let me write it here. This is an important formula.

On dimensional grounds, the g Yang-Mills square, I should be able to write it as gs. So on the Yang-Mills coupling, we will use the standard convention that A has the dimension of mass. And then this is a dimension 4 object. And this is a dimension p plus 1. And so the Yang-Mills coupling, you can deduce the dimension of the Yang-Mills coupling there.

And it just turns out to be p minus 3 dense-- again, because alpha prime in string theory is only length scale. So the alpha prime must come from here. And then times some constant. So again, dp is just some numerical constant.

And you can see explicitly that when p equal to 3, then you have d3 brane. Then the world volume theory is four dimensional. Then we recall that in four dimensions, Yang-Mills theory is dimensionless. QCD [INAUDIBLE]. Good? Any questions? Yes?

AUDIENCE: What to do with phi h?

HONG LIU: Hm?

AUDIENCE: What to do with phi h?

HONG LIU: What to do with phi h? What do you mean what to do with phi h?

AUDIENCE: Scalar color particles?

HONG LIU: Yeah. I forgot to mention here is the standard derivative, covariant derivative. So the alpha phi a is just the standard particle phi a minus i a alpha phi a. So it's the standard gauge covariant derivative.

AUDIENCE: Shouldn't phi be more fundamental presentation of--

HONG LIU: No. Everything is to the join because everything has two ends.

AUDIENCE: So how to interpret phi, then, in our--

HONG LIU: Hm?

AUDIENCE: From the present zoology of particles, where to put phi, the observed particles?

HONG LIU: Sorry?

AUDIENCE: If a is just the gauge boson [INAUDIBLE] or something, what is phi?

HONG LIU: Phi is some scalar field transformed on the join representation of the gauge group. It's a matter field describing the motion of the brane.

AUDIENCE: I have a question. There you say that we can reshuffle their indexes. So that symmetry should be permutation symmetry?

HONG LIU: No.

AUDIENCE: Why [INAUDIBLE]?

HONG LIU: But I reshuffle because I say it in the more heuristic way. But normally, if the indices is not-- these are the states. And you can just swivel-post them. A different i, they should be the same thing. They're just corresponding to relabeling, I'm just saying. The lateral action of anything on the state, of course, is the unitary transformation.

AUDIENCE: Is there anything to do with that?

HONG LIU: No. That thing is related to this star, why we put this star here. This is the reason why we put the star there, because the two endpoints, they should transform in opposite way. Yes?

AUDIENCE: Since the branes become dynamical, can't they fluctuate in different ways and so no longer coincide?

HONG LIU: Sorry?

AUDIENCE: Since the branes are dynamical, you put them all in one place at first. But can't they now start fluctuating and separate?

HONG LIU: They can certainly.

AUDIENCE: They're no longer symmetric. They become distinguishable.

HONG LIU: They can certainly start moving apart if you give them some initial motion. But the fluctuation, the system is isotropic. There's no place for them to-- they will fluctuate, but they will still at that point on average. There's no preferred direction for them to go unless you give them a direction. You say, I want them to go in that direction. Then you push them.

AUDIENCE: But you can still think of them fluctuating kind of separately, separating in their fluctuations?

HONG LIU: Sure. This a are their fluctuations. Phi are their fluctuations.

AUDIENCE: Is this commutative term belonging to the effective action an interaction between the D-branes or is it--

HONG LIU: Yeah. Good point. Let me just comment on this term. So if you think about it, as we said before, a D-brane, no matter what dimension of the D-brane, the story when we quantize them, they're almost the same. It's the same spectrum. If you have a space-feeling brane that everything is A alpha, and then if you have some lower dimensional brane, some of them become scalar field, et cetera. So essentially, they all have the same dynamics.

So this interaction can be considered just come from here. So you can start with a space time feeling brane, and then you go to lower dimensions. And then this just can be considered to come from there. It's part of the gauge theory.

AUDIENCE: So the whole low energy actions are the action of supergravity plus this D-brane interaction, you can think of the whole low energy action?

HONG LIU: You mean if you include the closed string?

AUDIENCE: Yeah.

HONG LIU: Yeah. That's right, if you have D-branes. That's right. This is effective action on the D-brane. This is the effective action on the D-brane.

AUDIENCE: I mean if you extend D-brane [INAUDIBLE], then where does this action come from?

HONG LIU: No, that's what you said.

AUDIENCE: It's just two actions crossed together?

HONG LIU: Yeah. Good?

AUDIENCE: Excuse me. How come it has the term in action that's proportional to phi without derivative?

HONG LIU: No. They have derivative. This is covariant derivatives.

AUDIENCE: The commutator.

HONG LIU: No, but phi have covariant derivatives.

AUDIENCE: But the next time.

HONG LIU: Yeah?

AUDIENCE: That thing appears to be proportional to phi without derivative.

HONG LIU: Yes?

AUDIENCE: But this is the Minkowski space or something.

HONG LIU: Yeah. That's a very good point. I'm going to mention that point. But the key is that this particular potential-- this is a very good point. I'm going to mention that. I will mention that in a few minutes. I do because I do want you to do your p-set.

So now let's consider separating the branes. Now we will consider separating the branes. Again, let's consider the situation we just have two of them. So at the beginning, they're coincidental. And then now let's separate them in some direction. Let's call that direction x. So let's say they separate by some distance, d. This is 1, this is 2.

So of course, this 1, 1 and 2, 2 string, nothing changes because they're just still ending on the same brane. But those strings are now different. So now 1, 2 and 2, 1 strings become different. So 1, 1, 2, 2, exactly the same as before.

And the 1, 2 string. For example, let's consider the 1, 2 string. Then the boundary condition changes in the way I have sigma equal to 0, say, in the x direction. Sigma equal to 0. Tau, say, is at some location. Let's call this x0. And then x sigma equal to pi tau then becomes x0 plus d.

So now it means when you contact this string, you have to do a slightly modified boundary condition. So you have start with x equal to 0. Now you must include a term depend on sigma.

And again, as before, we take the xL to be minus xR and periodic, et cetera. So for sigma equal to 0, then this boundary condition is satisfied. But in order to satisfy the boundary condition at pi, then you now need this w equal to d divided by pi. You need to develop d divided by pi.

So now, you have a sigma term. So that will change your [INAUDIBLE] condition. So remember the [INAUDIBLE] condition we had before is something like this, q alpha prime p minus, say, is p plus 4 pi alpha prime. Say for the open string. I'm writing the open string version, which is, up to some numerical factor, the same as closed string we wrote down before.

So now, because of this term, then there's additional contribution on the right hand side. It makes sense because now string has to be stretched over some distance. That costs energy. So take one minute to do it yourselves. Just plug this into here. You only need to look at the behavior of this term. Just plug in there. Take you, say, five seconds.

You find that the massless condition now has one more term plus the rest, as before. And that means now all the previously massless particle, this here with the corresponding a alpha and phi a, are no longer massless. Because previously, they were massless because those terms are 0. But now you have one more term. And they have a mass given by M divided by d divided by 2 alpha prime. Just take the square root of that.

And because this is precisely the d times the string tension, because 1 over 2 pi alpha prime is the string tension. So this is exactly the energy we expect, just from a classical picture. You have a string stretched between the two. Then this is the tension times the length of the string.

So now you can also easily understand what's going on from field theory point of view. And now you only have two sets of massless modes now rather than four. So what's happening is that now, the gauge symmetry is broken from u(2) to u(1) times u(1).

So the separation of the branes essentially corresponding to the Higgs mechanism in the following sense. When you separate the brane, I said before phi should be interpreted as the [INAUDIBLE] location of the brane. But you separate the brane. That means one of the five fields corresponding to the x-- the x is in the transverse direction for the brane. One of the five fields corresponding to the x must develop expectation value. And that expectation value then gives rise to this mass. So this is precisely a Higgs mechanism.

So now let's go back to the question what this potential term means, whether we can actually, as we said before, because of the translation symmetry, in principle, we can pull the brane anywhere. And now, I'm out of time. So let me just say, if you look at this potential, this becomes 0 precisely when phi a and phi b all become commutes.

So that means we can diagonalize the phi corresponding to all the transverse directions. You can diagonalize phi corresponding to all the transverse directions. And then they correspond to the location you put all your branes. Go do your p-set, and you will see a more explicit discussion of this.

And then one final remark. At the beginning, you started with n branes together. And then because of this, we can separate them. So we can find a solution which, say they commute. We separate them into different stacks, n1, n2, n3, et cetera.

So this corresponds to the configuration of phi. So let me just write all phi as a vector. Say there's a1, n1 of them at location a1, and n2 of them at location a2, and ak. If I separate them into n stacks, then will be like this. So n1 of them at location a1, and n2 of them at location a2.

You can check. So this is n1 times n1 identity matrix. This is n2 times n2. And you can check that such a configuration does satisfy this condition so that you actually can separate the brane into such configurations. And in this case, then the gauge symmetry is broken into u n1 times u n2 times u nk, because only those points [INAUDIBLE] parts survives. And all the other strings between them become massive. OK. Let's stop here.