Second Problem Set

1. Two firms dominate the international market for metal coatings. These are DuPont and BASF. Each has similar costs and production facilities. Each must set its production (measured in millions of tons) several months in advance so that the strategic variable for each is quantity. The BASF profit is given first in each payoff pair.

		DuPont Production			
		30	40	50	60
BASF Production	30	$1700, $1700	$1300, $1900	$1000, $2000	$950, $1850
	40	$1900, $1300	$1500, $1500	$1200, $1600	$1100, $1500
	50	$2000, $1000	$1600, $1200	$1300, $1300	$1000, $1200
	60	$1850, $950	$1500, $1100	$1200, $1000	$900, $900

    a. What is the Nash Equilibrium output for this game assuming that the two firms choose their production quantities simultaneously?

    b. What would be the equilibrium if DuPont could choose its output first and BASF chose second, taking DuPont's decision as given?

2. Oldton, Massachusetts organizes a children's soccer league for 12 weeks every fall. In turn, operation of this league requires that parents volunteer ahead of time (in the previous Spring) to act as a coach for a team. The more coaches, the smaller each team can be and the more playing time a child will have. However, coaching young children is both arduous and timing. The burden can be lightened though if other parents help the coach out. Rick Dans has coached for eight years. His neighbor, Faith Liss, has a soccer-age son and very much wants Rick to be his coach. However, Rick has to make the decision now (in the Spring) about whether to coach or not by signing a contract with the Oldton Soccer League. In an effort to coax Rick into coaching, Faith promises Rick that she will help him with all the soccer practices next fall. If Rick does not coach, then of course Faith does not need to help out. In this case, the payoffs (say in utility units) to Rick and Faith each next fall will be 0 and 0, respectively. If, however, Rick does coach and if Faith keeps her word, each gets a payoff of 2 units. On the other hand, if Rick commits to coaching and Faith does not keep her word, Rick is burdened with coaching without help and this gives him a payoff of –1. Meanwhile, Faith has a coach for her son and is free to pursue other activities in which case she earns a payoff of 3. Rick will not coach unless he thinks Faith will help.

    Evaluate this interaction as a three-stage game. In the first stage, Faith promises either to help or does not make such a promise. In the second stage, Rick either commits to coach or does not. In the final stage, Faith decides whether or not to help Rick. Given the game's structure, what is the likely (Nash) outcome of the game? Is Faith's initial promise to help credible? If not, what might she do to make it more believable?

3. Suppose that it is known in advance that, given the number of drivers/cars on the road, there will be 1000 crashes (one car hitting another) each year in the state of Massachusetts. Suppose further that the probability of being involved in any such crash is the same for all drivers/cars. However, the damage and cost of a crash is different depending on the cars involved. For simplicity, assume that there are two types of cars: Large ones and small ones. Large cars have the advantage that they offer a lot of protection to drivers. They have the disadvantage however that they are expensive to repair. A crash that involves two small cars typically results in a loss (injury and car damage) of $1,100 for each car. A crash that involves two large cars typically results in a loss of $2,500 for each car. Finally, in a crash that involves one small car and one large car, the driver of the large car suffers damage of $1,800, but the driver of the small car is at such a disadvantage that she actually

does even worse and incurs a cost of $3,000. Each driver has to think about what kind of car she wishes to drive recognizing that there is a given chance that she will be in an accident independent of her driving skills.

a. Treating the choice of car as the strategic choice variable, construct the payoff matrix for two drivers depending on the cars that each drives if a crash occurs. Use this matrix to identify the Nash equilibrium (or equilibria) that may exist in this game.

		Driver 2	
		Small Car	*Large Car*
Driver 1	*Small Car*	-$1100, $-1100	-$3000, -$1800
	Large Car	-$1800, -$3000	-$2500, -$2500

b. Based on your analysis, do you foresee any potential inefficiencies in the choice of automobiles? Explain.

4. The city of Cambridge has a problem with illegal parking. Currently, the city has a fine of $20 for illegal parking. But it only catches about 1/3 of the offenders. It is debating two proposals to deter illegal parking more effectively. One proposal is to raise the fine to $40, but to leave the probability unchanged at 1/3. The other proposal is to leave the fine at $20, but to hire more meter-readers and raise the rate of apprehension to 2/3. Assume that all offenders are risk-averse.

a. Evaluate the two policies. Which one do you think will have the greatest deterrent effect on tax cheating? Explain.

b. Which one do you think will be the most costly to implement? Explain.

5. Tiger-el is an upstream manufacturer of model electric trains that sells wholesale to The Great Toy Store (TGTS), the only store selling model trains in the area. Demand for the trains at the retail store level in inverse form is: $P = 1,000 – 2Q$, where P is the retail price of trains and Q is the total number of trains sold. The Great Toy Store incurs no service cost in selling the train. Its only cost is the wholesale price it pays for each train. Tiger-el incurs a production cost of $40 per train.

a. What wholesale price should Tiger-el charge for its trains? What price will these trains sell for at retail? How many trains will be sold?

b. What profit will Tiger-el and the retailer earn under the pricing choices found in part (a)?

b. How would the quantity of trains sold and the retail price change if Tiger-el merged with TGTS and sold the trains directly to the public through what is now its own subsidiary retailer? What would be the combined profit of the two firms now?

6. H. Simpson has a utility function defined over his level of overall wealth or general purchasing power, W as follows. $U = \sqrt{W}$. Currently, his net worth or wealth is $1,000,000. However, he now has a chance to make an investment of $200,000 in a bio-engineering firm, Bovinox, that is run by A. Fastou. Bovinox is in a tight race to develop a new gene therapy for fighting multiple sclerosis. If the company wins the race, Mr. Simpson will get back all of his $200,000 plus an additional $220,000 more bringing his total net worth to $1,220,000. If the company loses the race, Mr. Simpson will lose his $200,000 and have a net worth of only $800,000. The race will probably be decided in the next two weeks. Simpson reckons that Bovinox has a fifty-fifty chance of developing the gene therapy first.

a. If Mr. Simpson makes the investment, what is his expected net worth? What is his expected utility level?

b. Should Mr. Simpson make the investment?

7. Art Objektz owns a small camera shop and employs two people to perform much of the basic services. Each employee could stay unemployed and enjoy leisure which is worth €1,000 per month. Each employee also must choose between working hard putting forth effort (e = 1), and taking it easy (e =0). For each, an additional cost of €1,000 is incurred when she works hard (e = 1). The revenue generated by each person depends on her effort ***and*** on luck. If an employee takes it easy, (e = 0), and has bad luck (which happens with probability ½), the employee's work will generate €1000 per month. However, if an employee has good luck (again, probability ½) then even with low effort revenues will be €4,000 per month. What happens when an employee instead works hard (e = 1)? In this case, if she is unlucky, her hard work will again generate revenues of €4,000 per month. However, if an employee works hard and has good luck, she will now generate revenues of €9,000 per month. These outcomes are described in the table below.

		REPAIR SHOP REVENUE	
		Bad Luck (p=½)	Good Luck (p=½)
WORKER'S EFFORT	Low (e=0)	€1,000	€4,000
	High (e=1)	€4,000	€9,000

Art cannot observe either effort or luck. However, both he and the employees know that if luck is good (bad) for one worker it is also good (bad) for the other. Art has therefore adopted the following compensation scheme. He will pay each employee a base pay of €1,000 per month regardless of what happens in the store. However, at the end of each month he will also pay an additional €3,000 to that employee who generated the highest revenue. In the case of a tie, Art will flip a coin and the winner will get the €3,000.

   Evaluate this scheme. Will the employees be willing to work under this arrangement? If so, what level of effort employees put forth if they are risk neutral? What will be Art's expected profit?

8. Let there be three consumer types, 1, 2, and 3, and three kinds of widgets, red, blue, and yellow. Consumer type 1 values red widgets at $40, yellow widgets, at $20 and blue widgets at $15. Type 2 values red widgets at $15, yellow widgets at $40, and blue widgets at $20. For type 3, the respective valuations are $20, $15, and $40. Each consumer incurs a transport cost of $5.01 to visit the store. In other words, the willingness to pay of each consumer is as shown in the table below.

		Consumer Type		
		Consumer 1	Consumer 2	Consumer 3
Widget Type	Red	$40	$15	$20
	Yellow	$20	$40	$15
	Blue	$15	$20	$40

Once a consumer actually visits a store, the transport cost is sunk. The store incurs zero cost per widget but has only red widgets.

   Consider two advertising strategies. In the first, the store advertises that it has red widgets. In the second, the store advertises simply that it has widgets. Which strategy will the store prefer? Explain.

9. In the 1970's and 1980's, car theft was a major crime in the United States. Much of this illegal activity was part of an organized effort in which cars stolen cars were driven to "Chop Shops" and turned into about valuable components or given false new registration numbers and shipped overseas. Since 1991, however, auto theft has declined every year and is down about a 50% over the last decade. Many credit this decline to the use of Vehicle Recovery Systems (VRS) such as that marketed by LoJack (of Dedham, MA). These systems essentially make use of a homing device planted secretly in the car. As soon as the car is reported stolen, police can use a

transmitter to locate the vehicle's location. This has led to the recovery of millions of vehicles. More importantly, the homing device has often led police directly to the "Chop Shop" itself thereby enabling authorities to close the entire operation down. The elimination of these criminal organizations is of course good news for all car owners whether they have a LoJack type system or not. Demand for Vehicle Recovery Systems is estimated to be: $P = 4,800 - 0.3Q$, where P is the price per system and Q is the number (in thousands) of systems sold. That is, if Q equals 10,000 it really means 10 million systems were purchased. The cost of producing and maintaining a VRS is constant at about $600.

    a. What will be the price of a VRS and how many will be sold if the market is competitive?

    b. Is the competitive equilibrium described above likely to be efficient? Explain.

10. One thousand people live in Waterford, a small New England town with a water reservoir and pumping station. The total cost of delivering water is: $TC = Q^2$. Marginal cost is given by: $2Q$. Average cost is given by: Q. Here, Q is measured in thousands of gallons per week. Weekly water demand is given by: $P = 1200 - 4Q$ (where Q is again measured in thousands of gallons and P is the price per thousand gallon unit). The Waterford Water Board (which is hard to say) must set the same price for each gallon sold.

    a. What price should the Water Board set if it wants to maximize economic efficiency? How much water will it sell at this price? Explain.

    b. What price should the Water Board set if it wants to maximize the profit from selling water? How much water will it sell at this price? Explain.

    c. What price should the Water Board set if it wants to maximize the amount of water consumed while still covering its costs? How much water will it sell at this price? Explain.

    d. Over time, the Water Board vacillates over each of the three policies described above. Can you explain this outcome?

11. The central input used by Matrix Enterprises in manufacturing its warp drives is the dilithium crystal. The revenue that Matrix earns from these drives is such that it is willing to pay as much as $10,000 per crystal and it will buy at this price or any lower price, exactly 10 crystals. Currently, the only manufacturer of such crystals is Aloca. Aloca can manufacture dilithium crystals at a cost of $5,000 apiece. However, there is a possibility that a new firm, Rexull, will enter into the production and marketing of dilithium crystals next period. Rexull is hard at work right now trying to develop a production process that is cost efficient. The outcome of this process is, as noted, probabilistic. To be specific, Rexull's cost per crystal as the result of its research is uniformly distributed on the interval, 0 to $10,000. In other words, there is a 10% chance that it will develop a process to produce crystals at a cost of less than $1,000; a 100% chance that it will develop a process to produce crystals at a cost between $1,000 and $2,000; and so on.

    a. Assume that if Rexull does not enter the market, Aloca can extract the entire surplus from Matrix and sell at a price of $10,000 per crystal. Assume as well that Rexull knows Aloca's cost and will only enter if its own cost is below $5,000. In light of the probabilities associated with Rexull's research, and given the Matrix will buy crystals from the firm with the lowest price, what is the actual probability that Rexull will be able to enter the market successfully next period?

    b. If Rexull does enter, what price will it charge? (Think carefully.)

    c. Given your answers to 11a) and 11b) above, what price should Matrix expect to pay for each of its 10 dilithium crystals next period? What profit should Aloca expect to earn next period?

12. Return to Problem 11 above. Suppose now that Aloca offers Matrix a "long-term" contract. Specifically, suppose Aloca offers to sell Matrix dilithium crystals next period at a guaranteed price of $7,500 apiece. However, once signed, the contract is binding. Matrix must buy the 10 crystals and if it does not, it must pay a penalty of $5,000 per crystal ($50,000 overall).

    a. What is now the probability that Rexull will successfully enter the dilithium crystal market next period? What price will it charge Matrix if it does enter?

    b. Relative to the initial, no-contract scenario, is Matrix worse off, better off, or just as well off if it signs the contract? Relative to the initial scenario, does Aloca do better or worse if Matrix signs the contract?