15.053/8 April 30, 2013

Decision Trees 1

The example in the first half of today's lecture is a modification of the example in Bertsimas and Freund: Data, Models, and Decisions.

Quote of the Day

"Truly successful decision making relies on a balance between deliberate and instinctive thinking."

Malcolm Gladwell, in *Blink: The Power of Thinking Without Thinking*

"Every decision has a cost. Do I have to make this decision at all or can I move on to the next thing? What we decided to leave out is almost as important as what we put in."

Joshua Schachter

Overview of Lecture

- Introduction to Decision Trees: Juan Lee's job search

- Various aspects of the decision tree
  - construction
  - putting values in the tree
  - finding the optimal decisions
  - additional analysis

Juan Lee's Decision Problem

It is January 10th, and Juan Lee is currently a fourth year undergraduate in Management Science at Sloan. He has decided to seek out a job as a consultant. He already has received an offer from ABC consulting for $72,000 per year. He has until February 1st to decide whether to accept the offer. An old classmate of his, Mary Kumar, has told him that she has recommended him highly to her consulting firm, and feels that there is an excellent chance that they would give him an offer for $80,000. However, they are not prepared to make any decision until February 15th. If they made him an offer, he would need to decide by March 1. He also has the option of taking part in the consulting job fair in the middle of March. He is fairly certain that he could get a consulting job at that time, but is uncertain as to what he would be paid.

Juan's Goal

- Assume here that Juan views all of these consulting jobs as excellent opportunities, and that the only differentiating factor is money.

- Juan has decided to maximize his expected salary.

Juan's probabilities

- Juan does not know how to evaluate his "excellent chances" at Mary's firm, nor does he know what to expect from the consulting fair.

- Juan's best guesses
  - Probability of getting Job at Mary's firm: 60%
  - Different possibilities for consulting fair offers.
    - $90,000: 10%
    - $70,000: 50%
    - $60,000: 40%

On Juan's probabilities

- **Juan's best guesses are not probabilities in the sense of frequencies.**

- **They are called *Bayesian* or *subjective probabilities*.**

- **Most decision problems rely on "best guesses" both in terms of the outcomes (events) and the probabilities for those events.**
  - **Could be based on detailed analysis**
  - **Could be a first guess for a number that is not known at all.**

Probability as indifference between bets.

Event 1. There is no offer from Mary's firm.

Probability = p

Event 2. The 20 sided die shows an 8 or less.

Probability = .4

What is p if Juan is indifferent between betting on Event 1 or Event 2?

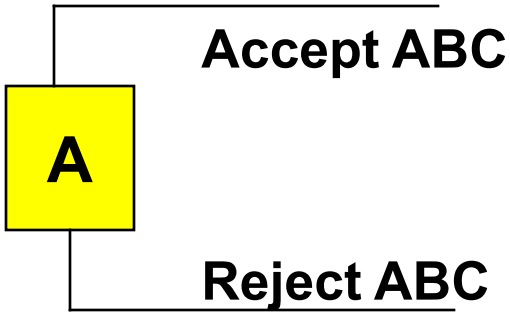
Under the probability as indifference hypothesis, p = .4

A 20-sided die

Photo courtesy of readerwalker on Flickr.

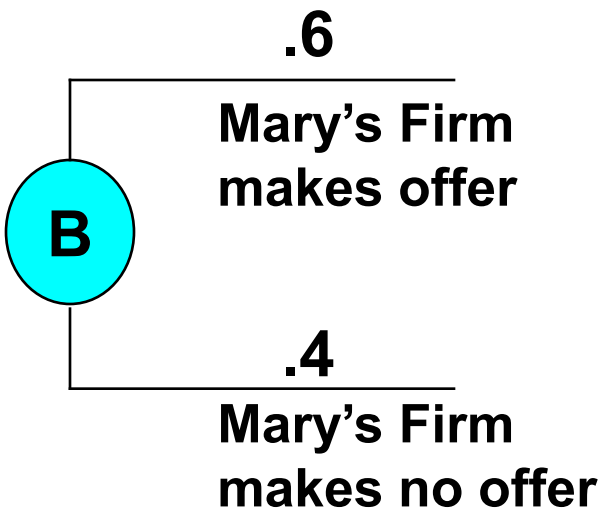
Decision Trees

- Method of organizing decisions over time in the face of uncertainties



Decision nodes:

- Represented as boxes
- lines coming from the nodes represent different choices.



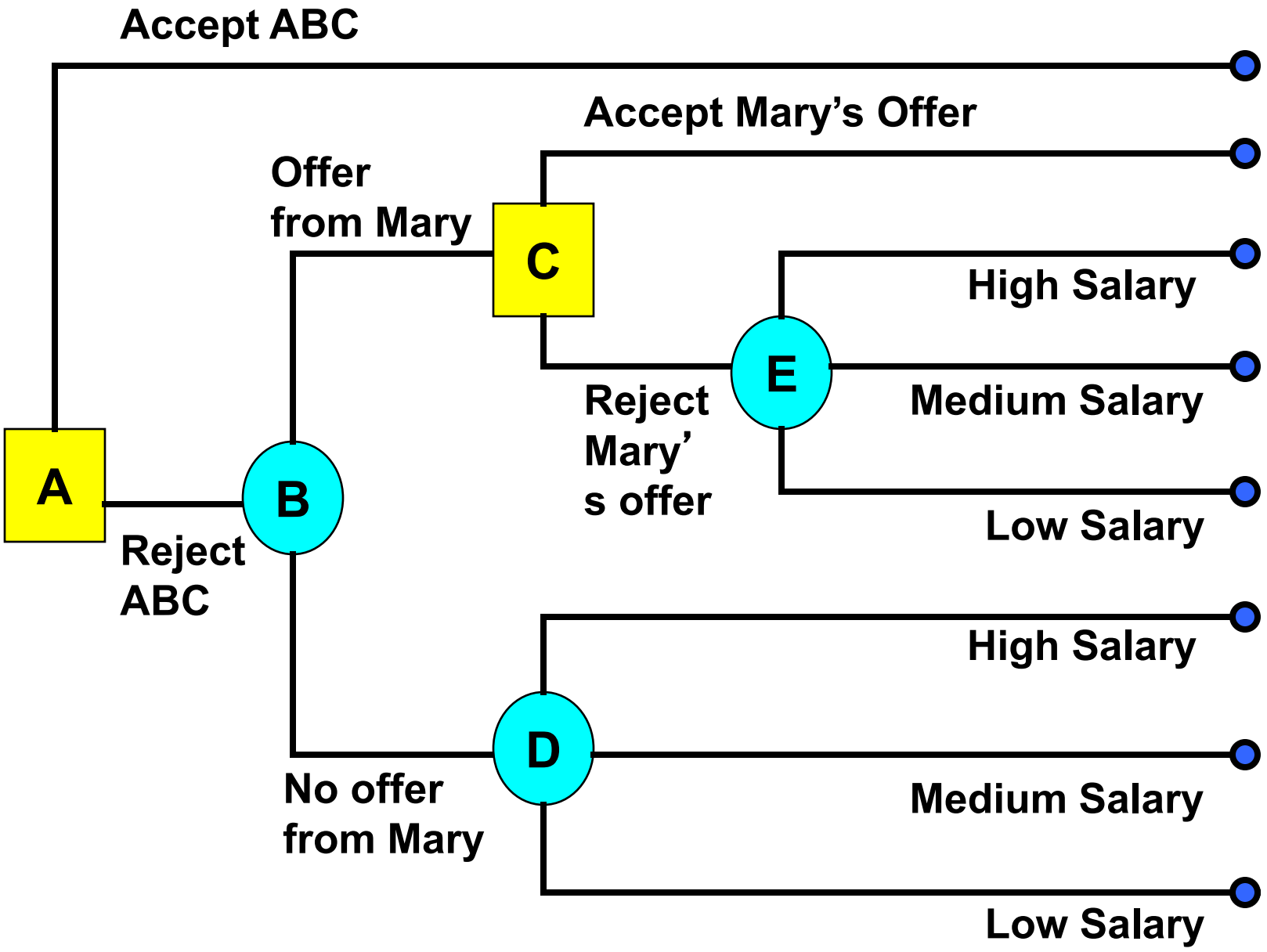
Event nodes:

- Represented as circles
- lines coming from the nodes represent different outcomes.

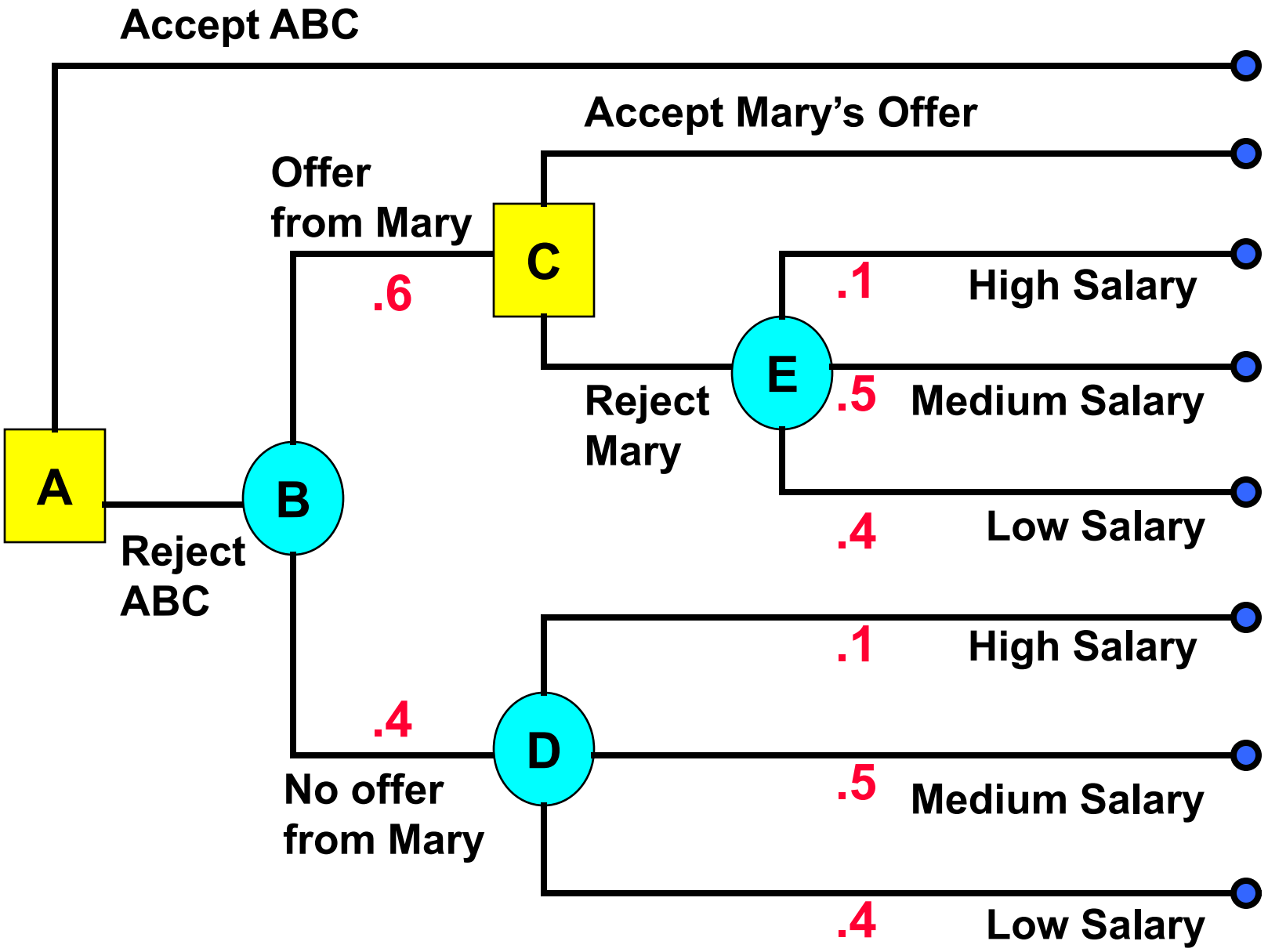
Which of the following is false?

1. A decision node represents a time at which a decision maker makes a decision.
2. Decision nodes are squares.
3. Event nodes (probability nodes) represents a time at which an uncertain event becomes known.
4. All probabilities are based on frequencies.

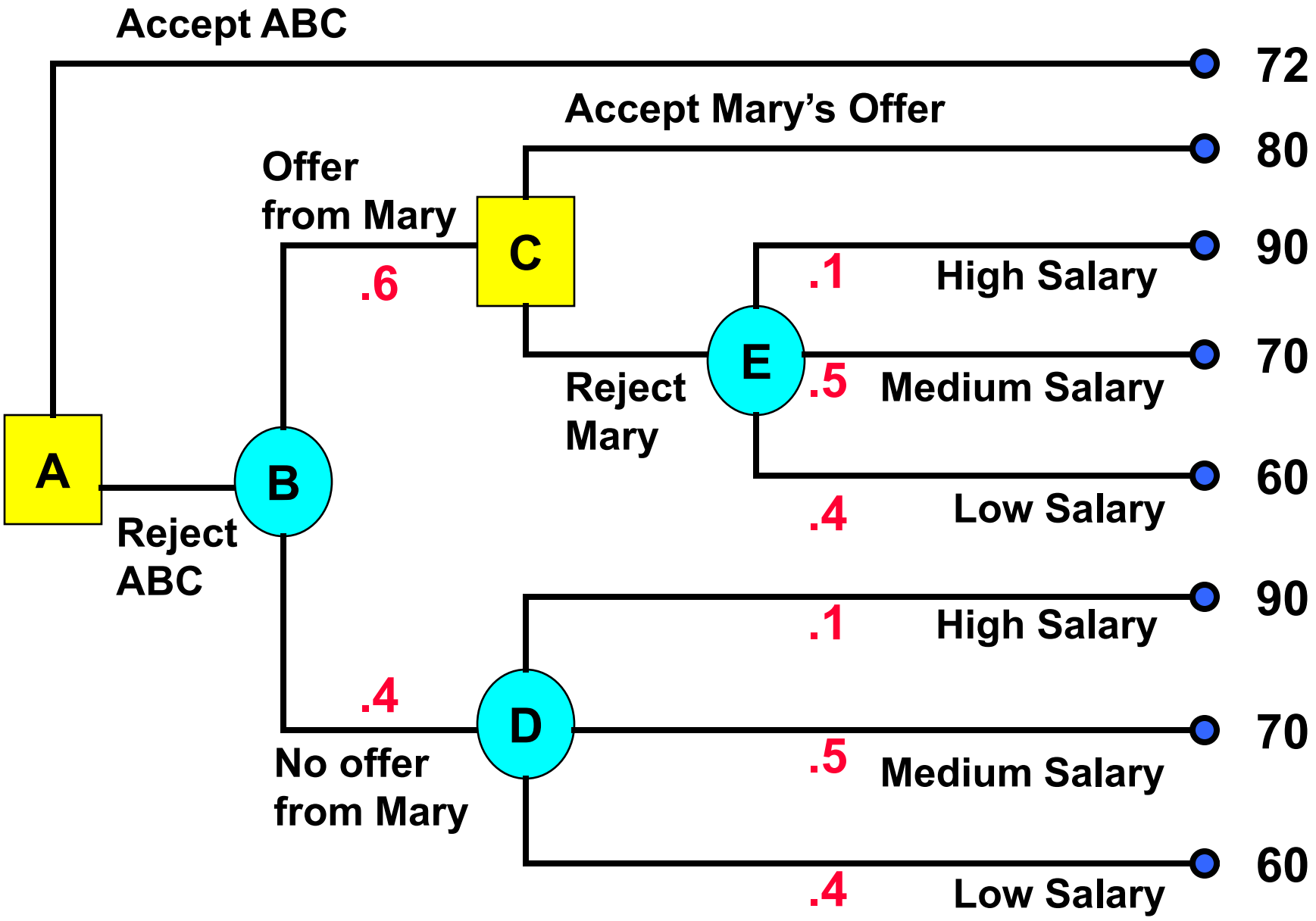
Step 1. Map out Juan Lee's Decision Problem



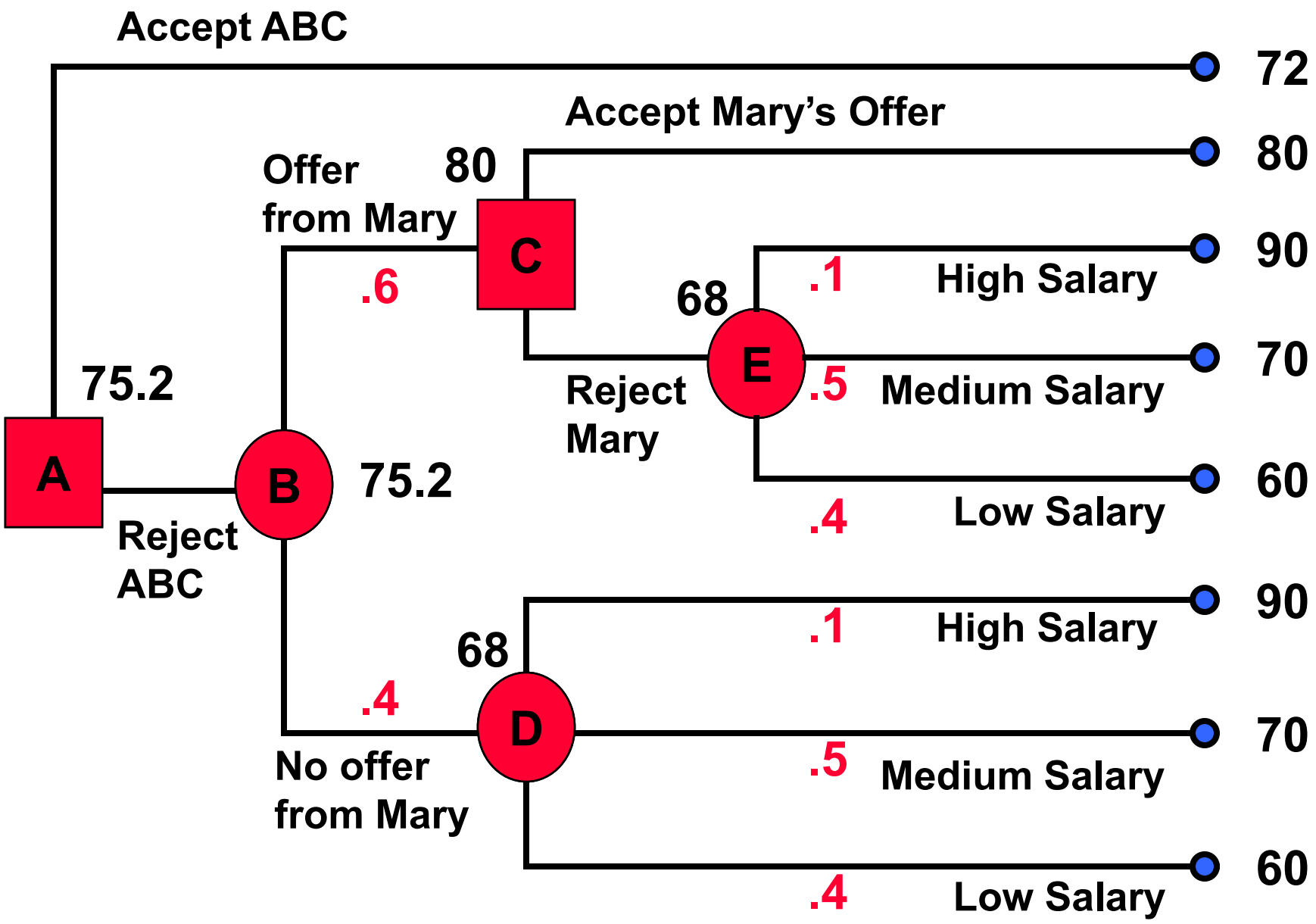
Step 2. Assign Probabilities to Events



Step 3. Evaluate the end nodes

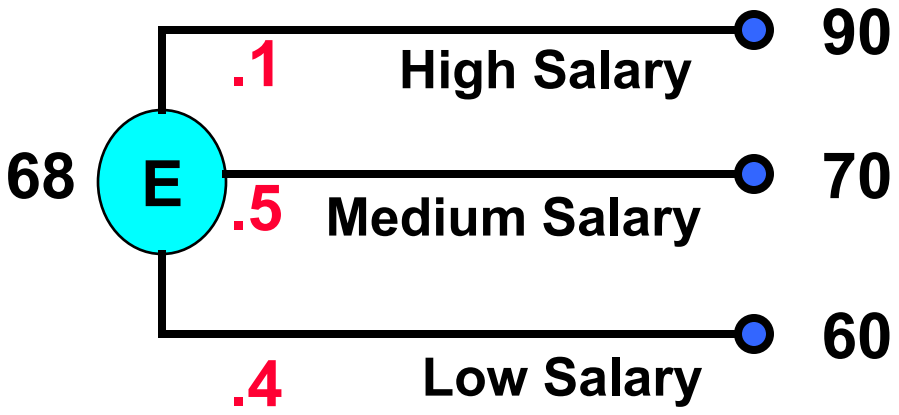


Step 4. Work Backwards and Evaluate



Evaluate Node E

Take the expected value.



.1 × 90

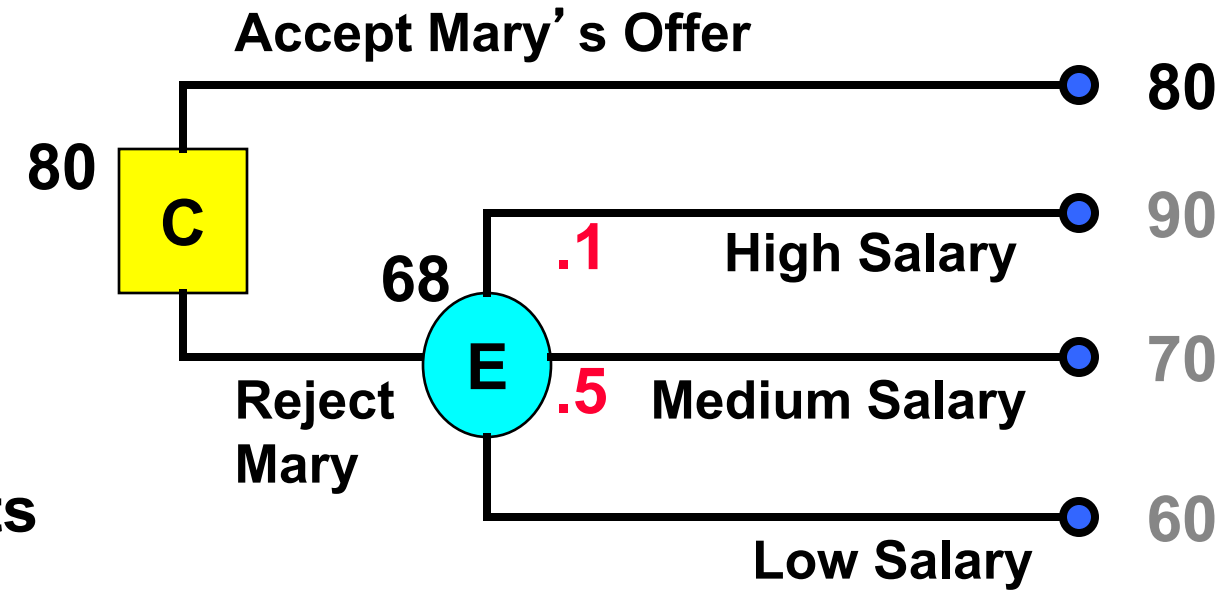
\+ .5 × 70

\+ .4 × 60

= 9 + 35 + 24 = 68

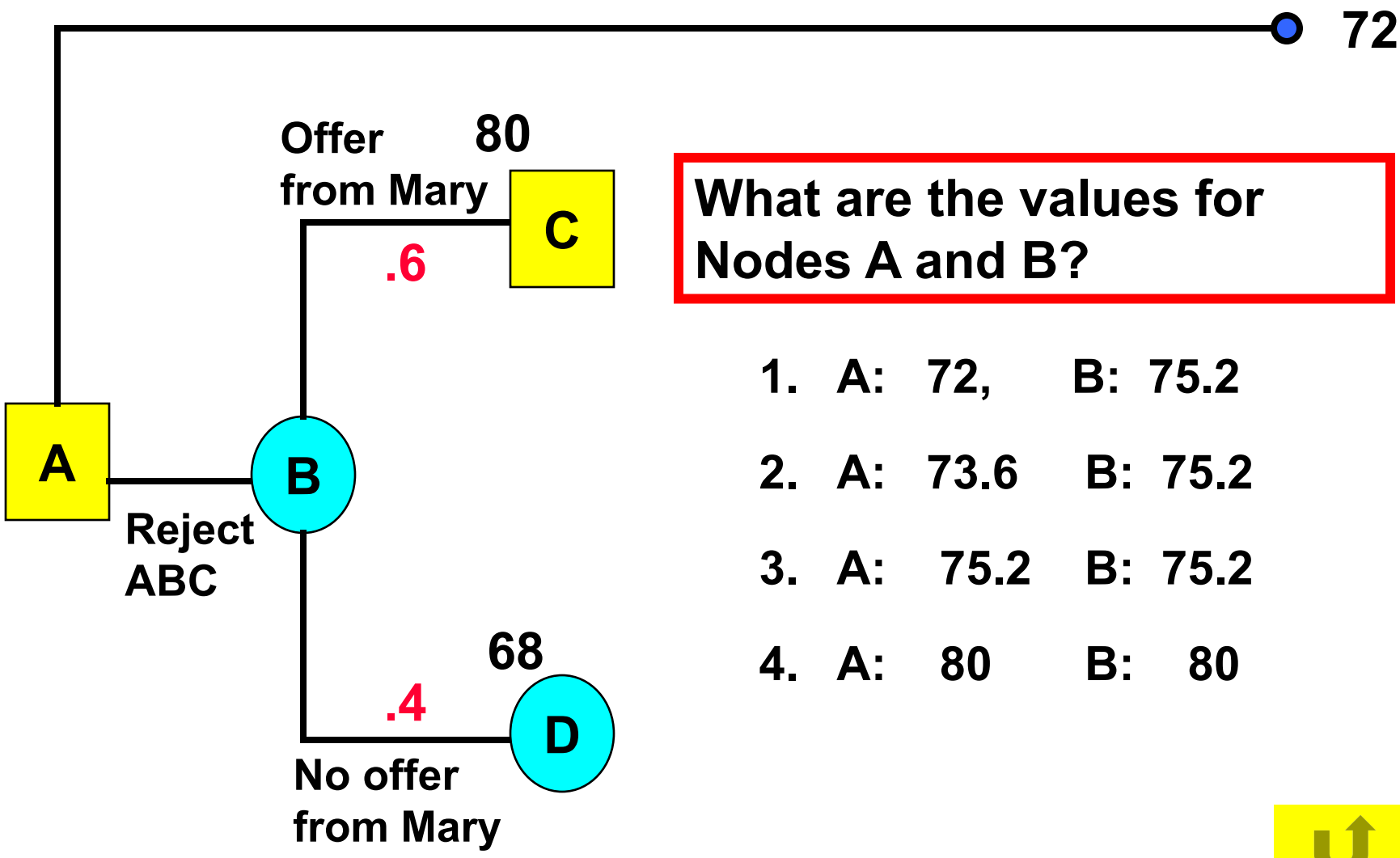
Evaluate Node C

For a decision node, take the best outcome leading from its branches.



Max (80, 68) = 80

Evaluate Nodes A and B



What are the values for Nodes A and B?

1. A: 72, B: 75.2
2. A: 73.6 B: 75.2
3. A: 75.2 B: 75.2
4. A: 80 B: 80

Brief summary so far

- Decision trees: a method for decision making over time with uncertainty.
  - Create the tree, one node at a time
  - Decision nodes and event nodes
  - Probabilities: usually subjective
  - Solve the tree by working backwards, starting with the end nodes.
    - Often we minimize expected cost (or maximize gain).

Mental Break

People who got it wrong.

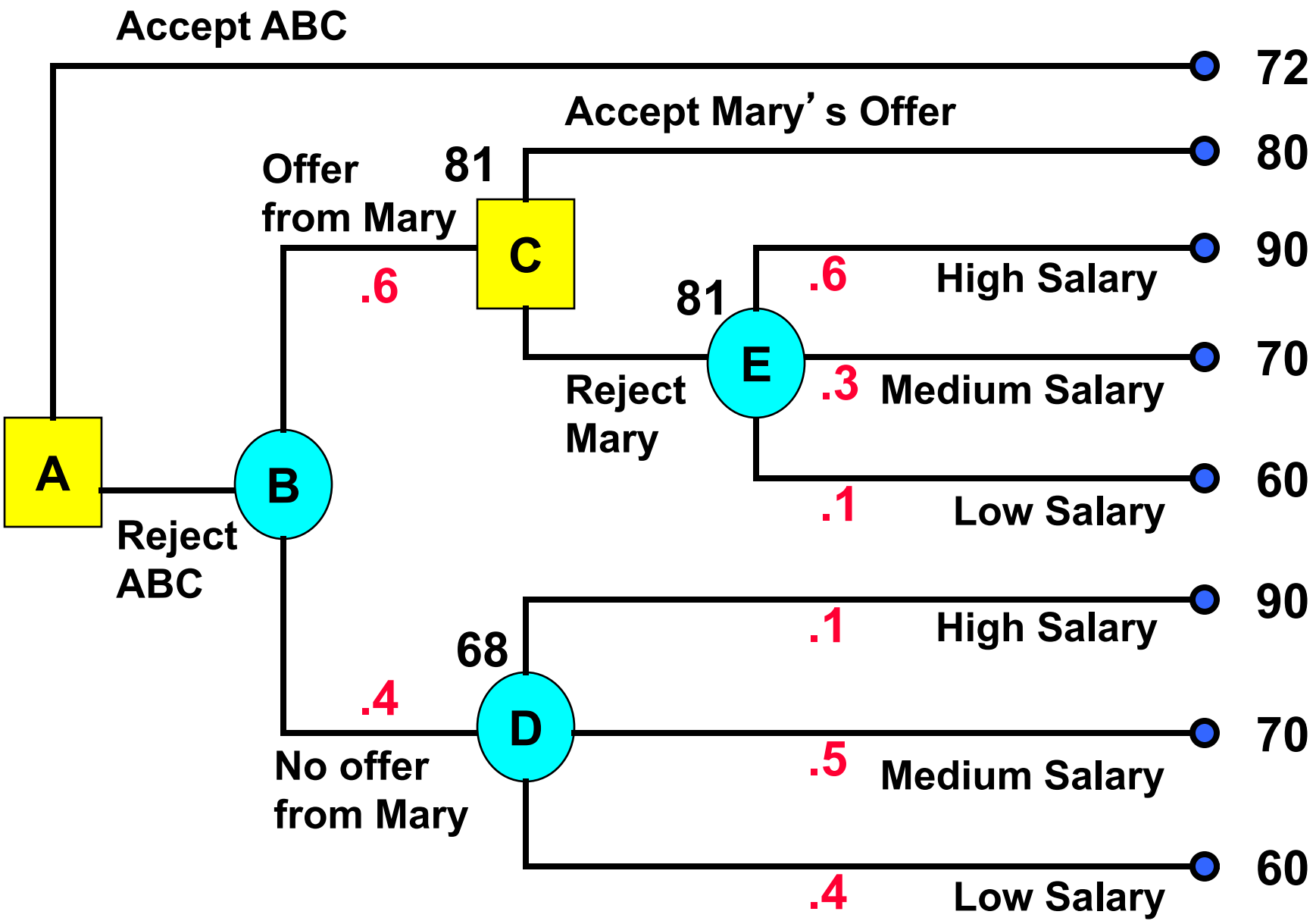
Key Aspects of a Decision Tree

Time flows from left to right

Branches from a decision node represent decisions and take into account all decisions or events leading to that node

- **Example. Juan expects to get Mary's job, but does not know how he is viewed in the job market. If he does get Mary's offer, he believes that he must be viewed highly. Accordingly, he would adjust his probabilities of getting salaries in the job fair as follows:**
  - **High: 60%**
  - **Medium: 30%**
  - **Low: 10%**

Illustration of new probabilities



Key Aspects of a Decision Tree

- Branches from an event node represent a set of mutually exclusive and collectively exhaustive
- Final nodes in the tree have an associated value
- Values of other nodes are computed working backwards. The value of an event node is the expected value of its endpoints. The value of a decision node is the highest value of its endpoints.

But the tree is only the beginning

- Typically in decision trees, there is a great deal of uncertainty surrounding the numbers.
  - Decision Trees work well in such conditions

- This is an ideal time for sensitivity analysis the "old fashioned way."
  - One varies numbers and sees the effect
  - One can also look for changes in the data that lead to changes in the decisions.

Microsoft® Excel

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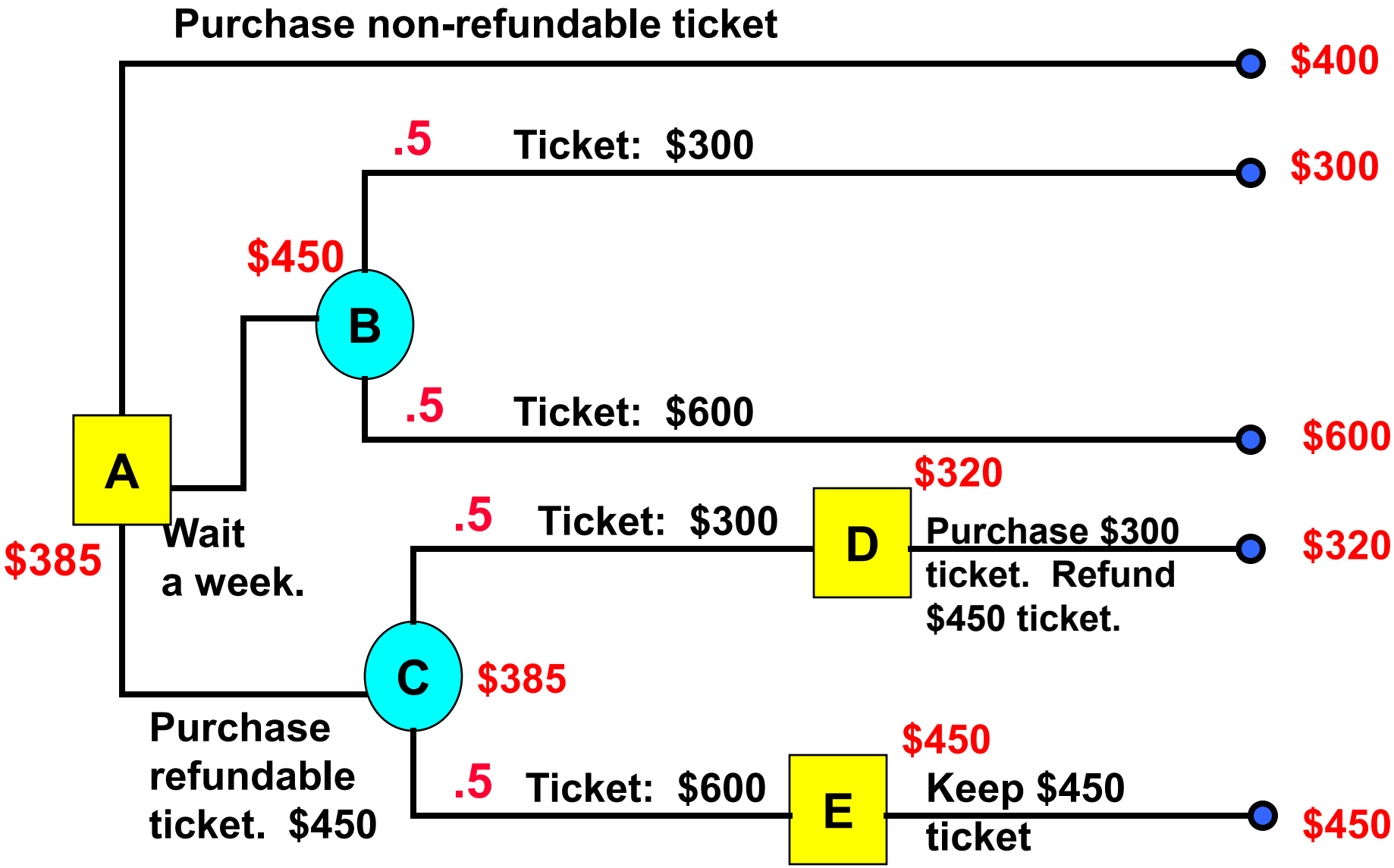
The Airfare Problem

You are trying to get the cheapest airfare that you can. You just called up and found that the ticket home will cost $400, and it cannot be refunded or exchanged. You can also buy a ticket for $450, which can be refunded for $430 (and thus costs you $20). The price of tickets will change in one week, and you will have one more chance to buy a ticket. There is a 50% chance that the ticket would cost $300, and a 50% chance that it would cost $600. What should you do to minimize the expected expense?

Exercise. Write the Airfare problem as a decision tree, and solve it.

The Air Fare Problem



Summary Conclusions

- Decision trees are a very useful technique for mapping out sequential decisions under uncertainty.
- They can be useful for considering the same problem from differing perspectives.
- Next Lectures:
  - Value of Information

15.053 Optimization Methods in Management Science

Spring 2013