17.812J / 14.296J Collective Choice I
Fall 2008

Problem 3.A

Consider the citizen-candidate model developed by Besley&Coate(1997), with the following assumptions. There are N voters (N odd). The set of policy alternatives is the unit interval $[0, 1]$. Each citizen i has Euclidian preferences over these alternatives with distinct ideal point ω_i and cares only about policy outcomes. Thus, for all i, the utility of i from policy x when citizen j is elected is:

$$V^i(x, j) = -|\omega_i - x| \tag{2}$$

Let $m = 1/2$ be the ideal point of the median individual.

Also, assume that there are only three potential candidates (C_1, C_2, C_3), where the ideal points are, respectively, $\omega_1, \omega_2, \omega_3$, with $\omega_1 < 1/2 = \omega_2 < \omega_3 = 1 - \omega_1$. These potential candidates may decide to run for office at a utility cost of $\delta < 1/2$. Voters have one vote (which they may abstain from using). The candidate who receives the most votes is elected, and in the event of ties, the winning candidate is chosen with equal probability from among the tying candidates. If only one candidate runs, he is automatically elected. If no candidate runs, a default policy x_0 is implemented. Assume here that $x_0 = 0$.

As Besley&Coate do, restrict attention to strategies that are not weakly dominated (p.89); and feel free to use assumptions 1 and AIV if necessary (pp.99,100).

Consider the following timing of the game:

1. Potential candidates simultaneously decide whether to enter the election
2. Citizens vote

We are looking for a Subgame-Perfect Nash Equilibrium in pure strategies.

Question 1

Characterize the subgame-perfect Nash Equilibria of this game (with sufficient conditions, a la Besley&Coate(1997, p.93), for an equilibrium with two entrants). In particular, find conditions on ω_1, if any exist, such that it is an equilibrium for candidates 1 and 3 to run, and for candidate 2 not to run.

Question 2

Revise the timing as follows:

1. Potential candidate 1 decides whether to run
2. Potential candidate 2 decides whether to run
3. Potential candidate 3 decides whether to run
4. Citizens vote

Characterize the subgame-perfect Nash Equilibria of this game (with sufficient conditions, a la Besley&Coate(1997, p.93), for an equilibrium with two entrants). In particular, find conditions on ω_1, if any exist, such that it is an equilibrium for candidates 1 and 3 to run, and for candidate 2 not to run.

Question 3

Revise the timing as follows:

1. Potential candidate 1 decides whether to run
2. Potential candidate 3 decides whether to run
3. Potential candidate 2 decides whether to run
4. Citizens vote

Characterize the subgame-perfect Nash Equilibria of this game (with sufficient conditions, a la Besley&Coate(1997, p.93), for an equilibrium with two entrants). In particular, find conditions on ω_1, if any exist, such that it is an equilibrium for candidates 1 and 3 to run, and for candidate 2 not to run. Comment on your answers to questions 1 through 3.