$$\frac{d\sigma}{dM^2 d\overline{M}^2} = \sigma_0 H(Q,\mu) \int dl^+ dl^- \, J_n(M^2 - Q l^+_{,\mu}) \, J_{\bar{n}}(\overline{M}^2 - Q l^-_{,\mu}) \, S(l^+, l^-) \tag{9.8}$$

9.3 Perturbative Results

9.4 Results with Resummation

10 SCET II

(**ROUGH)** When soft gluons interact with collinear particles, the resulting particle has momentum $Q(\lambda, 1, \lambda)$ and is therefore off the SCET mass shell.

$$q = q_s + q_c \sim Q(\lambda, 1, \lambda) \to q^2 = Q^2 \lambda >> (Q\lambda)^2 \tag{10.1}$$

Consequenlty, these offshell particles can be integrated out of the theory. Analogous to our definition of the ultra-soft wilson line, we can define a soft wilson line $S[n \cdot A_s]$ resulting from integrating out the offshell particles.

$$S_n = \left[\sum_{\text{perms}} \exp\left(-g \frac{1}{n\cdot} \mathcal{P} n \cdot A_{s,q} \right) \right] \tag{10.2}$$

Aslo, similar to the usoft case, gauge invariance restricts the placement of factors of S in operators. For example, we use our canonical heavy to light (soft to collinear) current. under soft and collinear gauge transformations, the fields transform as

$$\text{soft: } h_v \to U_s h_v \qquad \xi_{n,p} \to \xi_{n,p} \tag{10.3}$$

$$\text{collinear: } h_v \to h_v \qquad \xi_{n,p} \to \mathcal{U}_{p-Q} \xi_{n,Q} \tag{10.4}$$

The fact that our standard current $J = \overline{\xi}_{n,p} W_n \Gamma^\mu h_v$ is not gauge invariant under the soft transformations suggests that it is an incomplete description of the physics of this process. We can make this current soft gauge invariant by including the soft Wilson line. The soft Wilson line S_n transforming as

$$S_n \to \mathcal{U}_s S \tag{10.5}$$

makes the current

$$J = \overline{\xi}_{n,p} W \Gamma^\mu S^\dagger h_v \tag{10.6}$$

gauge invariant. We may also build up this current by a diagrammatic analysis. Necessary to the procdure is the fact that only $n \cdot A_{us}$ component of the usoft gluon builds up S (EXPLANATION) and only the $\bar{n} \cdot A_{n,q}$ component of the collinear gluon build up W. The simplest diagram for soft- collinear coupling, where collinear and soft gluons take the quarks off-shell is given in

Diagram.

This diagram yields the current

$$\text{Fig ()} = -g^2 \frac{n^\mu}{n \cdot q_s} \frac{\bar{n}^\nu}{\bar{n} \cdot q_c} \overline{\xi}_{n,q} T^a \Gamma T^b h_v. \tag{10.7}$$

But we have a probelm. This current appears to have the color factors a and b in the wrong order. With a representing soft gluons and b representing collinear gluons this current appears to be derived form the

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