8.851 Homework 7

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Problem 1) One-loop Exact β-function in NN EFT

In lecture we computed the one-loop diagram with two $C_0(\mu)$ insertions in the offshell momentum subtraction scheme (OS) and the PDS scheme and determined the one-loop counterterm $\delta^1 C_0 \propto \mu C_0(\mu)^2$. From this we computed the beta function by differentiating the explicit μ, but not the implicit μ in C_0. To prove this is correct requires computing the counterterms for the entire bubble sum chain of C_0 insertions. Lets check that this statement is consistent at two loops: Compute the two-loop counterterm $\delta^2 C_0$ in either the OS or PDS scheme and consider how it effects the β-function. Note that the two-loop graph is not hard, its just a product of one-loop diagrams.

Problem 2) Deuteron Electromagnetic Form Factor

a) In lecture we wrote down the LSZ formula for a bound state form factor and then derived a result in terms of irreducible diagrams. Repeat the steps and convince yourself of the validity of each one. In particular derive the result that the bound state Z factor is given by $i\Sigma^2/(d\Sigma/dE)$ at the pole, where Σ is the irreducible 2-pt function.

b) [Extra credit, but not required.] Verify the computation of the two irreducible one-loop diagrams whose answers I wrote down and combine them to get the form factor. Feel free to use the literature quoted on the website.

Problem 3) Counting operators with Group Theory

The nucleon has two degrees of freedom for spin and two for isospin, so lets think of it as a fundamental in SU(4). Our EFT might not be SU(4) symmetric, but we can still classify all operators by their SU(4) transformation properties. The nucleon is a fermion so we need antisymmetric products of fields in our operators which restricts the allowed representations. Combine the representations for two N's and two $N^\dagger$'s with no derivatives and count the number of operators from counting the representations. (Of these the 15 is actually ruled out by noting that it has no spin-singlet SU(2) subgroup.) Repeat for three body operators (three N's, and three $N^\dagger$'s) and then for four body operators (four N's, and four $N^\dagger$'s). [Hint: the last one is the easiest.]