

# Solution to Practice Exam, 15.040 Spring 2004

## **Problem 1**

*Problem 1 on the final exam, will be very similar to this, though the conditions will be altered.*

### **Thoughts on how to approach this problem:**

Problem 1 has two parts. First, you must come up with a game having certain features. Second, you must analyze the equilibria of this game. The second part is more straightforward, since you can use the blueprint provided by the analysis of examples in the textbook, in the Lecture Note on Strategic Substitutes and Complements, and in the lecture slides.

How then to approach the first part? The question asks us to think of a game in which (a) both players regard the strategies as strategic complements and (b) player A wants player B to be more aggressive whereas (c) player B wants A to be less aggressive. One natural way to approach this problem is to think first of games we have seen in which both players regard the strategies as strategic complements, and then to think about how a simple variation on one of those games might satisfy the other conditions. In this case, the Cooperation Game from the Lecture Note is an example in which both players view the strategies as strategic complements, though both players prefer that the other be more aggressive (i.e. do more work).

Is it possible to plausibly modify the Cooperation Game so that one of the players prefers the other to be less aggressive, while keeping the strategic complementarity? Strategic complementarity in this game means that each worker wants to work hard if the other works hard but not otherwise. In the original Cooperation Game, the reason for this is that each worker is only willing to put in effort if that will lead to the job getting done. Thus, you prefer that the other player works harder since then more of the task gets

done. But another natural reason to work only if the other player works is to avoid looking bad to the boss. In that case, you will prefer that the other player works less since then you will look relatively better. Let's try to create a very simple game based on this idea. (Having a simple game, obviously, simplifies the analysis in the second part.)

## Solution

*The Game: "Working with a Slacker".* Two workers. Each has a simple choice, Work or Not. If both Work the job gets done well, but if one or both of them do not work then the job gets done poorly. Worker **A** (Alice) only wants to work if doing so will lead to a well-done job. In particular, having a well-done job is "worth 10" and working "costs 5". Worker **B** (Bob) only wants to avoid looking bad. In particular, while working "costs 5", being the only one not to work "costs 10". (Note that, holding her own action fixed, Alice prefers for Bob to work harder whereas, holding his action fixed, Bob prefers that Alice work less.<sup>1</sup>) Putting this together, we get the payoff matrix

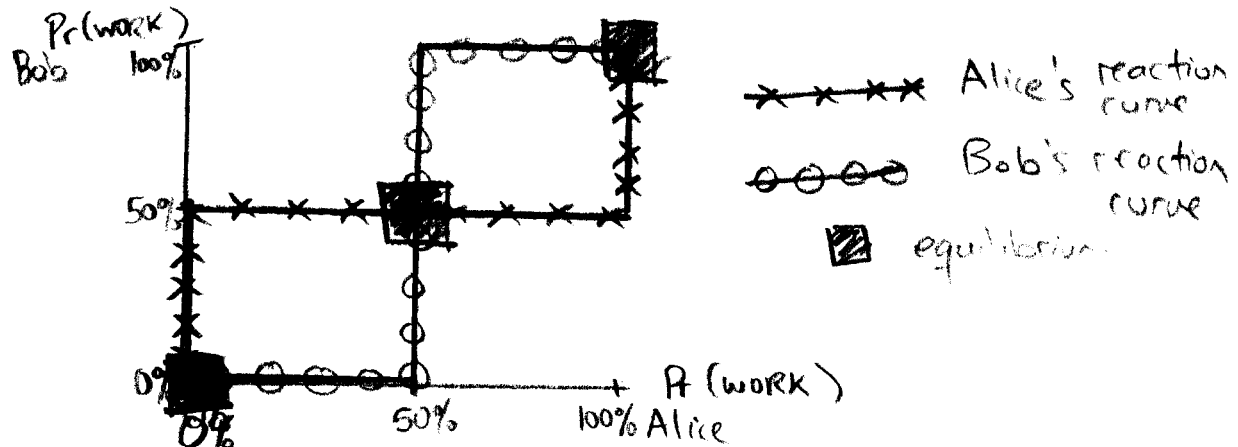
		Bob	
		WORK	NOT
Alice	WORK	5, -5	-5, -10
	NOT	0, -5	0, 0

(a) Reaction curves. For Alice:  $R_A(WORK) = WORK$  and  $R_A(NOT) = NOT$ . For Bob:  $R_B(WORK) = WORK$  and  $R_B(NOT) = NOT$ .

(b) Nash equilibria given simultaneous moves. Clearly, (WORK, WORK) and (NOT, NOT) are Nash equilibria. What about mixed strategies? By working, Alice is "risking 5 (payoff -5 rather than 0) to gain 5 (payoff 5 rather than 0)" so for her to be indifferent Bob must work 50% of the time. Similarly, by working Bob is "risking 5 (payoff -5 rather than 0) to gain 5

<sup>1</sup>In the Practice Final and in the Lecture Note on Strategic Substitutes and Complements, I routinely refer to games in which "player A wants player B to be more aggressive", etc. This means that, holding player A's action fixed, player A is never worse off (and sometimes better off) when player B chooses a higher ("more aggressive") strategy.

(payoff -5 rather than -10)" so he is indifferent when Alice works 50%. So, (WORK 50%, WORK 50%) is the (only) mixed-strategy equilibrium. We can also see the three equilibria by drawing the players' reaction curves.



(c) Subgame-perfect equilibria given sequential moves. *When Alice goes first:* As the follower, Bob's strategy is determined by his reaction curve: WORK if Alice WORKs and NOT if Alice NOTs. Given this, Alice chooses to WORK since she prefers (WORK, WORK) over (NOT, NOT). *When Bob goes first:* As the follower, Alice's strategy is determined by her reaction curve: WORK if Bob WORKs and NOT if Bob NOTs. Given this, Bob chooses NOT since he prefers (NOT, NOT) over (WORK, WORK).

*Note:* to be fully correct, you must specify a player's action at every one of his information sets. For example, when Alice is leader Bob has two information sets: (1) Alice WORKs and (2) Alice NOTs. In equilibrium, we don't observe what Bob was planning to do if Alice didn't work, but his plan of action in that unrealized event is an important part of the equilibrium.

(d) Preferences over moving first vs. last. *For Alice:* When Alice moves first, we have the outcome (WORK, WORK), whereas when Alice moves last we have the outcome (NOT, NOT). Thus, moving first is better than moving last. *For Bob:* When Bob moves first, we have the outcome (NOT, NOT), whereas when Bob moves last we have the outcome (WORK, WORK). Thus, moving first is better than moving last.

The results of the previous paragraph are to be expected. Take Alice: since Bob views the strategies as strategic complements and Alice wants Bob to work more, we know that she will tend to work more as the leader than when moving simultaneously. This makes Bob worse off and explains why he

prefers not to be the follower. Similarly, since Alice views the strategies as strategic complements and Bob wants Alice to work less, we know that Bob will tend to work less as the leader than when moving simultaneously. This makes Alice worse off and explains why she prefers not to be the follower.

## **Problem 2**

*Problem 2 on the final exam will be similar to this, but may involve a different type of game.*

### **Solution**

(a) This is an example of the Chicken Game; see Slides for Lecture #5 and the reading in the textbook. We know that there are two pure strategy equilibria, (Back Off, Fight) and (Fight, Back Off), as well as a mixed strategy equilibrium. What are the fighting probabilities in this mixed-strategy equilibrium? Each Elk is "risking 30% (80% - 50% if the other fights) to gain 10% (100% - 90% if the other backs off)". So to be indifferent between fighting and backing off, the probability of fighting must be 25%. (Because  $25\%/75\% = 1/3$ , the opposite ratio of the risk and reward from fighting.) Thus, the mixed strategy equilibrium is (25% Fight, 25% Fight).

(b) Since this is the Chicken Game, we know from Lecture that the answer to this question depends on whether male Elk evolve as one population or as two populations. What does this mean? Remember that evolution works by rewarding players who do relatively well compared to an average population, but what population is that? If it is the population of all players (both those who act as player *A* and those who act as player *B*), then there is one population. If it is just the population of players in the same role (only the player *A*'s if you are player *A*) then there are two populations. In this case, male elk are successful evolutionarily if they have more offspring than other male elk – which elk "acts as player *A*" doesn't matter. So, there is evolution of just one population. In this case, theory tells us that *only the mixed strategy equilibrium is evolutionarily stable*.

## **Problem 3**

*Problem 3 on the final exam will be similar to this, in that it will be based on a real (or "real") business strategy problem for which you will be asked to*

*provide strategy advice.*

## **Solution**

There are several issues that might be raised in the solution to this problem. I will focus on surprise and how strategic substitutes and complements relate. (See Lecture Note on Strategic Substitutes and Complements as well as the slides for Lecture 6.)

In our firm's future competition with Sony, we want Sony to be less aggressive. (In the terminology introduced in Lecture 6, our competition with Sony is a "competitive game" rather than a "reliance game".) Suppose first that this game is one in **strategic substitutes**. From page 14 of the Lecture Note, then, we prefer moving simultaneously to surprise. Since we also prefer moving first to moving simultaneously (page 11), it's clear that we should reveal ourselves and - if possible - commit to our strategy as soon as possible.

What if the game is one in **strategic complements**? Here we know that we prefer surprise to moving simultaneously. The only question, then, is whether surprise is better than leadership as well. This question is not addressed in the Lecture Note, so to make progress we need to think about why leadership is better than moving simultaneously. As the leader, we will commit to a strategy that is less than our Nash equilibrium strategy, thereby inducing Sony to also play a strategy that is less than his Nash equilibrium strategy. But wait! In a world with surprise, Sony acts as if we have committed to the least aggressive strategy of all! There is no way, through leadership, for us to induce Sony to play a less aggressive strategy than what they are already planning to do. On the other hand, surprise gives us the flexibility to play a best response to Sony's strategy (rather than committing to a strategy that is not a best response). Putting this together, surprise is definitely better than either moving simultaneously or moving first when future competition is in strategic complements.

## **Further discussion of strategic substitutes / complements**

This is all well and good, but it is very abstract. What will future competition be in, strategic substitutes or complements? First, some generic examples of types of competition that tend to fit into either category:

- *Compete on capacity/quantity*: Tends to be strategic substitutes. Example: Georgia Pacific and other lumber companies decide how much capacity to build to make processed lumber products, and then produce up to capacity, selling what they make at the market price.
- *Compete on price*: Tends to be strategic complements. Example: FedEx and other package delivery services set their own prices and then delivery packages for all customers who choose them.
- *Compete on advertising*: Tends to be strategic complements. Example: Pfizer (Viagra) and other makers of drugs for erectile dysfunction attract business through extensive marketing campaigns aimed at doctors as well as campaigns aimed at potential patients.
- *Compete on research*: Can go either way. In a winner-take-all patent race, for instance, the nature of competition depends on how close the race is. When the race is close, each player tends to view the strategies as strategic complements ("I can't let them take the lead"). When one player is way ahead, however, the player who is behind tends to view them as strategic substitutes ("why put in effort if other is putting in huge effort") while the player who is ahead tends to view them as strategic complements ("I must maintain my lead"). For examples of competition on research, think not only of the pharmaceutical market but the film industry, the toy market, etc...

We have a student from Sony in our class, who can answer this question better than I can, but here is my stab at an analysis. In today's market, Sony, Nintendo, and Microsoft's short-run competition seems to be mainly in prices and in advertising, which would correspond to strategic complements. The problem, however, asks us to think about announcing entry that is five years away = what's important for this is not the nature of short-run but of long-run competition. Here, I would say that they compete primarily on research and that this research is always a "close race", i.e. probably best viewed as being in strategic complements. Consequently, the firm should not announce its intentions at this stage. Not surprisingly, this advice has a very natural intuition: "if we announce our future entry, Sony will put its research engine into overdrive and reduce our advantage by developing more new games, etc., even if they can't match our quantum-tunnel production technology".