

8.851 Homework 5

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Problem 1) Renormalization of $c_F(\mu)$

Draw the diagrams needed to compute the anomalous dimension of the coefficient $c_F(\mu)$ which appears in the magnetic moment Lagrangian $\mathcal{L}_F^{(1)}$. Discuss whether the kinetic energy Lagrangian $\mathcal{L}_K^{(1)}$ mixes with $\mathcal{L}_F^{(1)}$ under renormalization. Argue that the anomalous dimension vanishes in the abelian case (and therefore is proportional to the adjoint Casimir C_A) without computing any integrals. (Hint: think about Coulomb gauge.)

Problem 2) Heavy-to-Light Currents in HQET

Consider the $\mathcal{O}(1/m_Q)$ heavy-to-light vector currents

$$\begin{aligned} O_1 &= \bar{q} \gamma^\mu i \not{D} Q_v, & O_4 &= \bar{q} (-iv \cdot \overleftarrow{D}) \gamma^\mu Q_v, \\ O_2 &= \bar{q} v^\mu i \not{D} Q_v, & O_5 &= \bar{q} (-iv \cdot \overleftarrow{D}) v^\mu Q_v, \\ O_3 &= \bar{q} i D^\mu Q_v, & O_6 &= \bar{q} (-i \overleftarrow{D}^\mu) Q_v, \end{aligned} \tag{1}$$

with coefficients B_1 to B_6 . Using reparameterization invariance determine which of these coefficients are fixed by the coefficients C_1 and C_2 of the leading order vector heavy-to-light currents $\bar{q} \gamma^\mu Q_v$ and $\bar{q} v^\mu Q_v$.

Problem 3) Non-perturbative form factor corrections

In Chapter 4 of the book do problem 3 parts a), b), and c). Feel free to use results given in problems 6-9 in Chapter 2.