

Last Compiled December 23, 2013

# Lectures on the Soft-Collinear Effective Theory

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EFT Course 8.851, SCET Lecture Notes  
Massachusetts Institute of Technology  
2013

(These notes are also part of a dedicated review with Christian W. Bauer.)

(The original version of these notes were typeset by Mobolaji Williams.)

# Abstract

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8.851 Effective Field Theory  
Spring 2013

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