

# Optimization Methods in Management Science

MIT 15.053, Spring 2013

## PROBLEM SET 1

SECOND GROUP OF STUDENTS (WITH FIRST LETTER OF SURNAMES **I–Z**)

DUE: FEBRUARY 12 , 2013

### Problem Set Rules:

1. Each student should hand in an individual problem set.
2. Discussing problem sets with other students is permitted. Copying from another person or solution set is *not* permitted.
3. Late assignments will *not* be accepted. No exceptions.
4. The non-Excel solution should be handed in at the beginning of class on the day the problem set is due. The Excel solutions, if required, should be posted on the website by the beginning of class on the day the problem set is due. Questions that require an Excel submission are marked with EXCEL SUBMISSION . For EXCEL SUBMISSION questions, only the Excel spreadsheet will be graded.

## Problem 1

(35 points total) The first problem asks you to formulate a 3-variable linear program in three different ways (four ways if you also count the algebraic formulation). Both of the first two ways are fairly natural. The third way is a bit obscure. And the algebraic formulation may seem overly complex. In practice, there are advantages to formulating linear programs in different ways. And there are huge advantages in the algebraic formulation. (One can express huge problems efficiently on a computer using a modeling language, which is based on the algebraic formulation.) In addition, formulating an LP in multiple ways provides insight into the LP models.

Accessories & co. is producing three kinds of covers for Apple products: one for iPod, one for iPad, and one for iPhone. The company's production facilities are such that if we devote the entire production to iPod covers, we can produce 6000 of them in one day. If we devote the entire production to iPhone covers or iPad covers, we can produce 5000 or 3000 of them in one day. The production schedule is one week (5 working days), and the week's production must be stored before distribution. Storing 1000 iPod covers (packaging included) takes up 40 cubic feet of space. Storing 1000 iPhone covers (packaging included) takes up 45 cubic feet of space, and storing 1000 iPad covers (packaging included) takes up 210 cubic feet of space. The total storage space available is 6000 cubic feet. Due to commercial agreements with Apple, Accessories & co. has to deliver at least 5000 iPod covers and 4000 iPad covers per week in order to strengthen the product's diffusion. The marketing department estimates that the weekly demand for iPod covers, iPhone, and iPad covers does not exceed 10000 and 15000, and 8000 units, therefore the company does not want to produce more than these amounts for iPod, iPhone, and iPad covers. Finally, the net profit per each iPod cover, iPhone cover, and iPad cover is \$4, \$6, and \$10, respectively.

The aim is to determine a weekly production schedule that maximizes the total net profit.

- (a) (5 points) Write a Linear Programming formulation for the problem. Start by stating any assumptions that you make. Label each constraint (except nonnegativity). For this first formulation, the decision variables should represent the proportion of time spent each day on producing each of the two items:

$$\begin{aligned}x_1 &= \text{proportion of time devoted each day to iPod cover production,} \\x_2 &= \text{proportion of time devoted each day to iPhone cover production,} \\x_3 &= \text{proportion of time devoted each day to iPad cover production.}\end{aligned}$$

(Different formulations will be required for parts (b) and (c).)

- (b) (5 points) Write a second Linear Programming formulation for the problem. Label each constraint (except nonnegativity). For this second formulation, the decision variables should represent the number of items of each type produced over the week:

$$\begin{aligned}y_1 &= \text{number of iPod covers produced over the week,} \\y_2 &= \text{number of iPhone covers produced over the week,} \\y_3 &= \text{number of iPad covers produced over the week.}\end{aligned}$$

The problem data is the same but you must make sure that everything matches the new decision variables.

- (c) (5 points) Write a third Linear Programming formulation for the problem. Label each constraint (except nonnegativity). Assume that each working day has 8 working hours. For this third formulation, the decision variables should be:

$$\begin{aligned}z_1 &= \text{number of hours devoted to the production of iPod smart covers in one week,} \\z_2 &= \text{number of hours devoted to the production of iPhone smart covers in one week,} \\z_3 &= \text{total number of production hours employed during the week.}\end{aligned}$$

Express the objective function in thousands of dollars. The problem data is the same but you must make sure that everything matches the new decision variables.

- (d) (5 points) What is the relationship between the variables  $z_1, z_2, z_3$  of part (c) and the variables  $x_1, x_2, x_3$  of part (a) of this problem? Give a formula to compute  $z_1, z_2, z_3$  from  $x_1, x_2, x_3$ .
- (e) (5 points) EXCEL SUBMISSION Solve the problem using Excel Solver, following the guidelines given in the Excel Workbook that comes with this problem set. Pay attention to the formulation in the Excel Workbook: it is similar to the one required for part (b), but it is not exactly the same.
- (f) (10 points) Write an algebraic formulation of the weekly production schedule problem described above using the following notation:
- $n$  is the number of product types,
  - $x_j$  is the number of days devoted to the production of products of type  $j$ ,
  - $p_j$  is the number of items of type  $j$  that can be manufactured in one day, assuming that the process is devoted to products of type  $j$ .

- $P$  is the number of production days in one week,
- $s_j$  is the storage space required by *one* item of type  $j$ ,
- $S$  is the total storage space available for the week's production,
- $r_j$  is the unit profit for each product of type  $j$ ,
- $d_j$  is the weekly maximum demand for an item of type  $j$ .
- $b_j$  is the weekly minimum demand for an item of type  $j$ .

## Problem 2

(10 points total). Problem 2 reviews the transformations from nonlinear constraints or objectives into linear constraints and objectives, as mentioned in the second lecture and discussed in the tutorial “LP Transformation Tricks”.

Identify which of the following optimization problems can be reformulated as a single Linear Program, and write the corresponding equivalent LP formulation. For those problems that cannot be reformulated, the answer should identify all the troublesome constraint(s) and/or objective.

(a) (5 points) Problem formulation:

$$\begin{array}{rcl}
 \min & \max\{2.3x_1 + x_2, 4.3x_1 - 0.5x_2, 2.5x_1 + 3.5x_2\} & \\
 \text{s.t.} & & \\
 \text{Constr1 :} & x_1/(x_1 + x_2) \leq 0.5 & \\
 \text{Constr2 :} & 10x_1 + 28x_2 = 3.4 & \\
 \text{Constr3 :} & x_1 + x_2 \geq 0 & \\
 & x_1 \text{ free} & \\
 & x_2 \text{ free} & 
 \end{array} \quad (1)$$

(b) (5 points) Problem formulation:

$$\begin{array}{rcl}
 \min & |0.8x_1 + 0.9x_2| & \\
 \text{s.t.} & & \\
 \text{Constr1 :} & |0.9x_1 + 1.2x_2| \leq 10 & \\
 & x_1 \geq 0 & \\
 & x_2 \text{ free} & 
 \end{array} \quad (2)$$

## Problem 3 (Second group of students)<sup>1</sup>

(55 points total) Charles Watts Electronics manufactures the following six peripheral devices used in computers especially designed for jet fighter planes: internal modems, external modems, graphics circuit boards, USB memory stick, hard disk drives, and memory expansion boards. Each of these technical products requires time, in minutes, on three types of electronic testing equipment as shown in the following table:

<sup>1</sup>This problem is based on Problem B.29 of Operations Management by Heizer and Render (2010).

	<b>Internal Modem</b>	<b>External Modem</b>	<b>Circuit Board</b>	<b>USB Stick</b>	<b>Hard Drives</b>	<b>Memory Boards</b>
Test device 1	7	3	12	6	18	17
Test device 2	2	5	3	2	15	17
Test device 3	5	1	3	2	9	2

The first two test devices are available 130 hours per week. The third (device 3) requires more preventive maintenance and may be used only 100 hours each week. Watts Electronics believes that it cannot sell more than 2000, 1500, 1800, 1200, 1000, 1000 units of each device, respectively. Thus, it does not want to produce more than these units. The table that follows summarizes the revenues and material costs for each product:

<b>Device</b>	<b>Revenue per unit sold (\$)</b>	<b>Material Cost per unit (\$)</b>
Internal modem	200	35
External modem	120	25
Circuit board	180	40
USB memory stick	130	45
Hard disk drive	430	170
Memory expansion board	260	60

In addition, variable labor costs are \$16 per hour for test device 1, \$12 per hour for test device 2, and \$18 per hour for test device 3. Watts Electronics wants to maximize its profits.

- a) (10 points) Write a linear program for this problem.
- b) (10 points) EXCEL SUBMISSION Write a spreadsheet for the problem and solve the problem using Excel Solver, following the guidelines given in the Excel Workbook that comes with this problem set. (Hint: the optimal value is \$211666.9 .)
- c) Use the Excel spreadsheet to answer the following questions:
  - (i) (4 points) What is the value of an additional minute of time per week on test device 1? Test device 2? Test device 3? Should Watts Electronics add more test device time? If so, on which equipment?
  - (ii) (4 points) Suppose that Watts Electronics is considering to increase the availability time of test device 2 for the next week. What would be the increase in the profit if the availability time increases to  $t$  for  $t = 131, 132, \text{ and } 133$ . (Assume that there are still 130 hours of test device 1 and 100 hours of test device 3.) The increase is the difference between the new profit and the profit from Part (a).
  - (iii) (4 points) Based on your answer to part (ii), what do you think will be the contribution if the availability time of test device 2 increases to 135? (Verify that you are correct.) What is the formula for the optimum profit if the availability time increased by  $130+t$ ? (You may assume that  $t$  is between 1 and 10).
  - (iv) (4 points) Based on your formula in part (iii), what is the contribution if the availability time of test device 2 increases to 150. Use Excel solver to see if the formula is correct. Use Excel solver to determine the maximum value of  $t$  for which your formula is correct (Be accurate to within an hour).

- (v) (4 points) How would the optimal solution in Part (b) change if the labor costs increased to \$18 per hour for test device 1, \$13 per hour for test device 2, and \$20 per hour for test device 3?
- (vi) (5 points) Over what range of the labor cost for test device 3 will the optimal production-mix in Part (b) remain optimal? (Be accurate to within one dollar.)
- d) (10 points) Write an algebraic formulation for the problem using the following notation:
- $m$  is the number of peripheral devices,
  - $n$  is the number of test devices,
  - $a_i$  is the maximum availability time of test device  $i$  ( in hour),
  - $c_i^L$  is the labor cost per hour of test device  $i$  (in \$),
  - $c_j^M$  is the material cost per unit for device  $j$  (in \$),
  - $r_j$  is the revenue cost per unit for device  $j$  (in \$),
  - $b_j$  is the maximum availability of material  $j$ ,
  - $q_{ij}$  is the amount of time in minute on test service  $i$  that is required in device  $j$ ,

MIT OpenCourseWare  
<http://ocw.mit.edu>

15.053 Optimization Methods in Management Science  
Spring 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.