8.851 Homework 3

Iain Stewart, February 26, 2003

Problem 1) Mixing and 4-quark operators

a) Compute the divergent part of the one-loop graphs that renormalize the operators in Eq.(1.125) of the text and derive the matrix Z_{ij} discussed in class.

b) Verify that in the case $\mu \to e\nu_\mu\bar{\nu}_e$ the four fermion operator is not renormalized at order e^2. Explain why.

Problem 2) Dimension 6 Operators and Short and Long Distance Contributions

For each process determine which operator(s) give the dominant contributions. Then discuss the sensitivity to long and short distance contributions (feel free to use references, this problem is partly a reading assignment).

a) decay rate for $K \to \pi\nu\bar{\nu}$

b) mass and lifetime differences in $K^0 - \bar{K}^0$ mixing

c) decay rate for $B \to X_s e^+ e^-$

Problem 3) CP violation in $B \to \pi\pi$

a) Explain what symmetry forbids B decays to a $|\pi\pi\rangle_{I=1}$ state, where I is isospin. Define $|\pi^+\pi^-\rangle_S = (|\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle)/\sqrt{2}$ and a similar equation for $|\pi^0\pi^-\rangle_S$, then work out the decomposition of these states and $|\pi^0\pi^0\rangle$ in terms of two pion states with $I = 0$ and $I = 2$.

b) Using isospin classify the QCD penguin and current-current operators for $B \to \pi\pi$. Next using your results in a) determine which operators can contribute for decays to the different charged states.

c) Ignoring the QCD penguin operators determine which weak phase is found by measuring $\operatorname{Im}\lambda$ for $B \to \pi\pi$, with λ as defined in class.

d) Add penguin contributions and explain what this does to your result in part c). What other information do we now need to know to extract the weak phase in c)?