

1.8 2D non-axisymmetric potentials

For orbits in a non-axisymmetric potential, no component of the angular momentum is conserved. There are nonetheless two distinct families of orbits that contribute to the shape of the galaxy. The families are generated by stable, closed *parent orbits*.

Starting at any point on the long axis of the galaxy, one can launch particles perpendicular to that axis. In general these will generate rosettes that do not close upon themselves. But for exactly one value of velocity at every position on the long axis there is an orbit which returns to that point on the axis with the same velocity – it closes on itself, producing an orbit that is roughly elliptical (see Figure 1.21). Small perturbations to this orbit produce orbits that fill a narrow ribbon straddling this orbit. Such orbits are called *tube orbits*. As the perturbations increase, the width of the ribbon increases, but the sign of the angular momentum of the orbit does not change.

A second family has a parent that oscillates back and forth on the long axis. Small perturbations to this parent produces orbits that fill a bow or box shaped region along the long axis. Bigger perturbations produce wider bows and boxes. But the angular momentum is relatively small and changes sign frequently. These are called *box orbits*.

Box orbits (sometimes called *centrophilic*) are characterized by an angular momentum that changes sign, while tube orbits (*centrophobic*) always have L_z that does not change sign. Box orbits *must* be present for a disk to be non-axisymmetric, since the tube orbits are elongated counter to the elongation of the potential.

In addition to these major families, there are also minor families. The parent closed orbits for these families are characterized by the stars coming to a complete halt and reversing direction. The names for these families are particularly apt: banana orbits and fish orbits look like bananas and fishes. Small perturbations from the parents produce similarly shaped orbits, but larger perturbations produce boxes and tubes.

A further complication can be added if the non-axisymmetric potential is allowed to tumble. Barred galaxies are thought to be such tumbling non-axisymmetric shapes.

1.8.1 3D non-axisymmetric potentials

The most straightforward case of a 3D non-axisymmetric potentials would be one that was ellipsoidal, with three unequal axes. Such potentials have three major families of orbits. There are *short-axis tubes* whose angular momentum is roughly aligned with the short axis, and *long-axis tubes* whose angular momentum is roughly aligned with the long axis. The projection of the angular momentum

along these axes does not change sign. There are also box orbits, for which the projection of the angular momentum along each of the three axes does change sign. The tube orbits are generically torii. The box orbits are quite boxy.

Stäckel potentials are remarkable in being triaxial but admitting for closed form descriptions of its orbits in terms of confocal ellipsoidal coordinates. Stäckel potentials are useful for studying the qualitative nature of orbits but less useful for constructing realistic models.

There is overwhelming evidence for central mass concentrations at the centers of galaxies that are thought to be black holes. These produce cusps at centers of potentials that destroy box orbits. It may be that galaxies with large central black holes cannot be triaxial.

As with 2D non-axisymmetric potentials, 3D potentials may tumble, adding another degree of complication.