Network Models

Assignment

Transportation

Intro to Modeling/Excel

How the Solver Works

Sensitivity Analysis

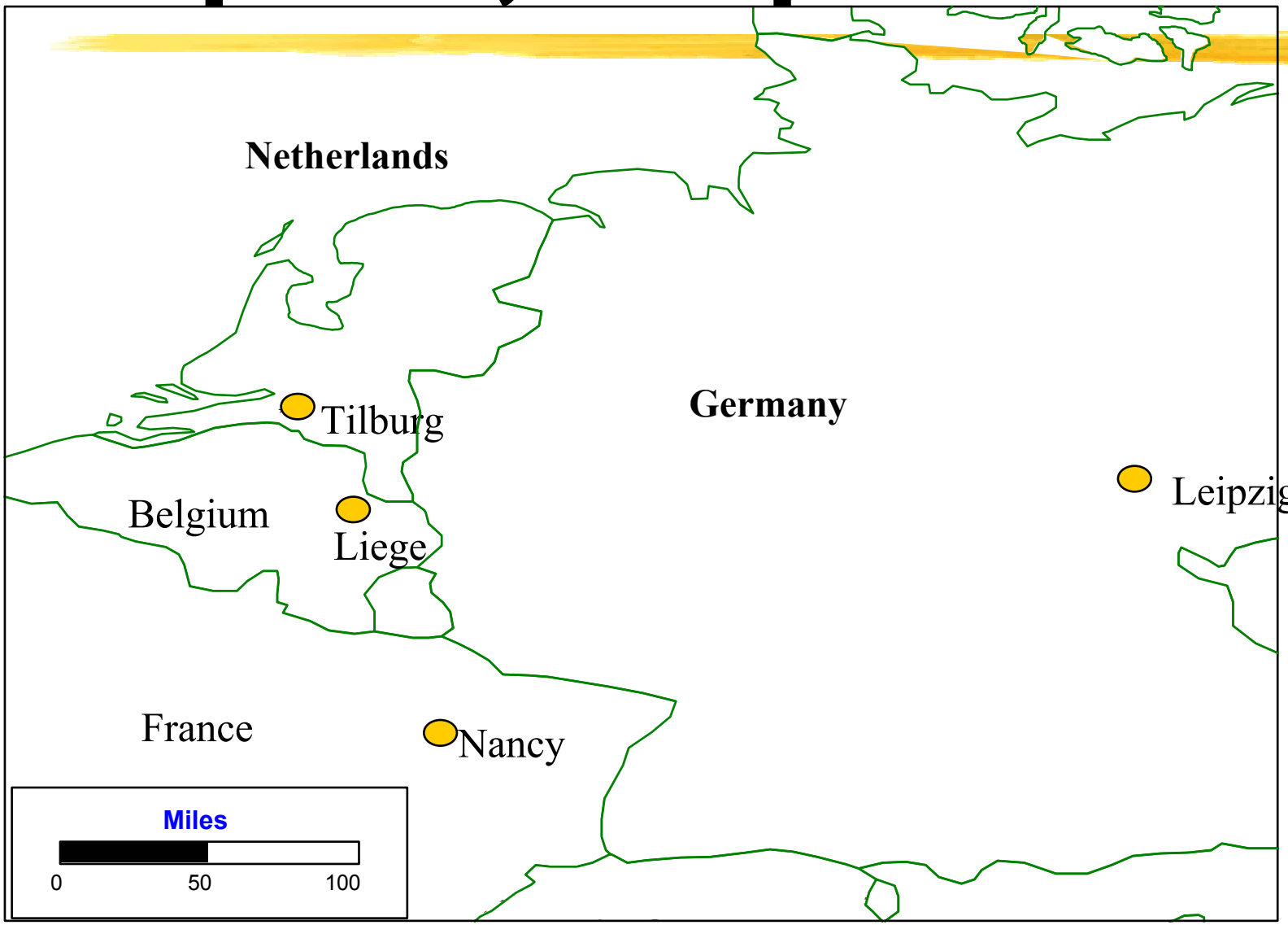
Objective

- Mini Course on Networks
  - Introduction to modeling
    - In Excel and AMPL
  - Intuitive description of solution approach
  - Intuitive description of sensitivity analysis
- Intuitive and visual context for covering technical aspects

Assignment Model

- Autopower Europe
  - Manufactures UPS for major installations
  - Four manufacturing plants
    - Leipzig, Germany
    - Nancy, France
    - Liege, Belgium
    - Tilburg, The Netherlands
  - One VP to audit each plant

Autopower, Europe



Assignment Problem

- Who's to visit whom?
  - VP's expertise and plant's needs
  - Available time and travel requirements
  - Language abilities
  - …

The Challenge

- Estimate costs (Done - Thoughts?)
- One VP to each plant
- One plant for each VP
- Minimize cost of assignments

01AssignmentModel.xls

Autoppower Europe: Assignment Model

Moore et al. pp224

Estimated Assignment Costs

VP	Leipzig 1	Nancy 2	Liege 3	Tilburg 4
Finance (F)	24	10	21	11
Marketing (M)	14	22	10	15
Operations (O)	15	17	20	19
Personnel (P)	11	19	14	13

Assignments

VP	Leipzig 1	Nancy 2	Liege 3	Tilburg 4	Plants Assigned
Finance (F)					0
Marketing (M)					0
Operations (O)					0
Personnel (P)					0
VPs Assigned	0	0	0	0	

Cost of Assignments

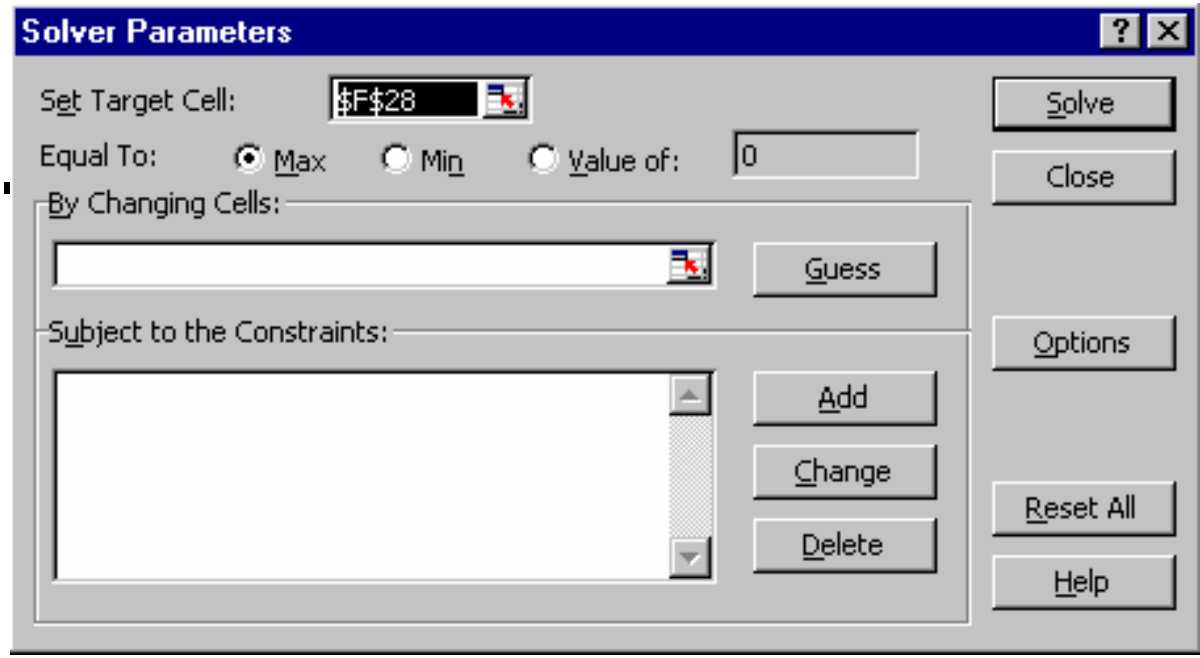
VP	Leipzig 1	Nancy 2	Liege 3	Tilburg 4	Total Cost
Finance (F)	0	0	0	0	0
Marketing (M)	0	0	0	0	0
Operations (O)	0	0	0	0	0
Personnel (P)	0	0	0	0	0
Total Cost	0	0	0	0	0

A Challenge

- Find best assignments

Building a Network Model

- In Excel
- Tools | Solver..



Set Target Cell: $F$28

By Changing Cells: $B$15:$E$18

The Constraints

- Each VP assigned to one plant
  - $F$15 = 1
  - $F$16 = 1
  - $F$17 = 1
  - $F$18 = 1

Shortcut
$F$15:$F$18 = 1

- Each plant assigned one VP
  - $B$19 = 1
  - $C$19 = 1
  - $D$19 = 1
  - $E$19 = 1

Shortcut
$B$19:$E$19 = 1

What's Missing

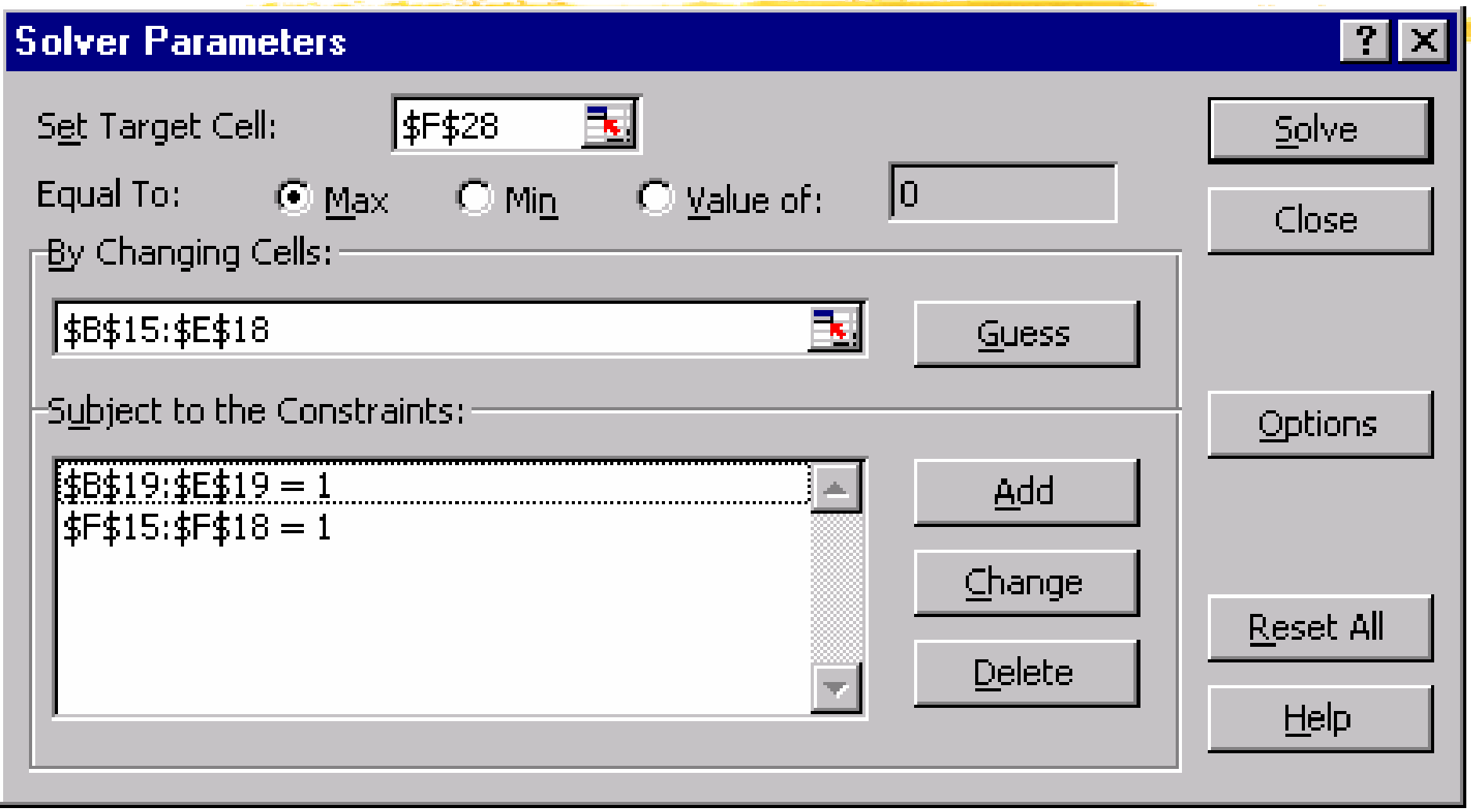
Additional Constraints...

- Non-negativity
  - The variables cannot be negative
  - Handled separately
- Integrality
  - The variables should have integral values
  - We can ignore these because this is a network model!!!

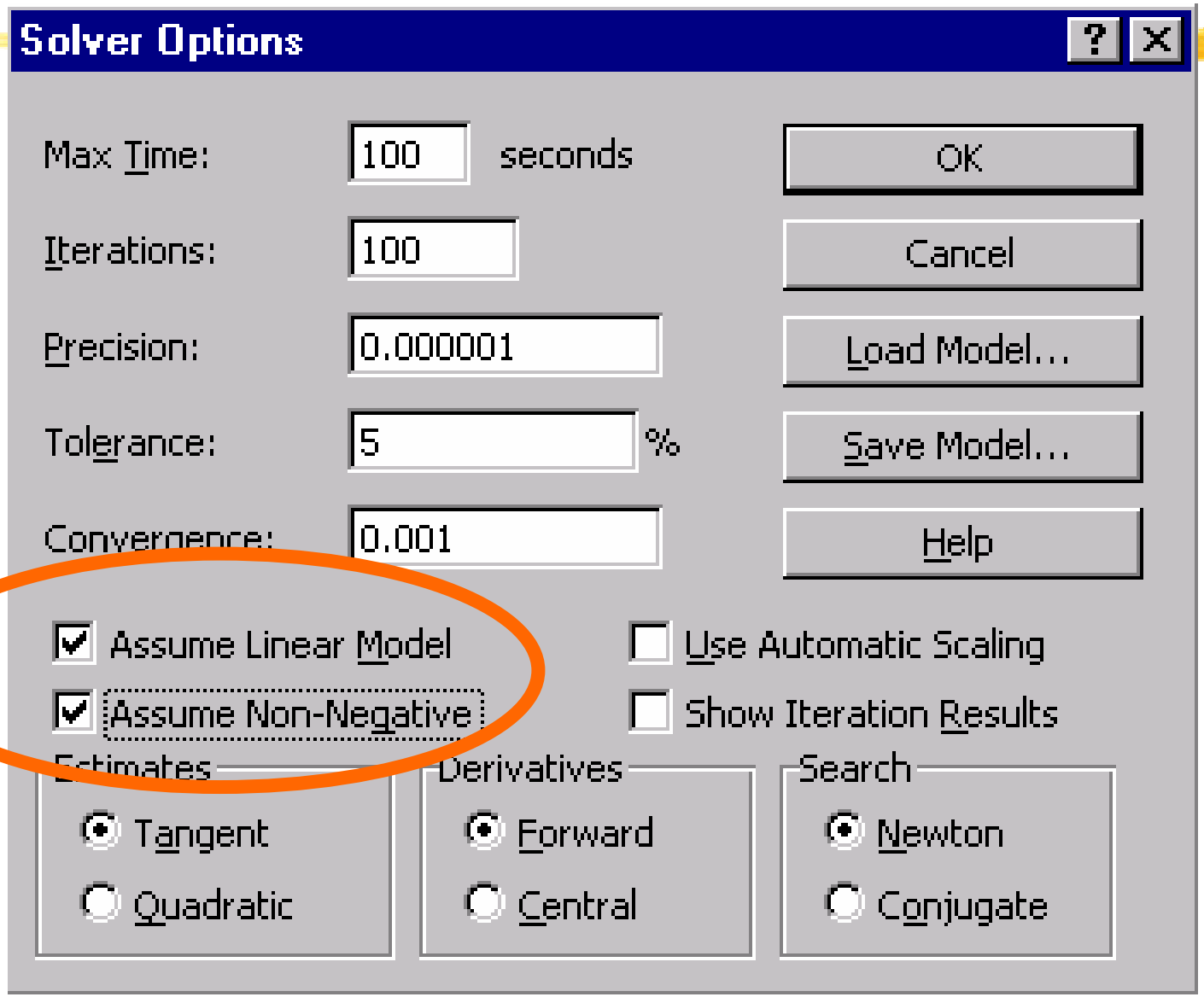
Model Components

- Set Target Cell: Objective $F$28
  - The value we want to minimize/maximize
- Equal to: Min
  - Min for Minimize or Max for Maximize
- By Changing Cells:
  Variables or Adjustables $B$15:$E$18
  - The values we can change to find the answer
- Subject to the Constraints ….
  - $B$19:$B$18 = 1
  - $F$15:$F$18 = 1

Excel Model


Solver Parameters
Set Target Cell:
$F$28
Equal To:
Max
Min
Value of:
0
By Changing Cells:
$B$15:$E$18
Subject to the Constraints:
$B$19:$E$19 = 1
$F$15:$F$18 = 1
Solve
Close
Guess
Options
Add
Change
Delete
Reset All
Help

Options


Solver Options
Max Time:
100
seconds
OK
Iterations:
100
Cancel
Precision:
0.000001
Load Model...
Tolerance:
5
%
Save Model...
Convergence:
0.001
Help
Assume Linear Model
Use Automatic Scaling
Assume Non-Negative
Show Iteration Results
Estimates
Tangent
Quadratic
Derivatives
Forward
Central
Search
Newton
Conjugate

Limits

- **Max time**: Limits the time allowed for the solution process in seconds

- **Iterations**:Limits the number of interim calculations. (More details to come)

Precision

- Controls the precision of solutions.
- Is 1/3 <= 0.3333? 0.333333?

Quality of Solutions

- **Tolerance:** For integer problems. Later
- **Convergence**: For non-linear problems. Later

Review & Terminology

- Objective: Target Cell
- Equal to: Max or Min
- Variables: By Changing Cells
- Constraints: Constraints
  - LHS: Reference Cell - a function of the variables
  - RHS: Constraint - a constant (ideally)
- Options:
  - Assume Linear Model
  - Assume Non-negative
- Solve

What do you think?

- Realistic?
- Practical?
- Issues?
- Questions…

First Kind of Network Model

- Sum across row = Const.
- Sum down column = Const.

Each variable in two constraints:
A "row" constraint
A "column" constraint

Estimated Assignment Costs

VP	Leipzig 1	Nancy 2	Liege 3	Tilburg 4
Finance (F)	24	10	21	11
Marketing (M)	14	22	10	15
Operations (O)	15	17	20	19
IT (I)	19	24	15	10
Personnel (P)	11	19	14	13

Assignments

VP	Leipzig 1	Nancy 2	Liege 3	Tilburg 4	Plants Assigned
Finance (F)	0	0	0	0	0
Marketing (M)	0	0	0	0	0
Operations (O)	0	0	0	0	0
IT (I)	0	0	0	0	0
Personnel (P)	0	0	0	0	0
VPs Assigned	0	0	0	0	

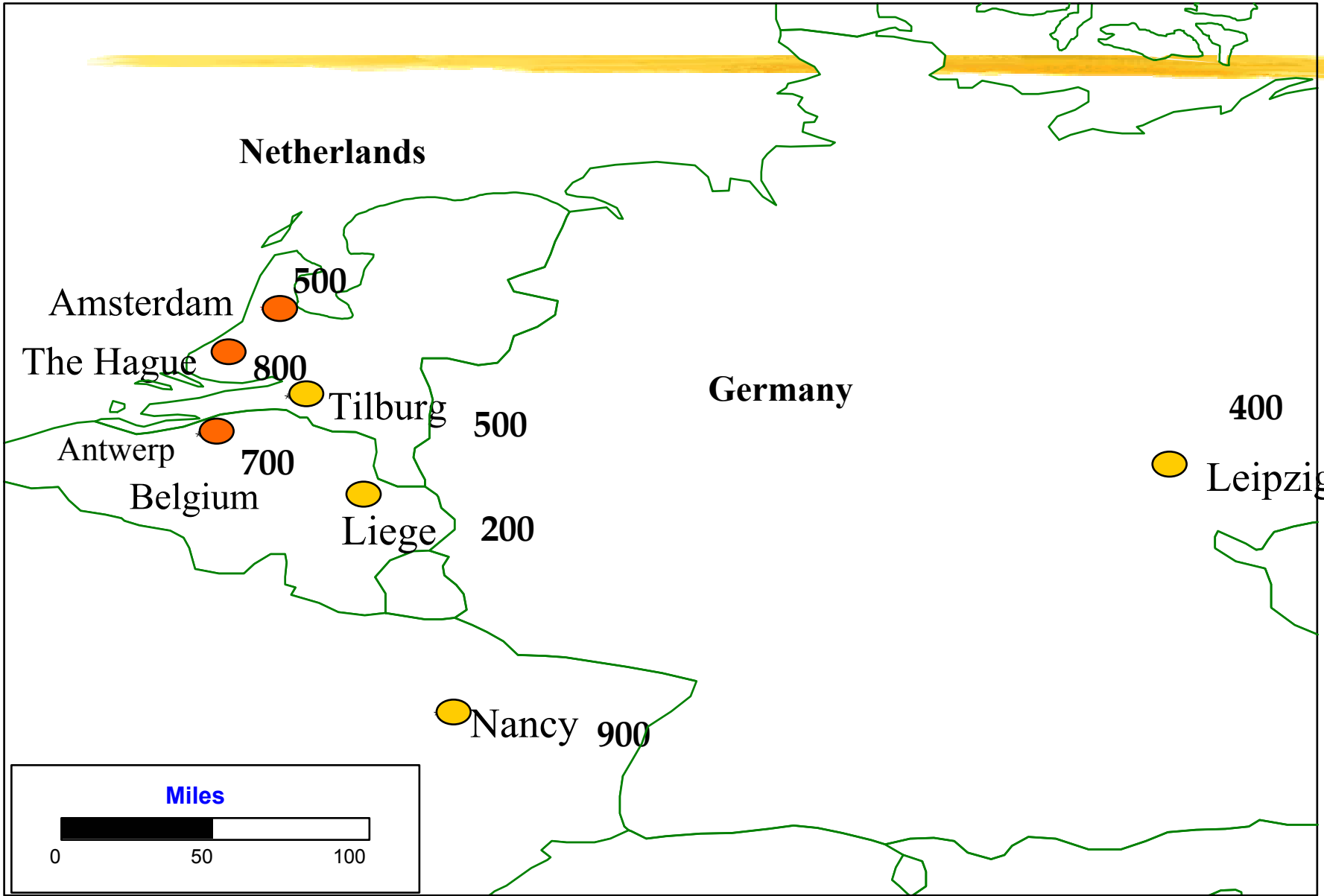
Cost of Assignments

VP	Leipzig 1	Nancy 2	Liege 3	Tilburg 4	Total Cost
Finance (F)	0	0	0	0	0
Marketing (M)	0	0	0	0	0
Operations (O)	0	0	0	0	0
IT (I)	0	0	0	0	0
Personnel (P)	0	0	0	0	0
Total Cost	0	0	0	0	0

Influence of Optimization

- Changes focus of "negotiation" about assignments
  - from emotion and personal preferences
  - to estimation of cost

Motor Distribution



Transportation Costs

From Origin	To Destination Leipzig	Nancy	Liege	Tilburg
Amsterdam	120	130	41	59.5
Antwerp	61	40	100	110
The Hague	102.5	90	122	42

Unit transportation costs from harbors to plants

Minimize
the transportation costs involved in moving the motors from the harbors to the plants

A Transportation Model

Autopower Transportation Model

Unit Cost

From/To	Leipzig	Nancy	Liege	Tilburg
Amsterdam	$ 120.0	$ 130.0	$ 41.0	$ 59.5
Antwerp	$ 61.0	$ 40.0	$ 100.0	$ 110.0
The Hague	$ 102.5	$ 90.0	$ 122.0	$ 42.0

Shipments

From/To	Leipzig	Nancy	Liege	Tilburg	Total	Available
Amsterdam	-	-	-	-	-	500
Antwerp	-	-	-	-	-	700
The Hague	-	-	-	-	-	800
Total	-	-	-	-	-	
Required	400	900	200	500		

Total Cost

From/To	Leipzig	Nancy	Liege	Tilburg	Total
Amsterdam	$ -	$ -	$ -	$ -	$ -
Antwerp	$ -	$ -	$ -	$ -	$ -
The Hague	$ -	$ -	$ -	$ -	$ -
Total	$ -	$ -	$ -	$ -	$ -

Challenge

- Find a best answer

Building a Solver Model

- Tools | Solver…
  - Set Target Cell: The cell holding the value you want to minimize (cost) or maximize (revenue)
  - Equal to: Choose Max to maximize or Min to minimize this
  - By Changing Cells: The cells or variables the model is allowed to adjust

Solver Model Cont'd

- Subject to the Constraints: The constraints that limit the choices of the values of the adjustables.
  - Click on Add
    - Cell Reference is a cell that holds a value calculated from the adjustables
    - Constraint is a cell that holds a value that constraints the Cell Reference.
    - <=, =, => is the sense of the constraint. Choose one.

What are the Constraints?

- Supply Constraints
  - Amsterdam: G9 <= H9
  - Antwerp: G10 <= H10
  - The Hague: G11 <= H11
- Demand Constraints
  - Leipzig: C12 => C13
  - Nancy: D12 => D13
  - Liege: E12 => E13
  - Tilburg: F12 => F13

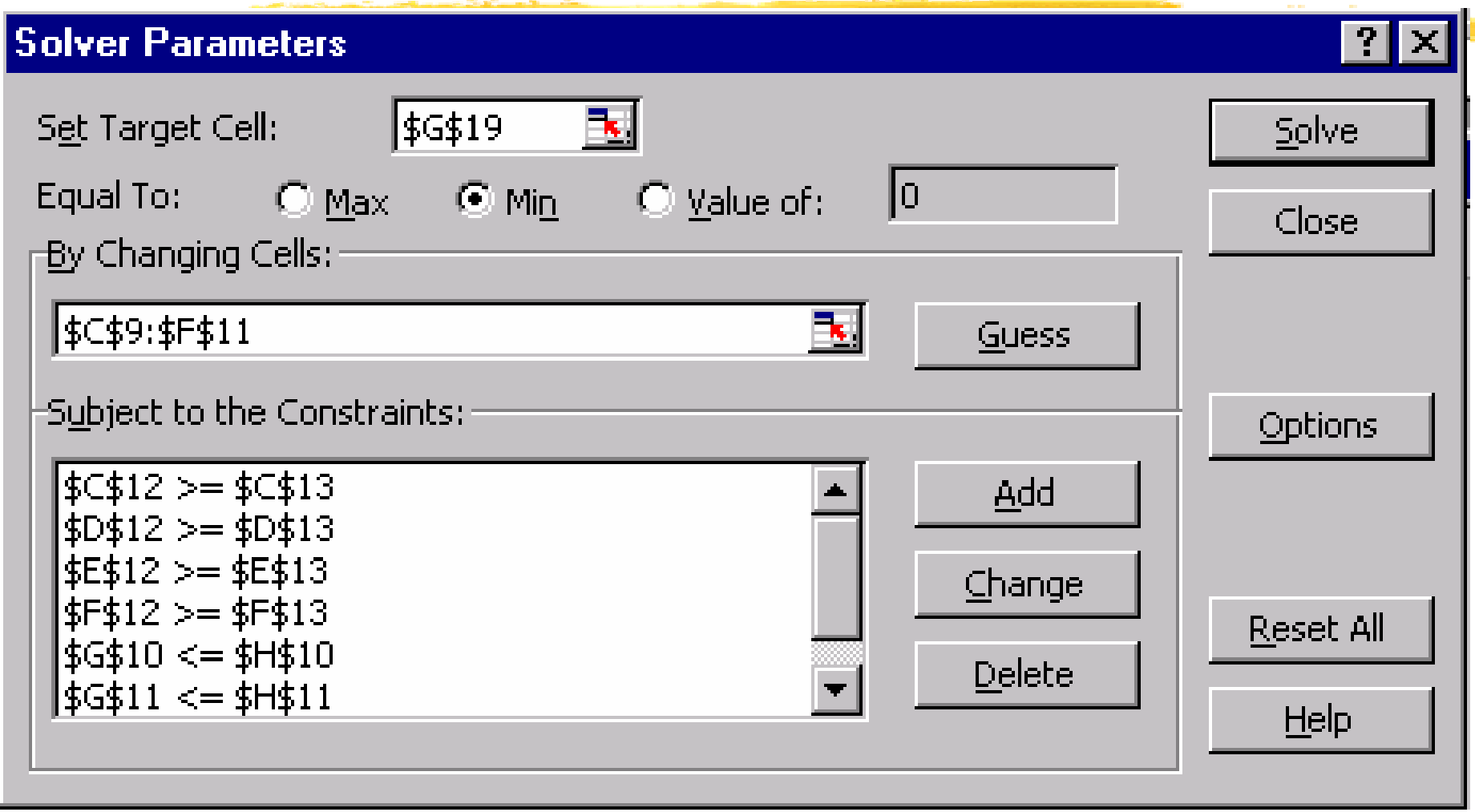
G9 is the total volume shipped from Amsterdam

Short cut:
G9:G11 <= H9:H11

C12 is the total volume shipped to Leipzig

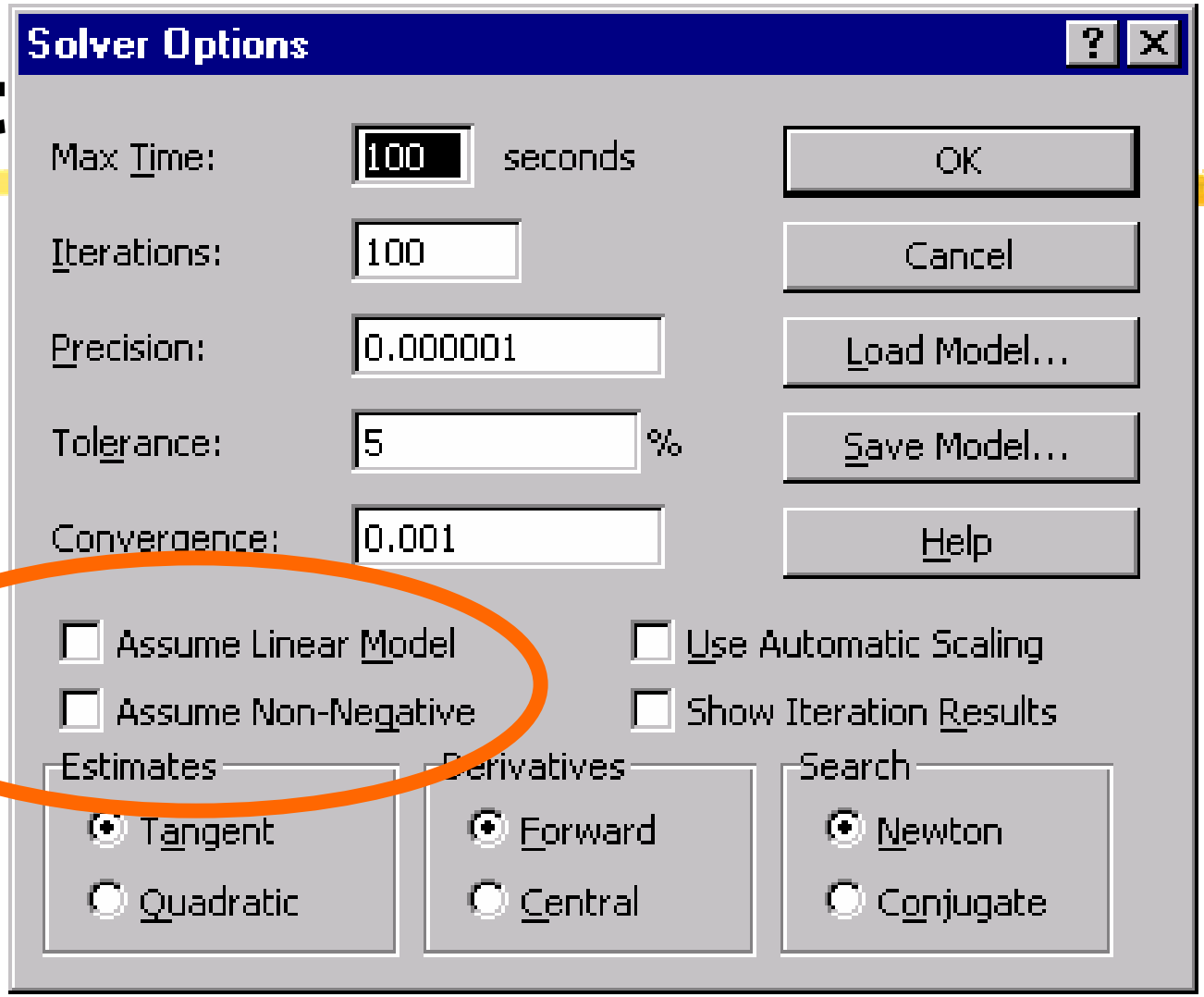
Short cut:
C12:F12 => C13:F13

The Model


Solver Parameters
Set Target Cell:
$G$19
Equal To:
Max
Min
Value of:
0
By Changing Cells:
$C$9:$F$11
Subject to the Constraints:
$C$12 >= $C$13
$D$12 >= $D$13
$E$12 >= $E$13
$F$12 >= $F$13
$G$10 <= $H$10
$G$11 <= $H$11
Solve
Close
Guess
Options
Add
Change
Delete
Reset All
Help

What's Missing?

Opt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Assume Linear Model
Use Automatic Scaling
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Search
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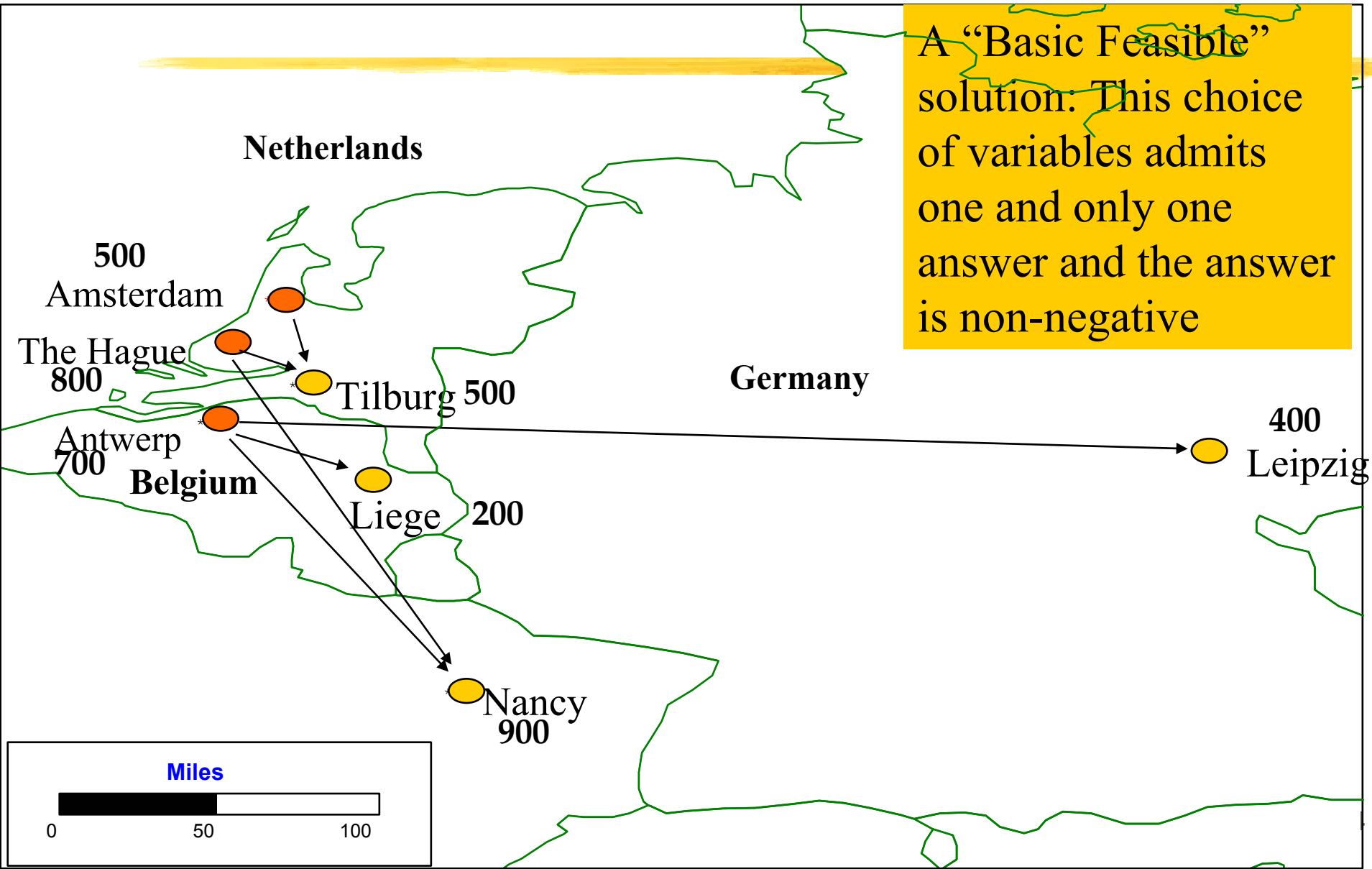
Conjugate

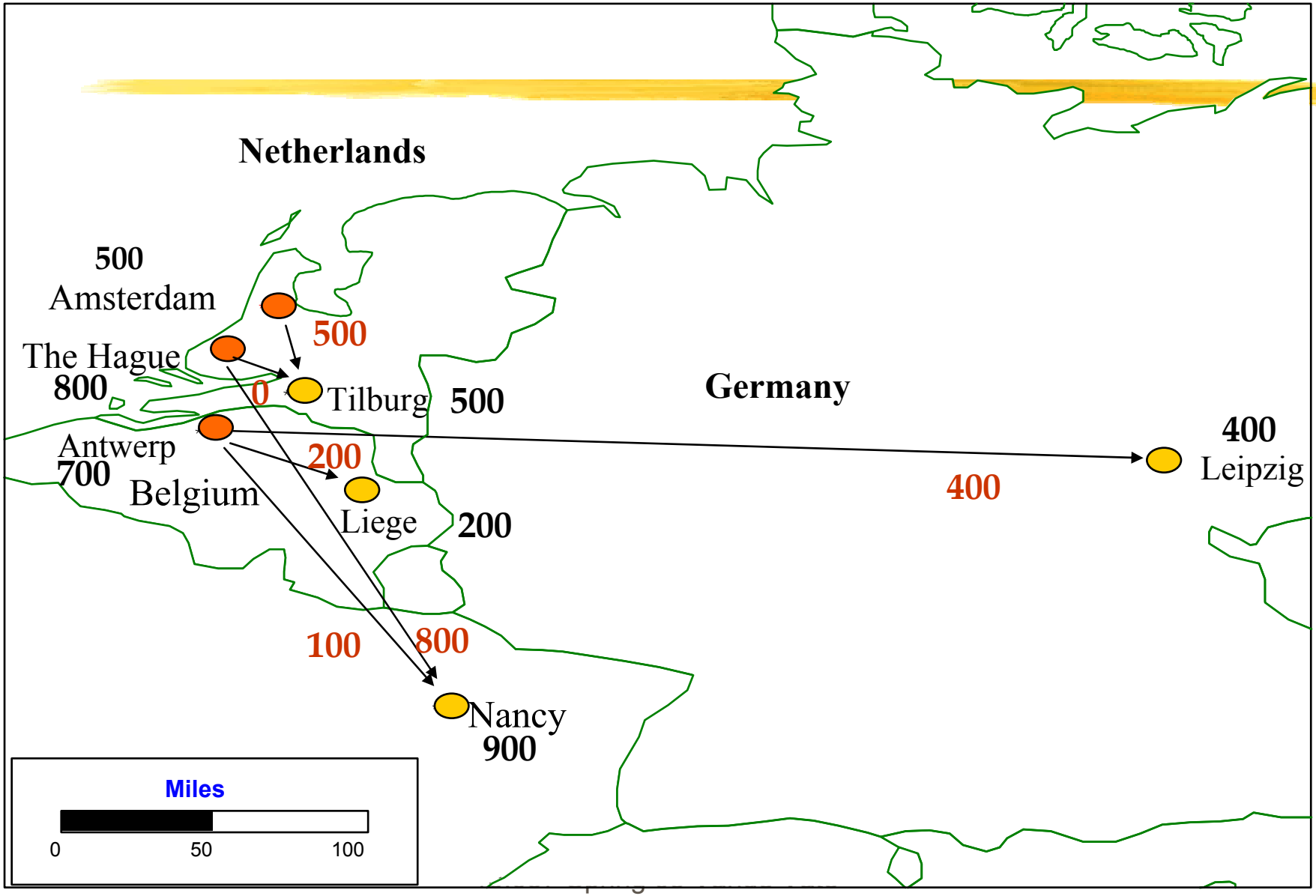
How the Solver Works

- Not magic
- Quick and intuitive
- Not comprehensive
- Basic understanding of tool and terms

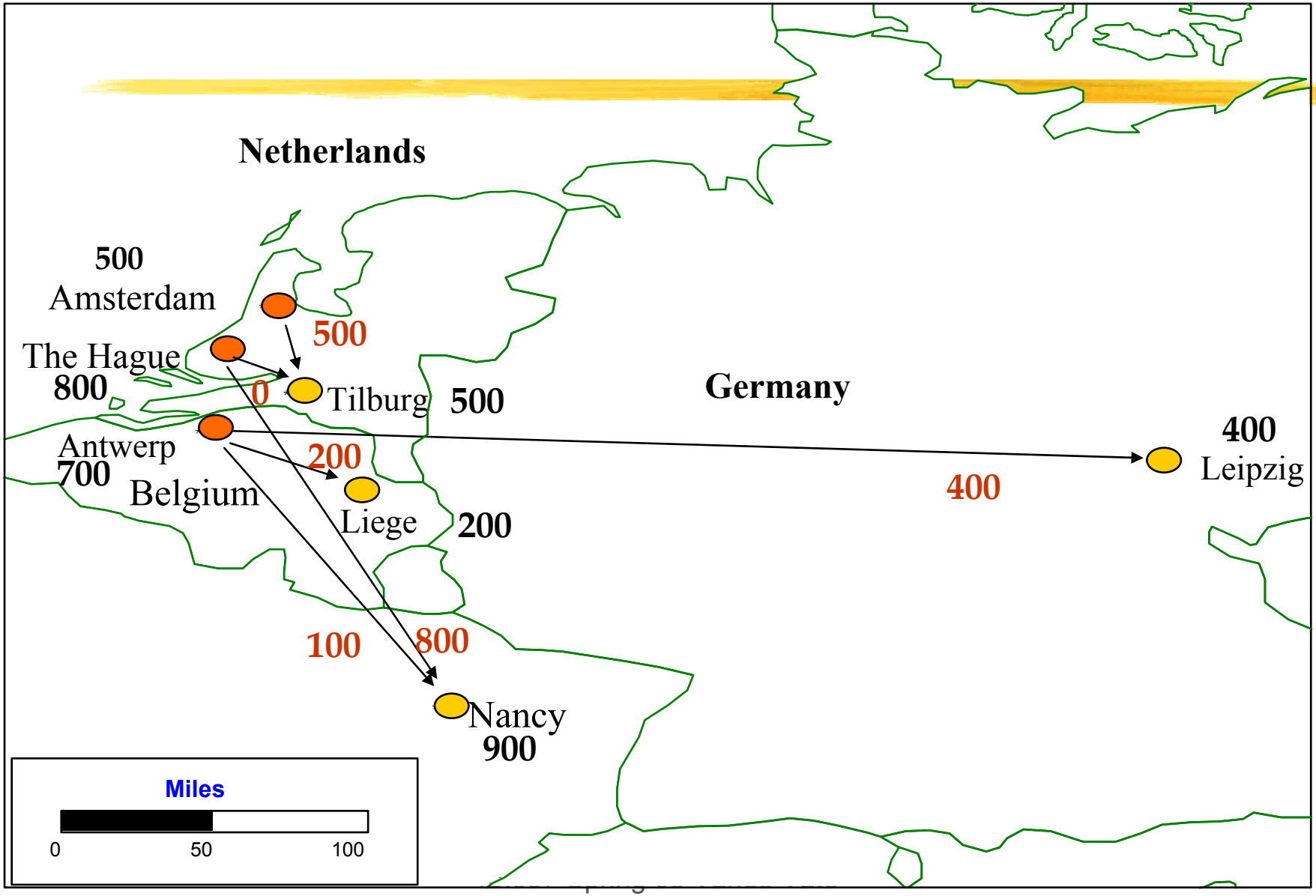
How the Solver works



A Basic Feasible Solution



More Technical Detail



Mathematically*

- z are the basic variables
- y are the non-basic variables
- Write the constraints as

$$Ax = Bz + Ny = b$$

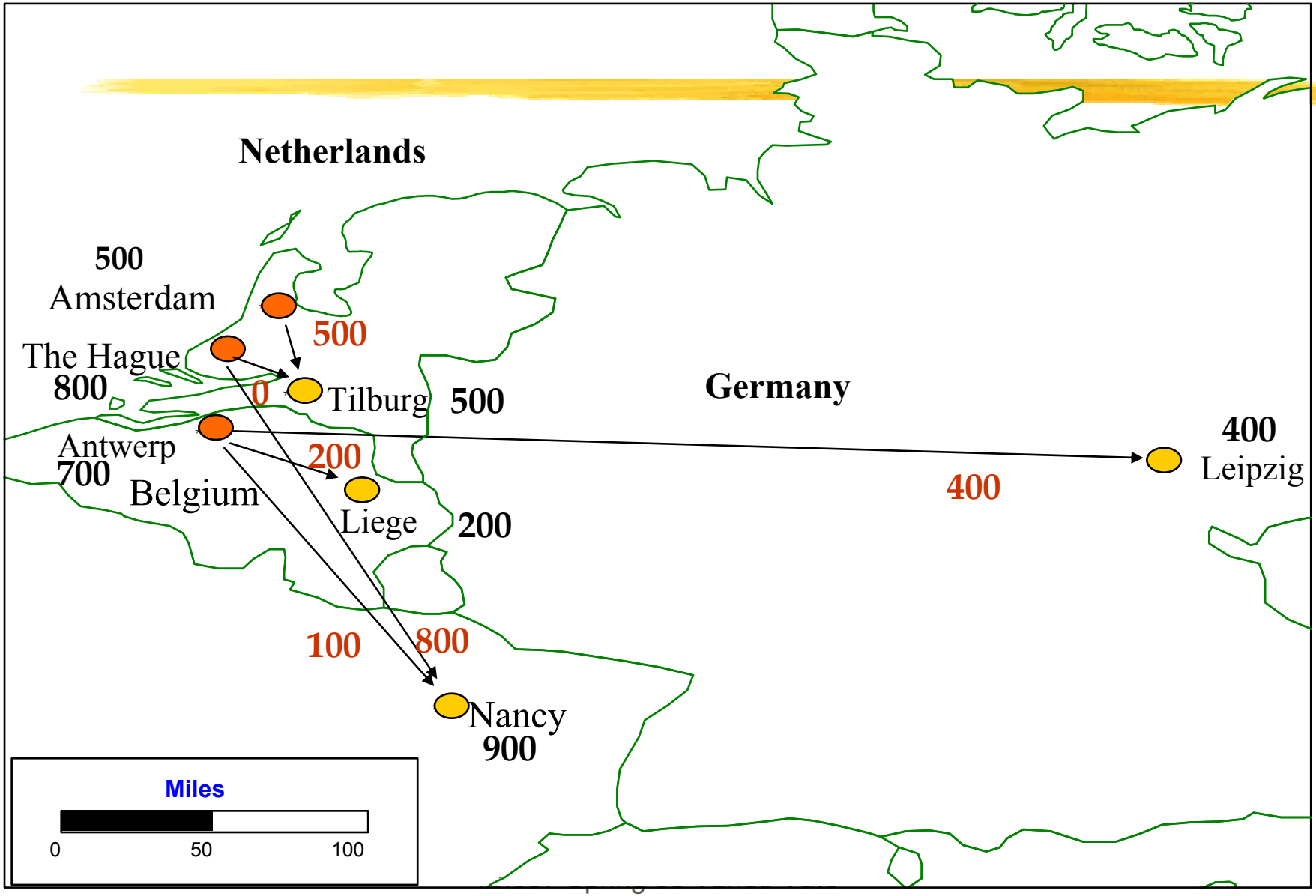
- Fix the non-basic variables to y*
- The unique solution for the basic variables

$$x = B^{-1}(b - Ny^*)$$

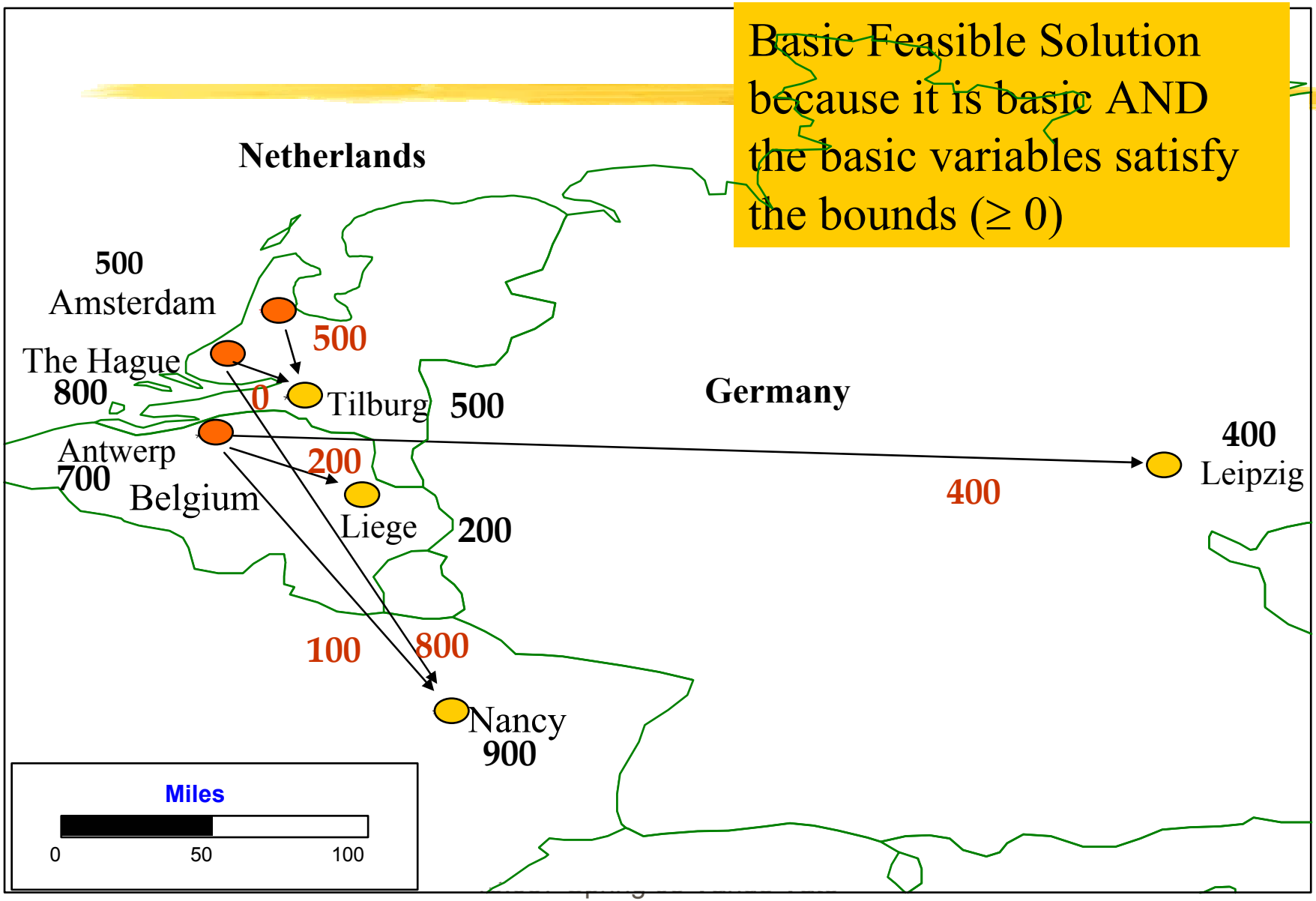
- B must be invertible and so square
- Question: We have 7 constraints (3 ports, 4 plants) and only 6 basic variables. How so?

* For those who care to know

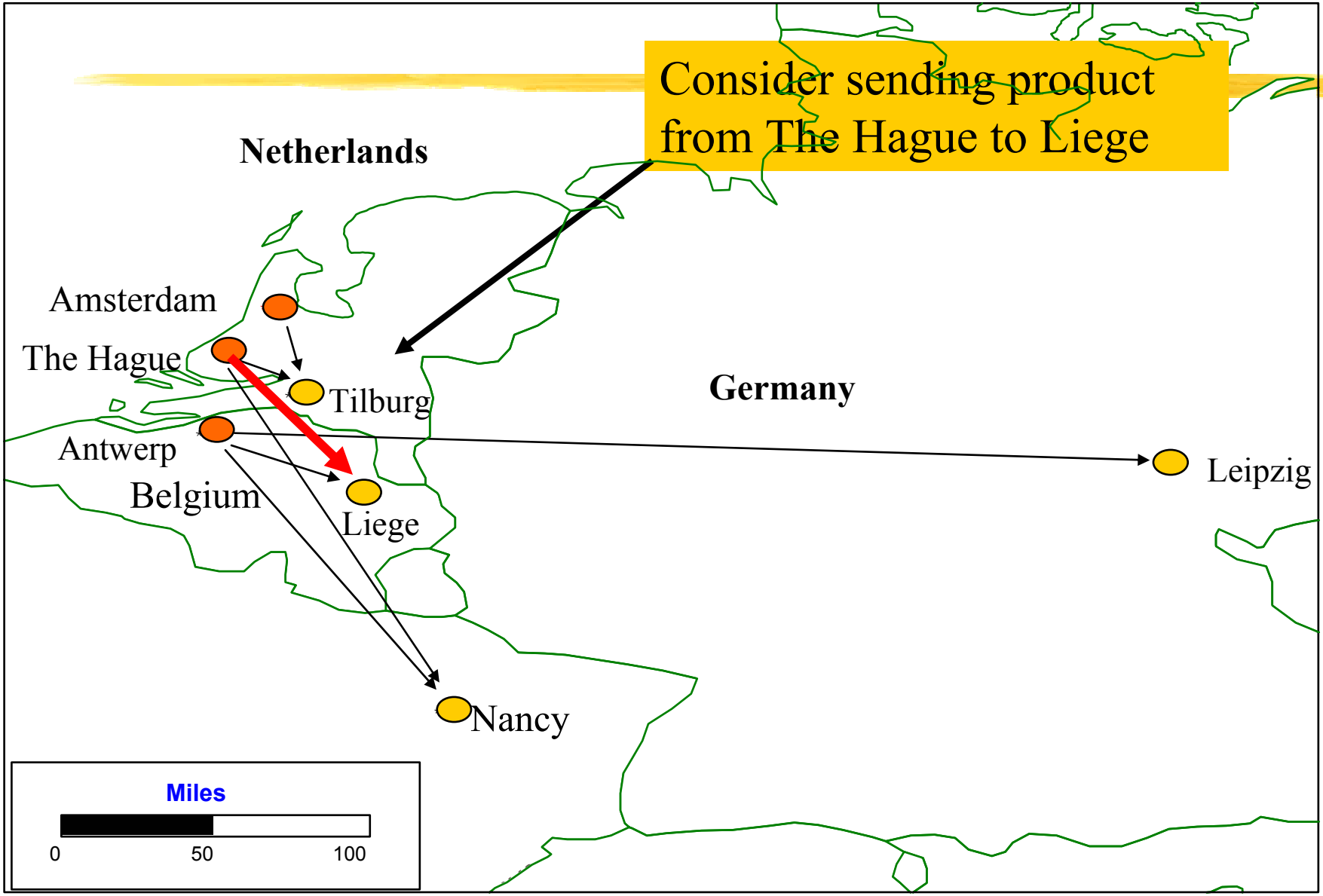
More Technical Detail



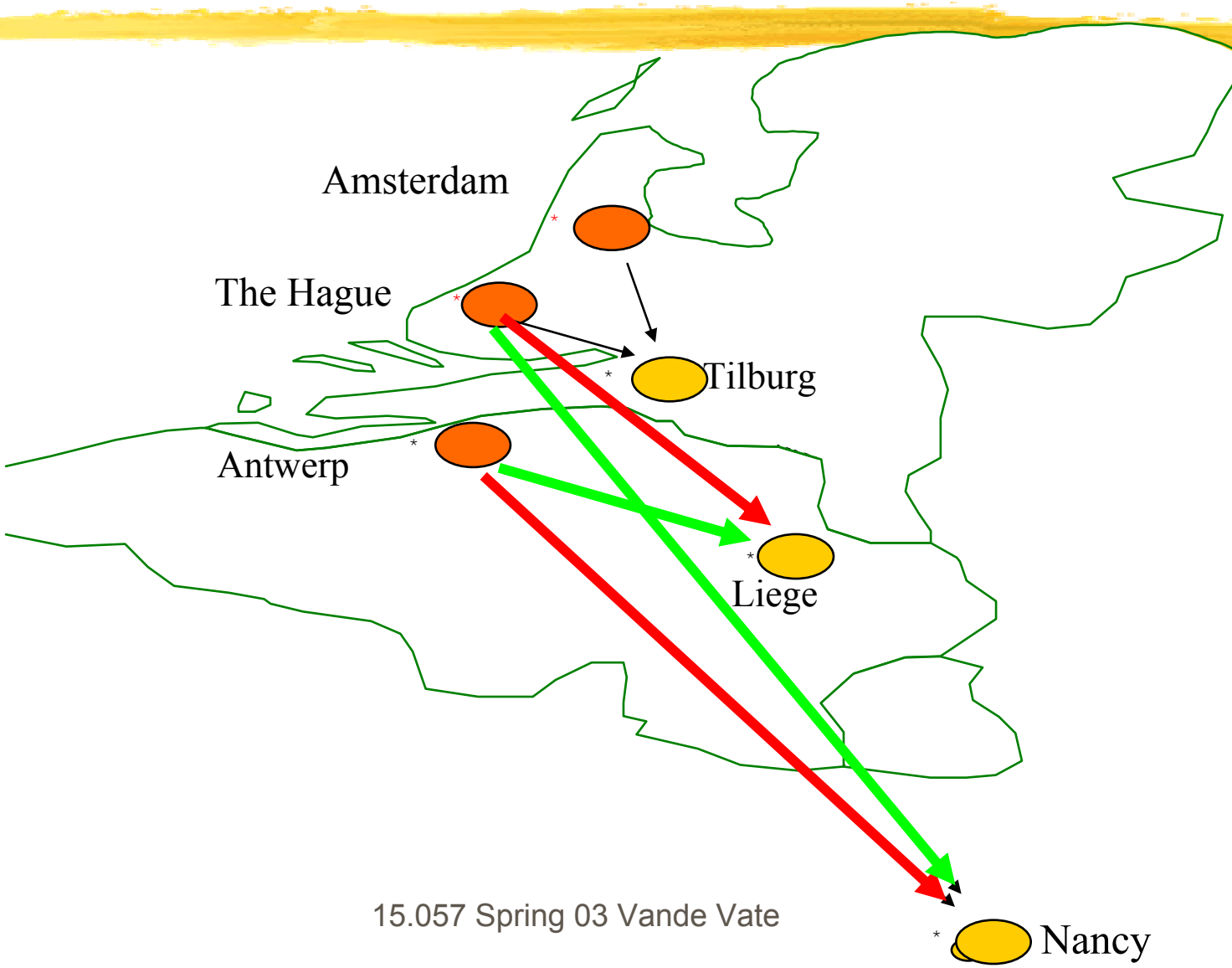
How Solver Works



Simple Improvement



Conserving Flow



Conserving Flow

Costs

$122

$ 40

$162

Saves

$100

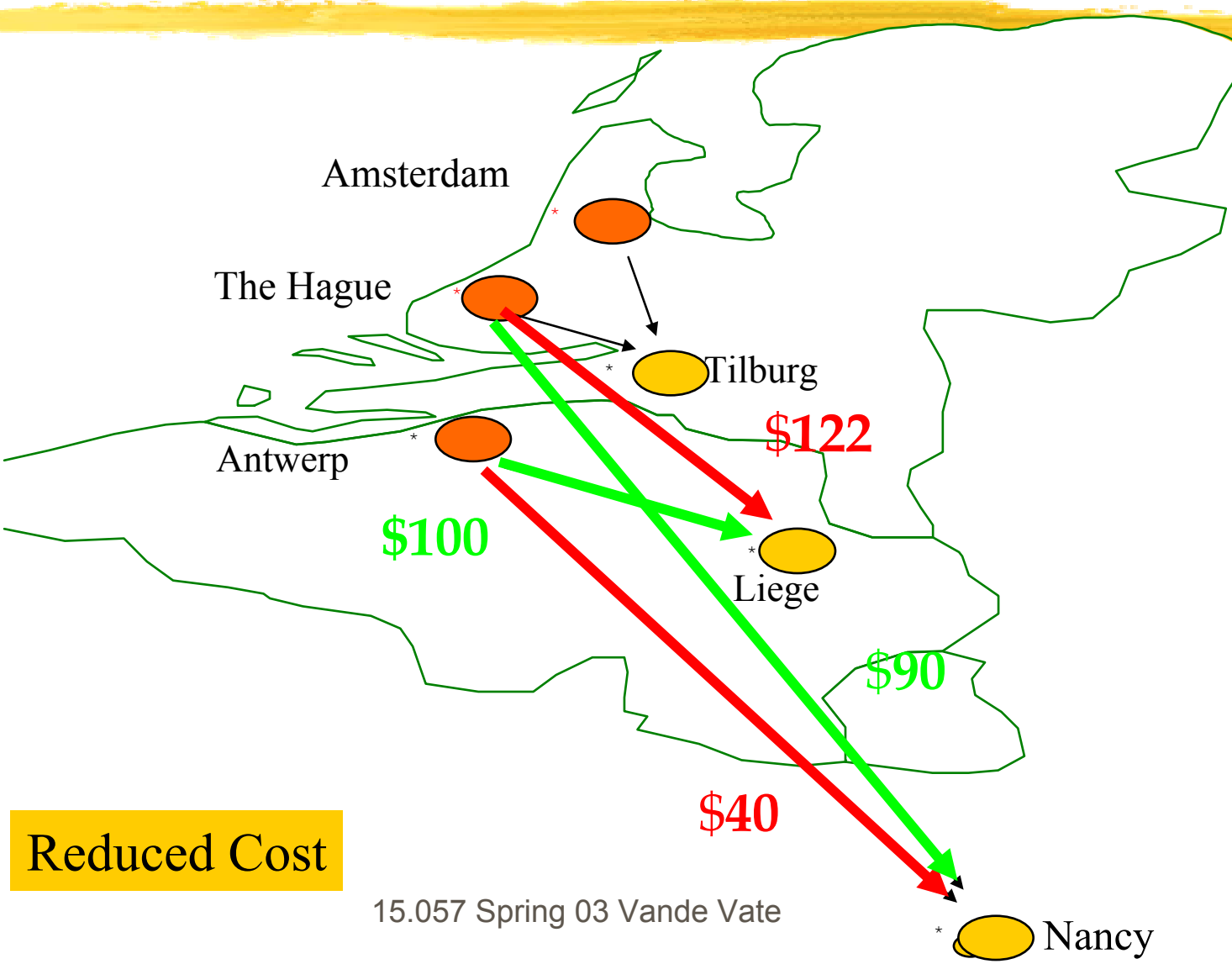
$ 90

$190

Net

$28

Reduced Cost

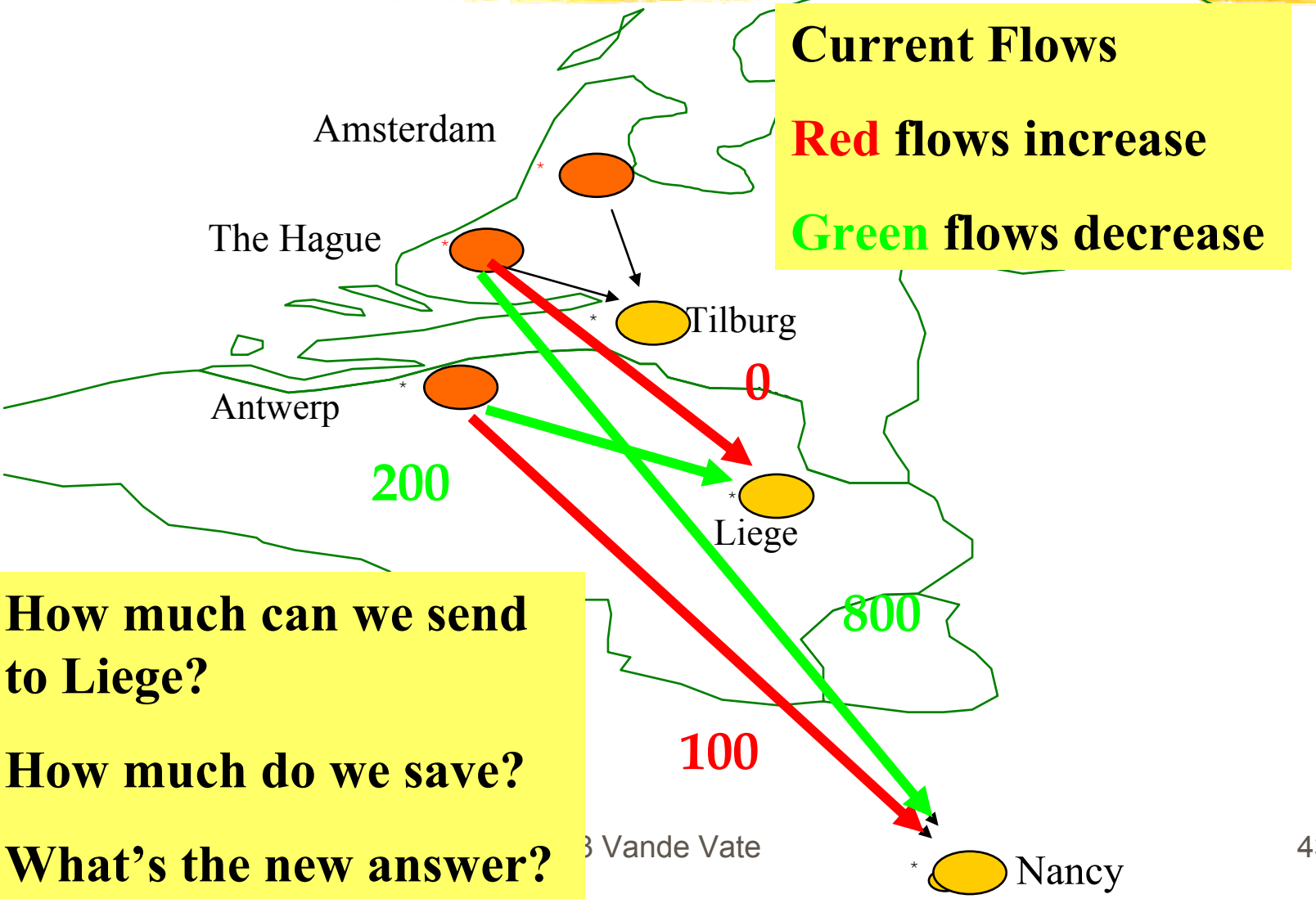


How Much Can We Save?

Costs
$122
$ 40
$162

Saves
$100
$ 90
$190

Net
$28



Current Flows

Red flows increase

Green flows decrease

How much can we send to Liege?

How much do we save?

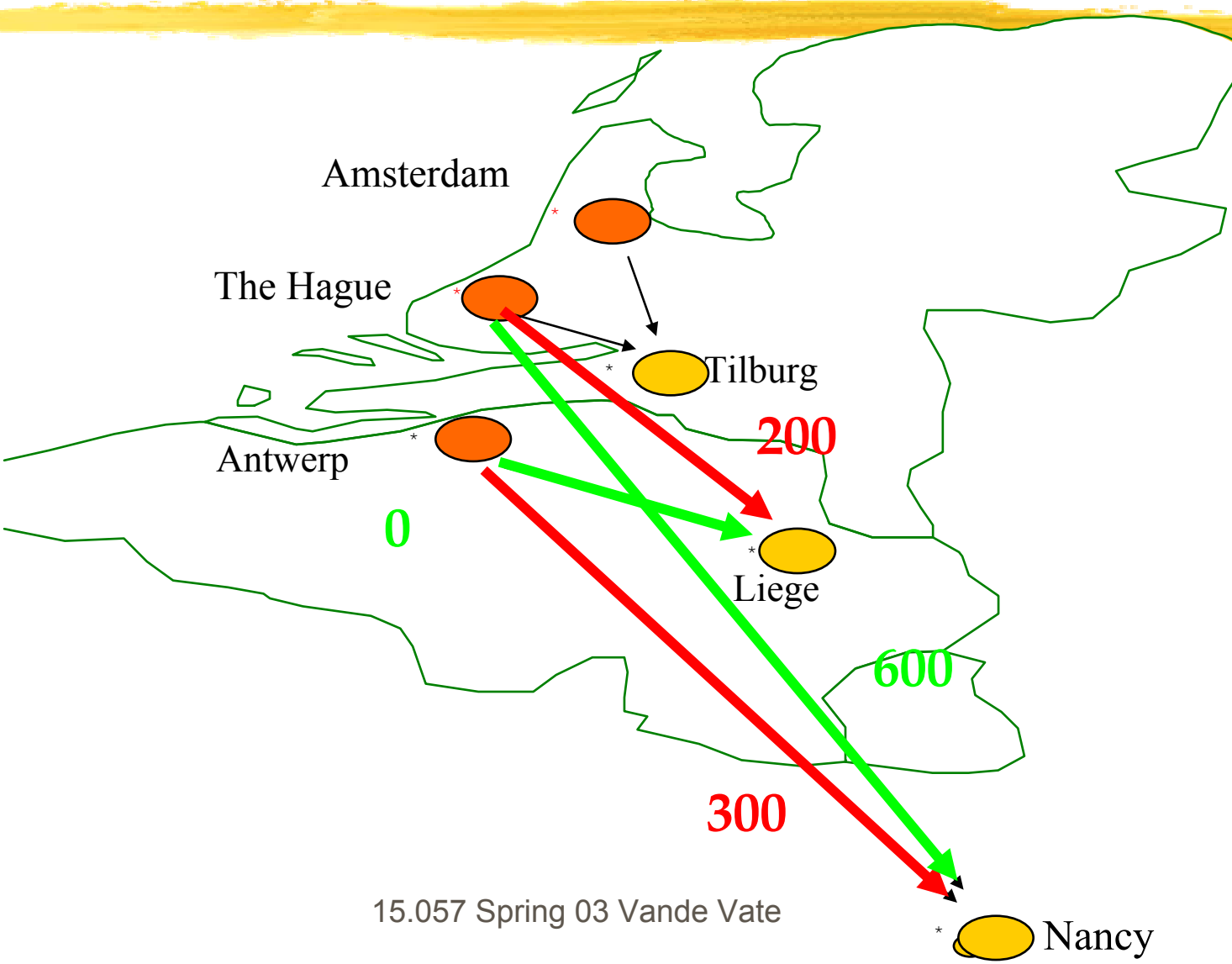
What's the new answer?

New Answer

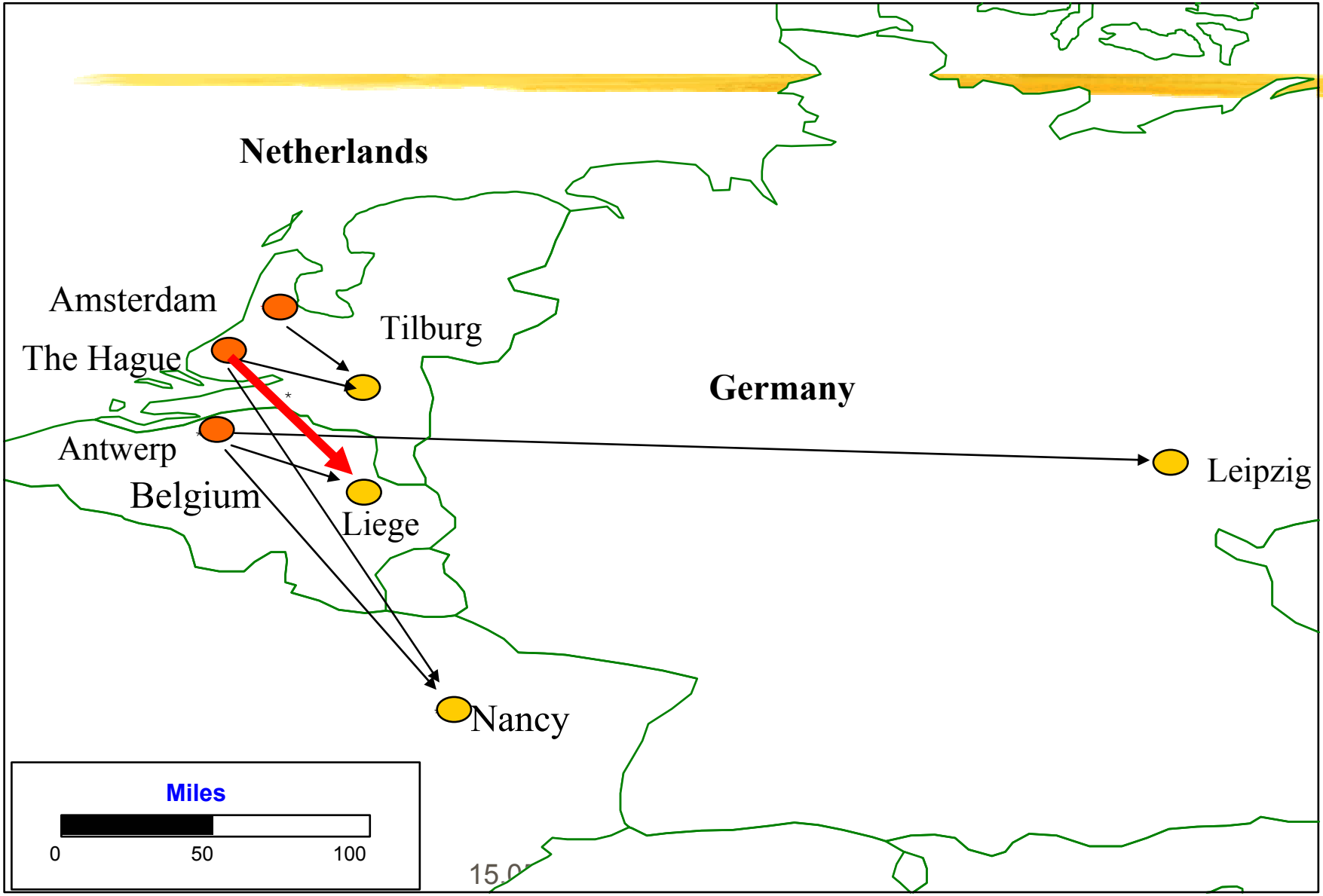
Costs
$122
$ 40
$162

Saves
$100
$ 90
$190

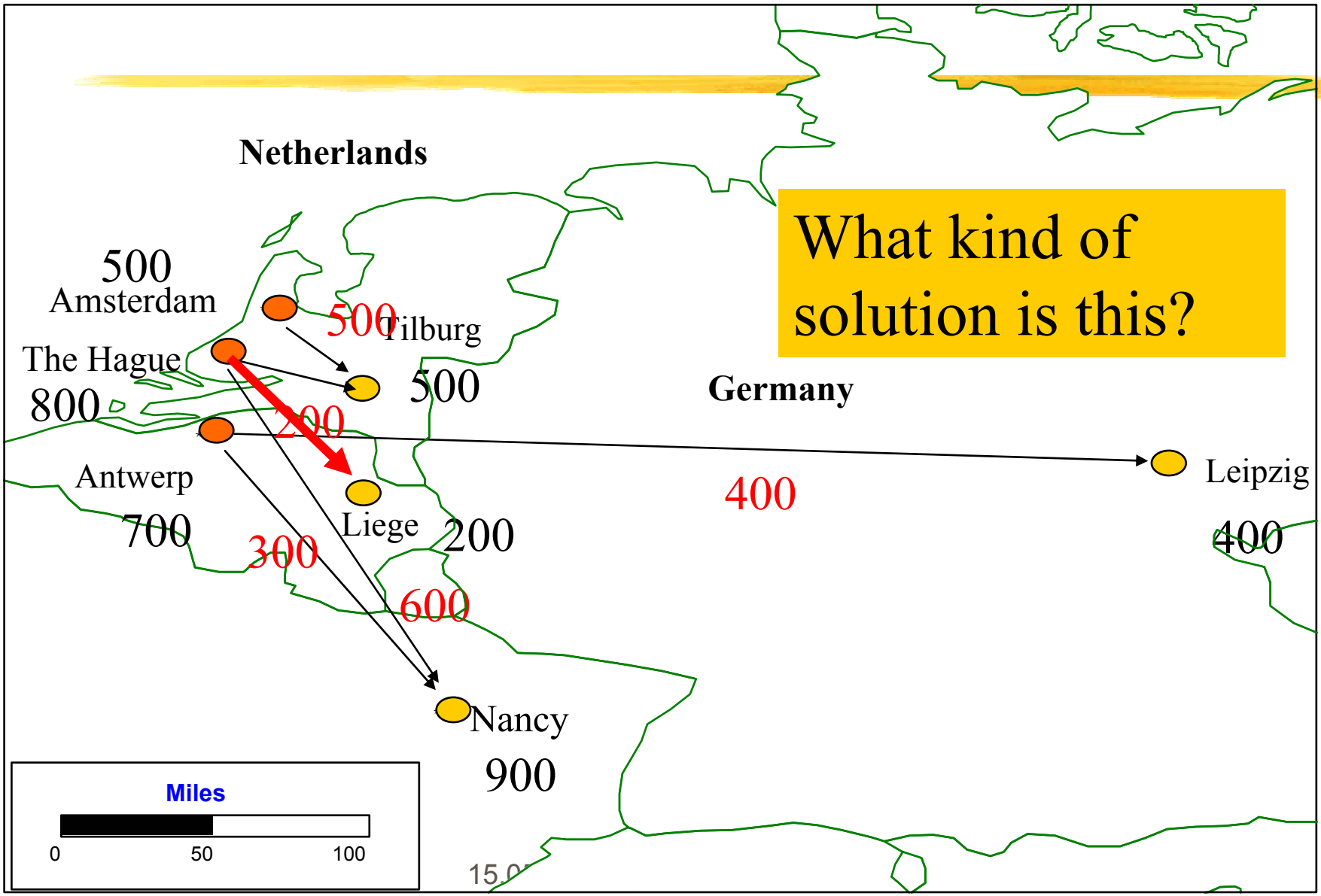
Net
$28



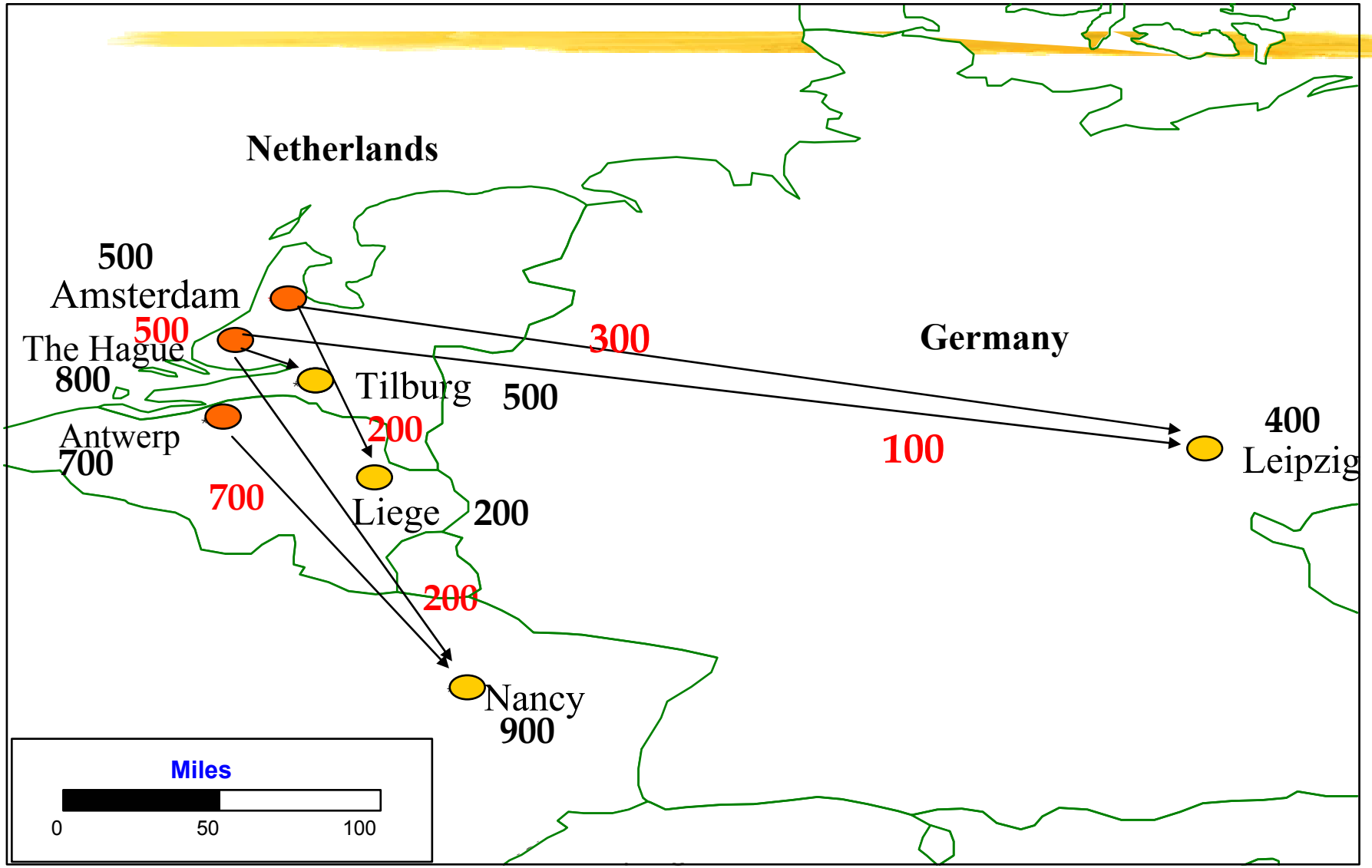
The New Answer



The New Answer



An Optimal Basic Feasible Solution



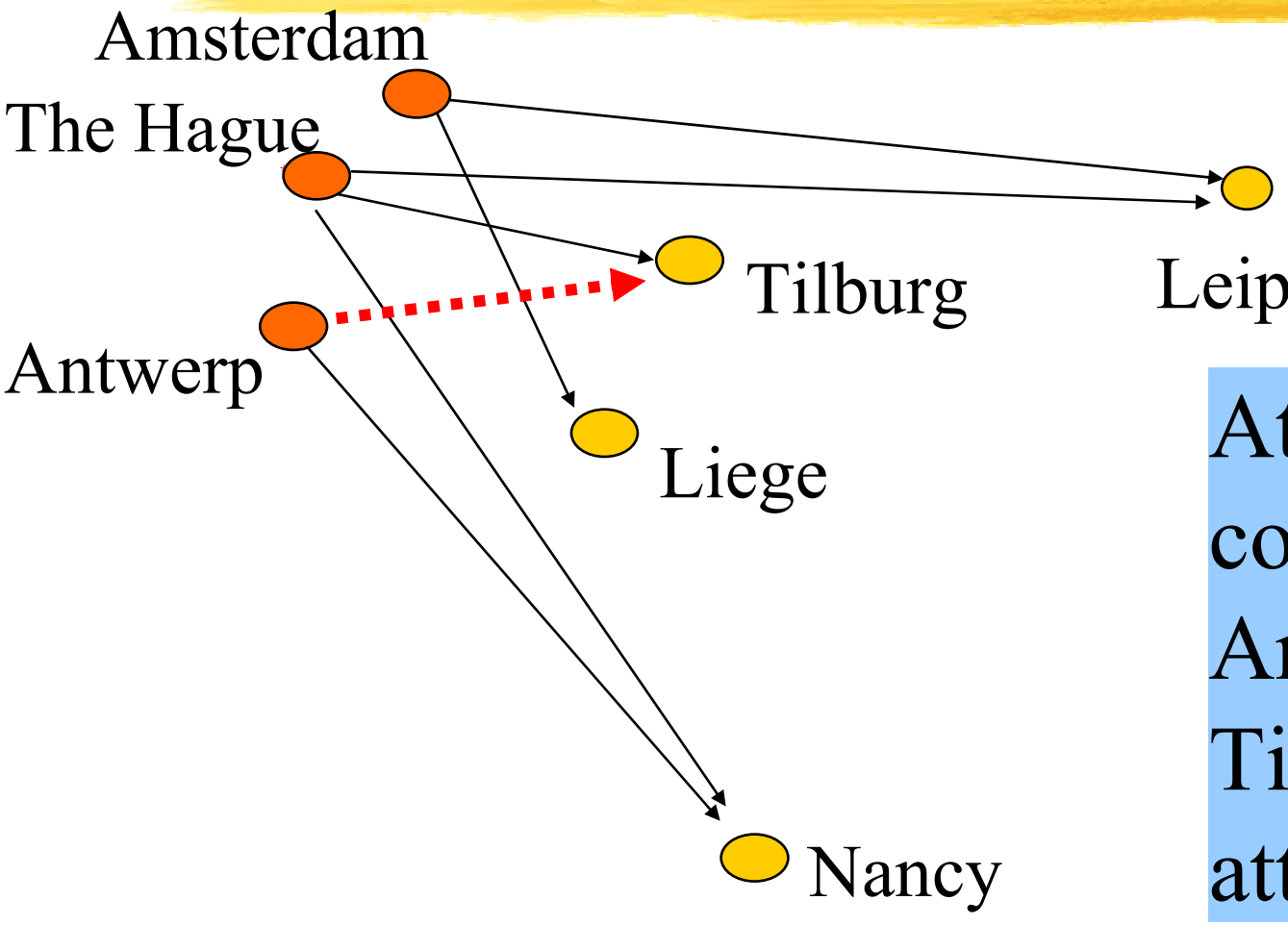
Summary

- Solver
  - Finds a basic feasible solution
    - Satisfies all the constraints
    - Using these variables there is just one answer
  - Computes reduced costs of non-basic variables one at a time
    - How would increasing the new variable affect cost?
  - Selects an entering variable
    - Increasing this non-basic variable saves money
  - Computes a leaving variable
    - What basic variable first reaches zero?
  - Repeats

Sensitivity Analysis

- How would the answer change if the data were a little different?
- Why is this important?
- Intuitive understanding

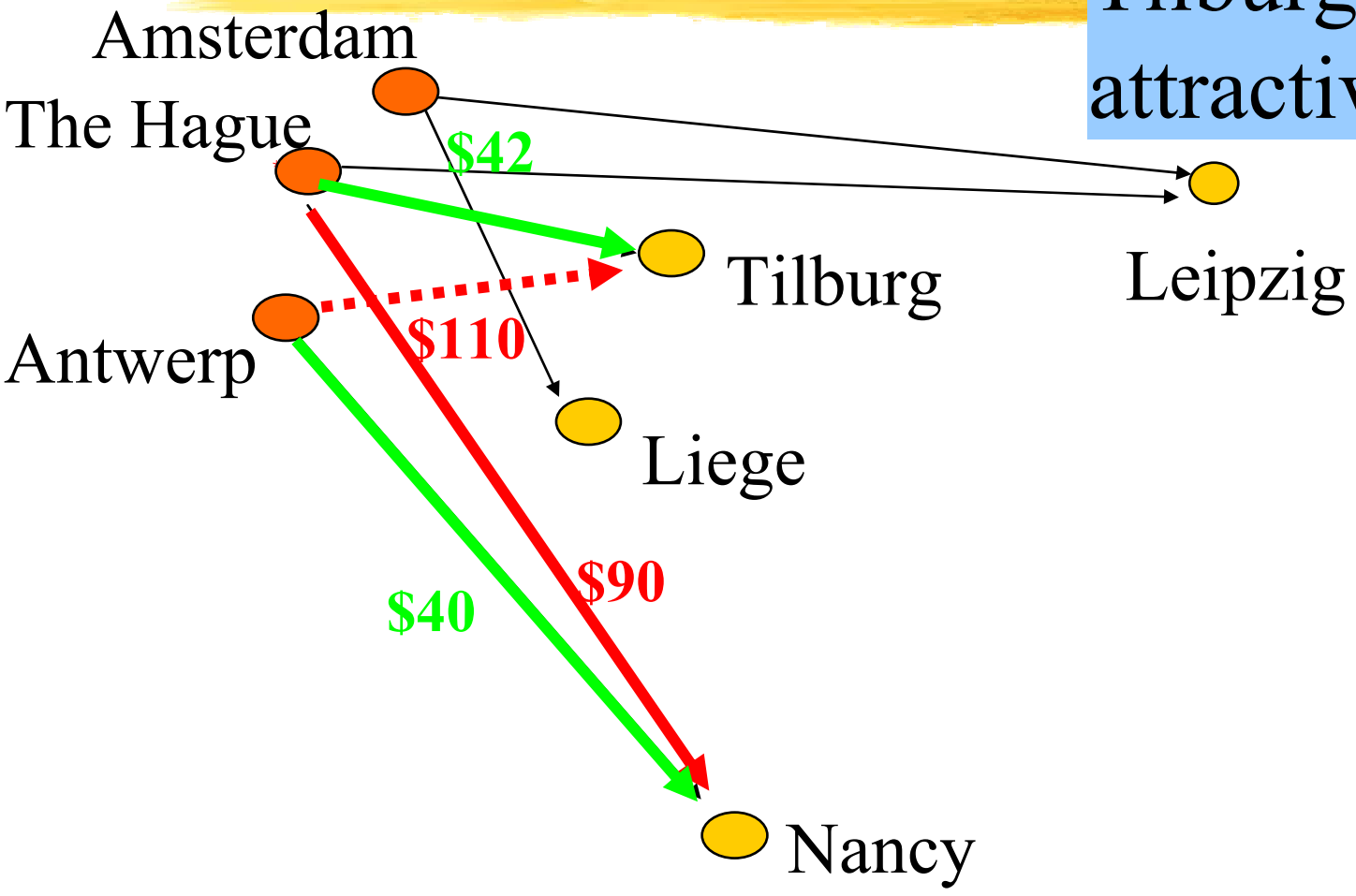
Price Sensitivity



At what unit cost would Antwerp to Tilburg be attractive?

Price Sensitivity

At what unit cost would Antwerp to Tilburg be attractive?



Costs:

$110

$ 90

$200

Savings:

$42

$40

$82

Net $118

The carrier would have to PAY us $8!

Try It!

Autopower Transportation Model

Unit Cost

From/To	Leipzig	Nancy	Liege	Tilburg
Amsterdam	$ 120.0	$ 130.0	$ 41.0	$ 59.5
Antwerp	$ 61.0	$ 40.0	$ 100.0	$ 110.0
The Hague	$ 102.5	$ 90.0	$ 122.0	$ 42.0

Shipments

From/To	Leipzig	Nancy	Liege	Tilburg	Total	Available
Amsterdam	-	-	-	-	-	500
Antwerp	-	-	-	-	-	700
The Hague	-	-	-	-	-	800
Total	-	-	-	-	-	
Required	400	900	200	500		

Total Cost

From/To	Leipzig	Nancy	Liege	Tilburg	Total
Amsterdam	$ -	$ -	$ -	$ -	$ -
Antwerp	$ -	$ -	$ -	$ -	$ -
The Hague	$ -	$ -	$ -	$ -	$ -
Total	$ -	$ -	$ -	$ -	$ -

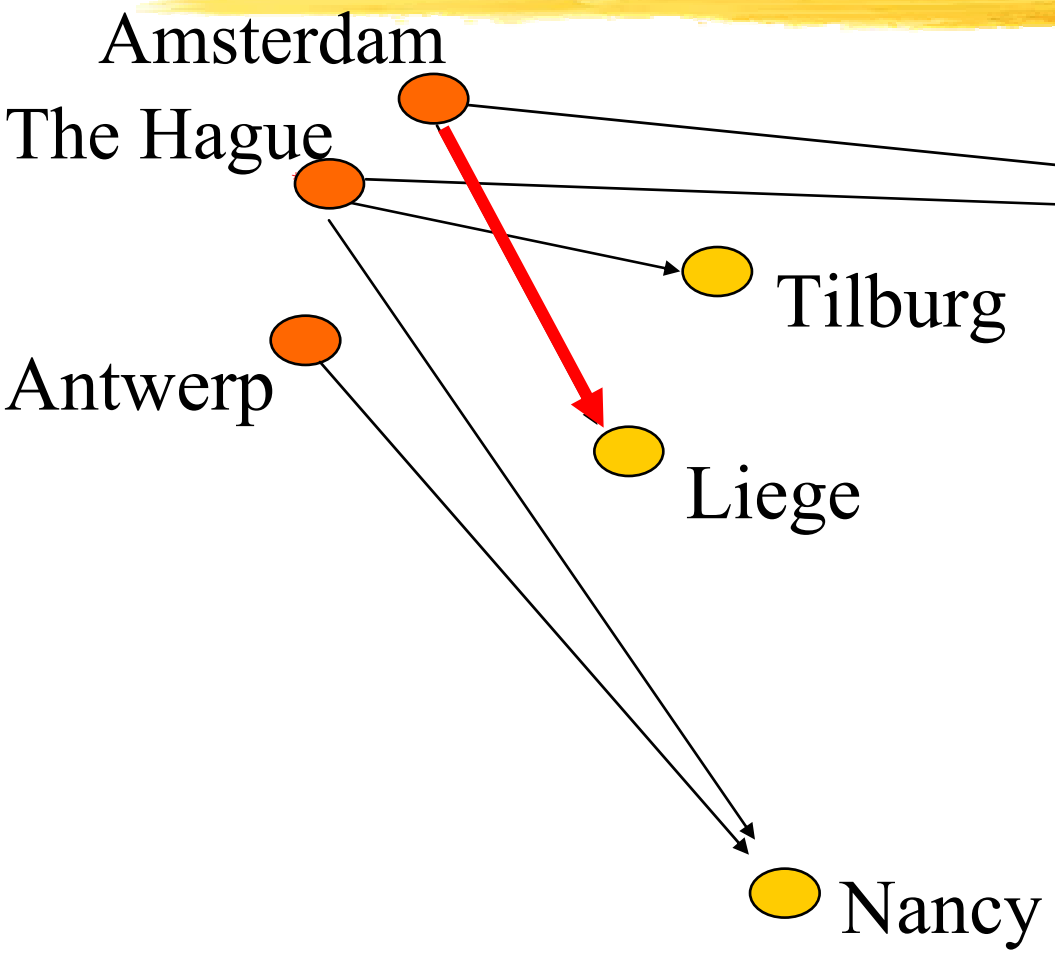
Reduced Costs are…

- The reduced cost of a variable is…

  The rate of change in the objective if we are forced to use some of that variable

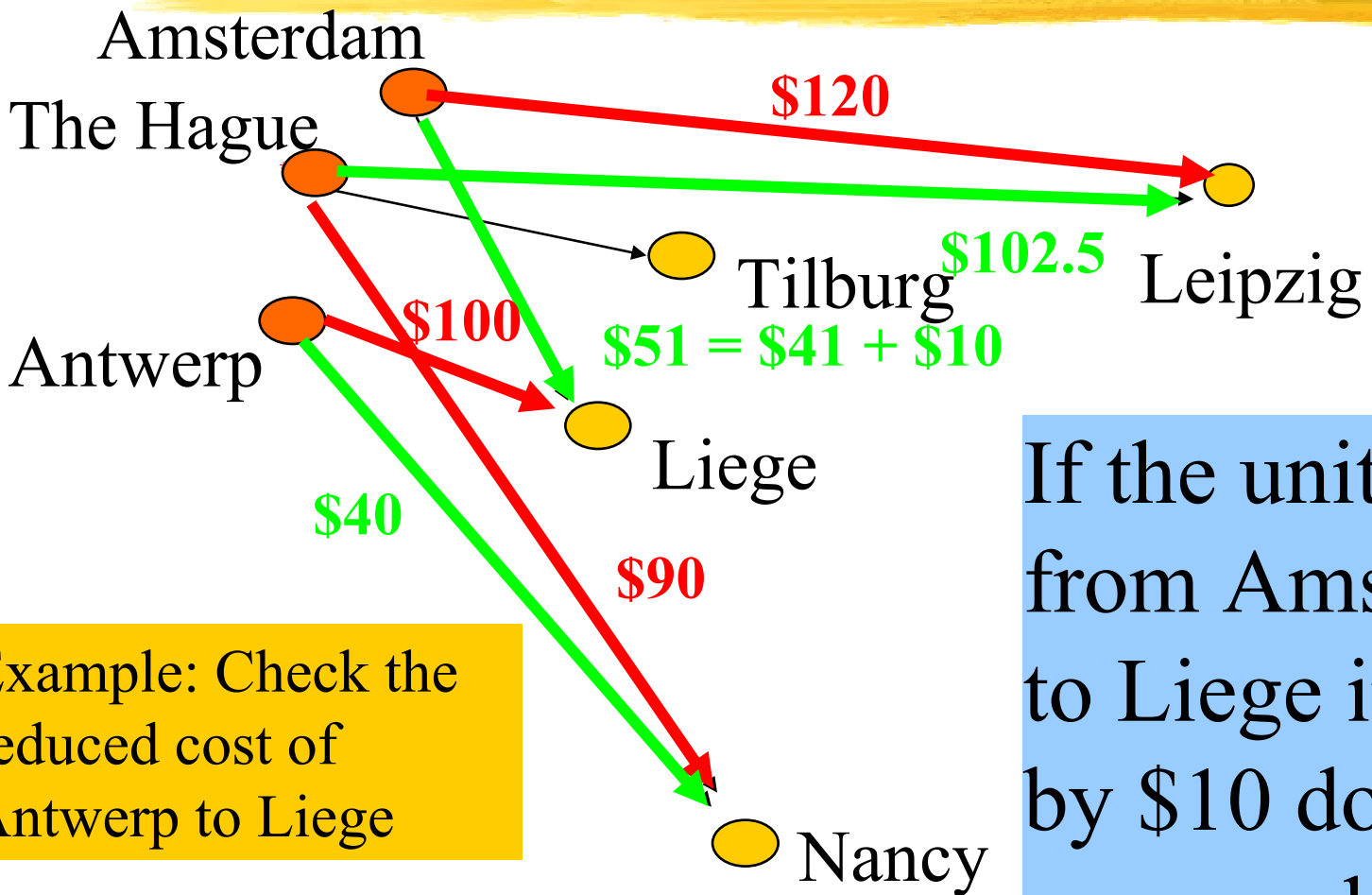
- The reduced costs of basic variables are 0

Price Sensitivity: Basic Variables



If the unit cost from Amsterdam to Liege increases by $10 does the answer change?

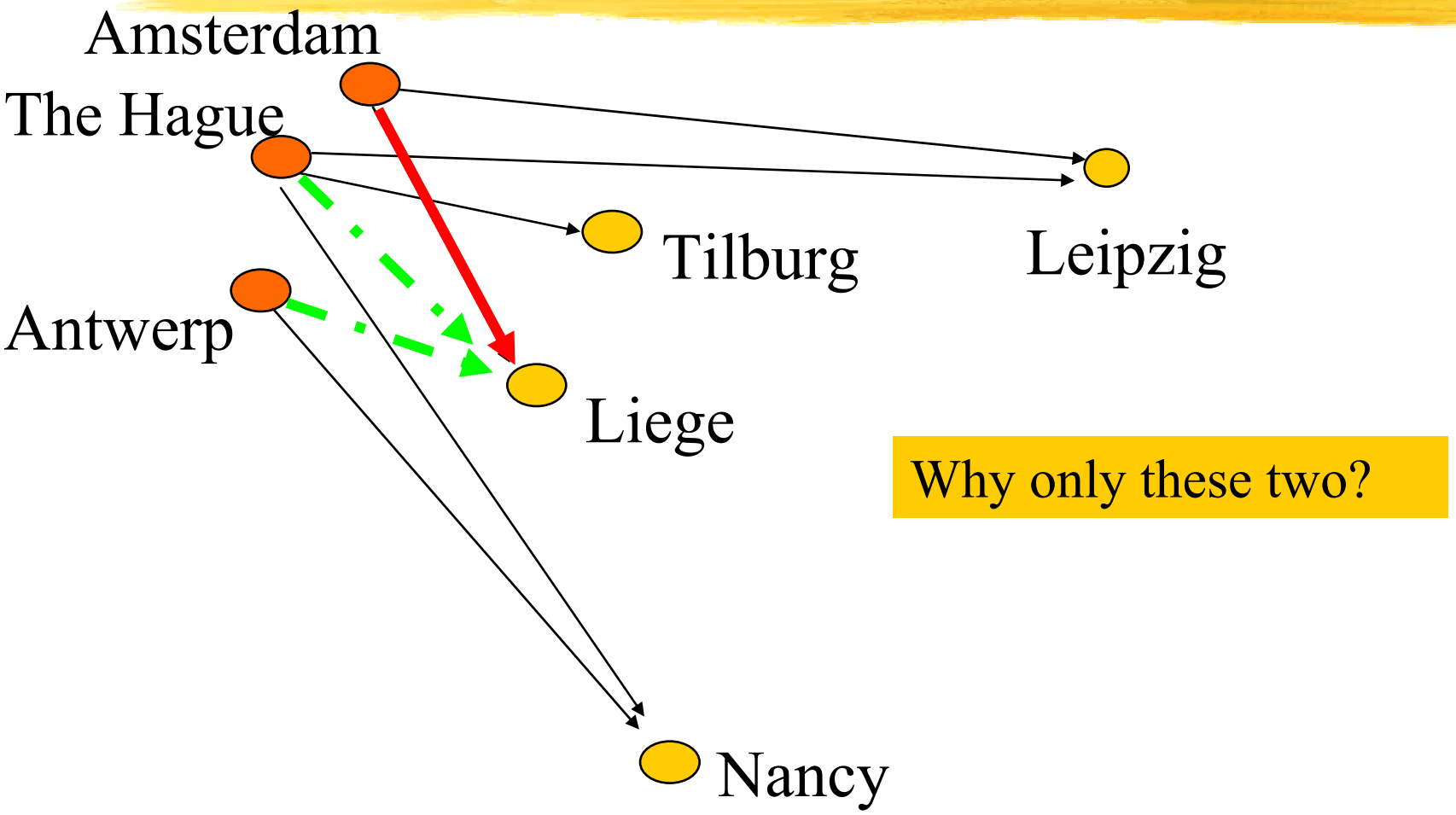
Checking Reduced Costs: Example



Example: Check the reduced cost of Antwerp to Liege

If the unit cost from Amsterdam to Liege increases by $10 does the answer change?

Check All Reduced Costs

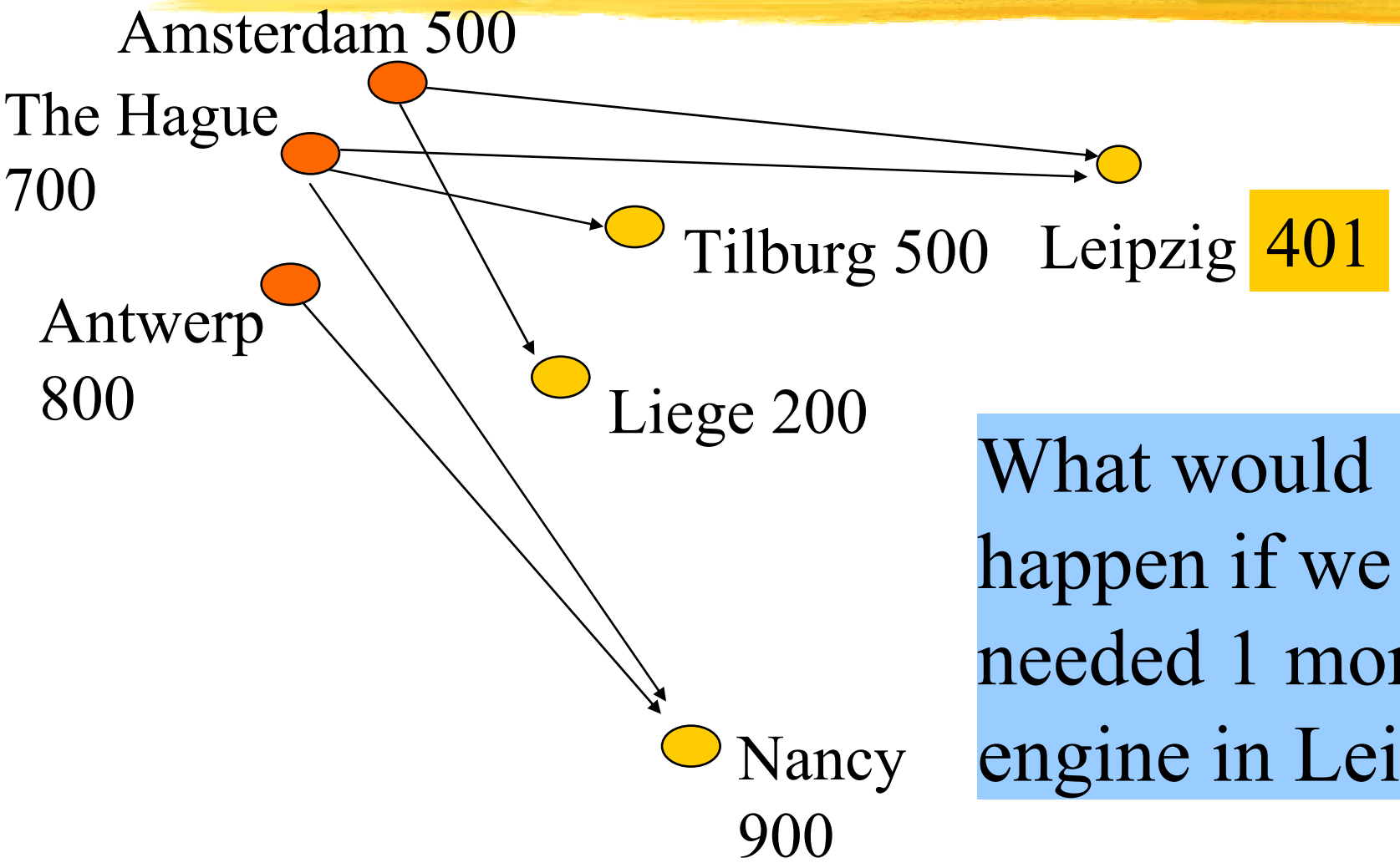


Value of Price Sensitivity?

Resource Sensitivity

- How would the objective value change if we had more of a resource
- Tells us the marginal value of that resource
- If the optimal solution doesn't use all of the resource, then…

Resource Sensitivity

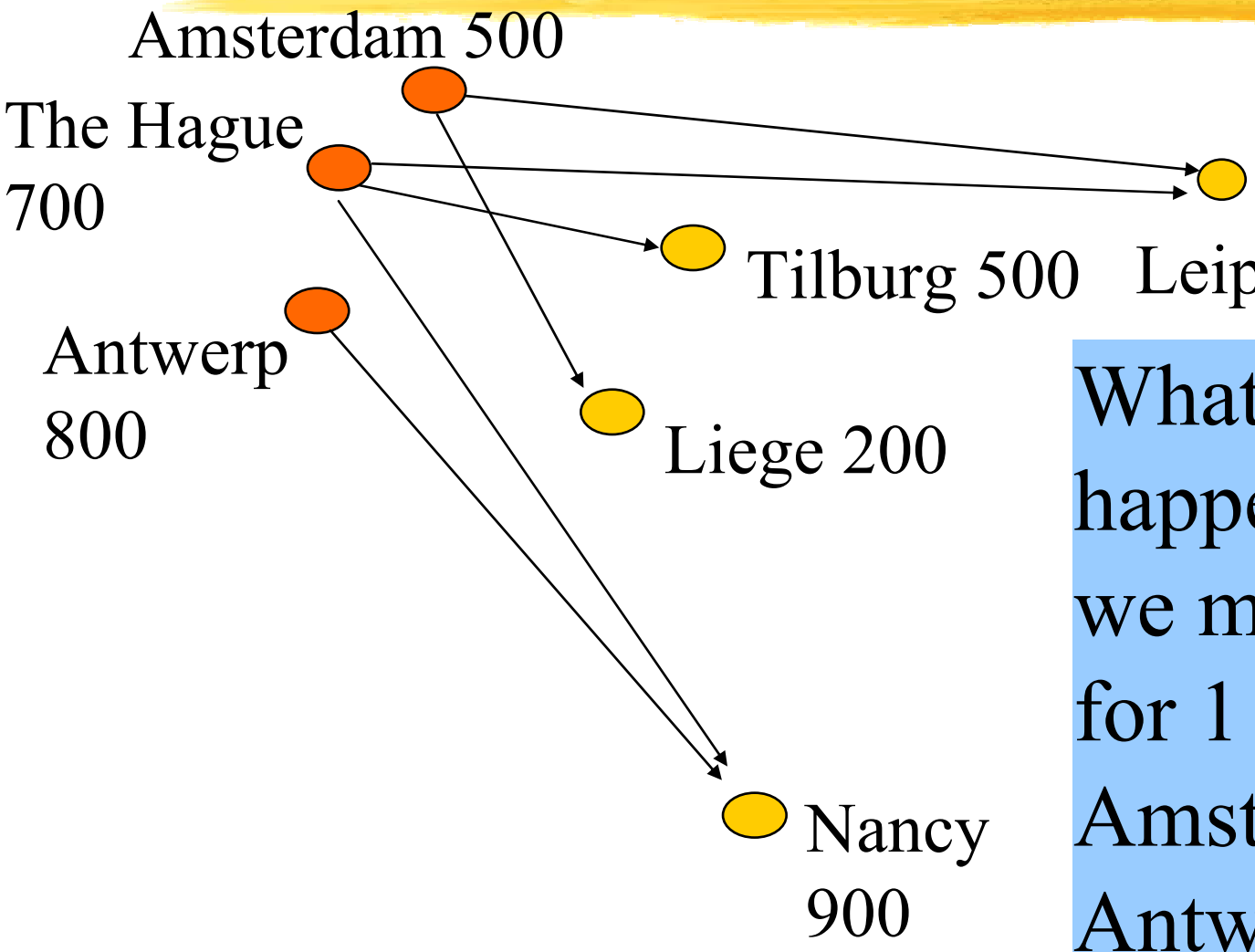


What would happen if we needed 1 more engine in Leipzig?

Infeasible

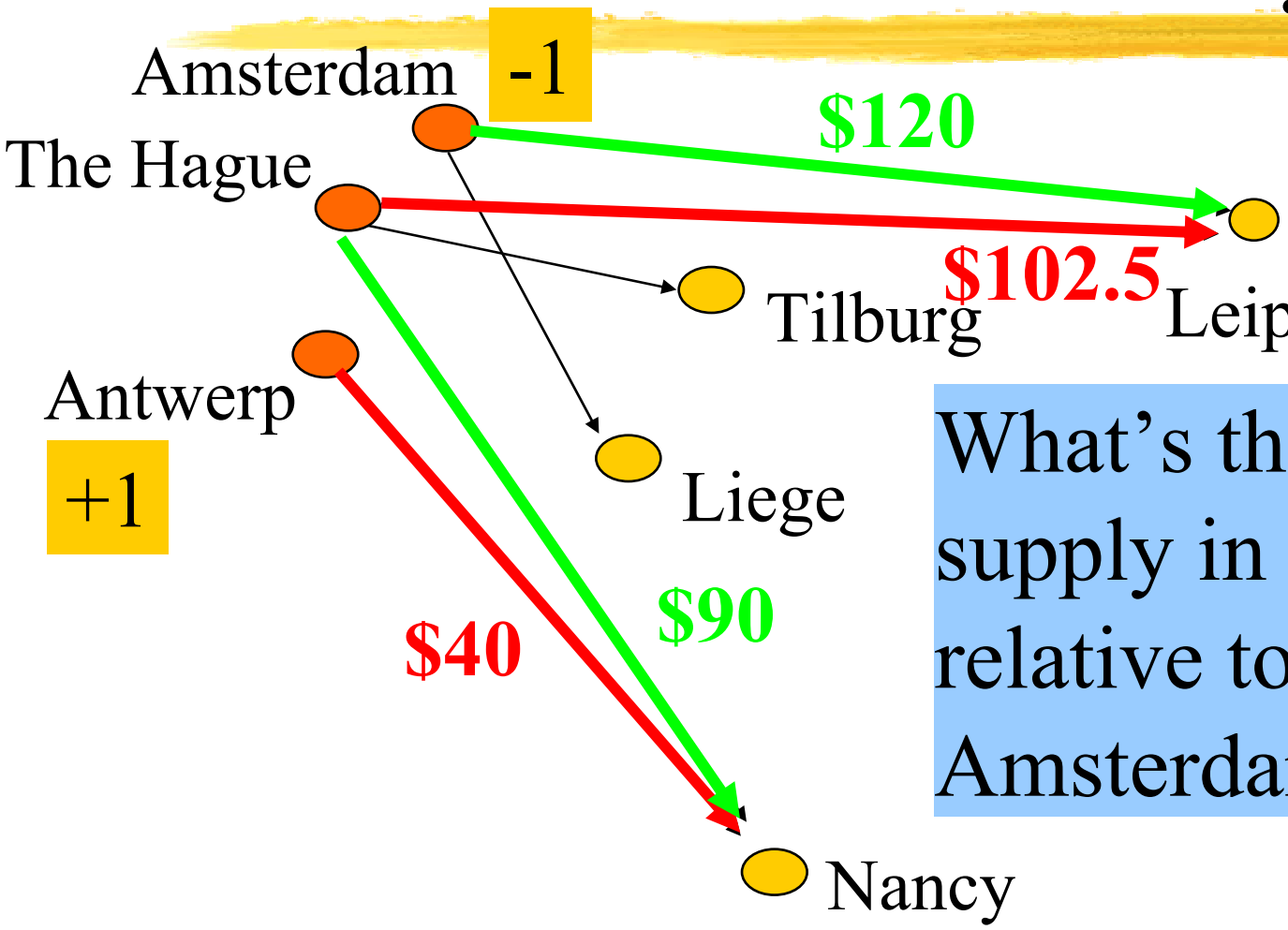
- Supply
  - Amsterdam 500
  - Antwerp 700
  - The Hague 800
  - Total 2,000
- Demand
  - Leipzig 400+1
  - Nancy 900
  - Liege 200
  - Tilburg 500
  - Total 2,000+1

Resource Sensitivity



What would happen to cost if we moved supply for 1 engine from Amsterdam to Antwerp?

Resource Sensitivity



What's the value of supply in Antwerp relative to supply in Amsterdam?

Value of Resource Sensitivity

A Special Feature

- We can eliminate any one of the constraints in this problem without changing the answers!
- Why?

Redundant Constraint

- Supply
  - Amsterdam 500
  - Antwerp 700
  - The Hague 800
  - Total 2,000
- Demand
  - Leipzig 400
  - Nancy 900
  - Liege 200
  - Tilburg 500
  - Total 2,000

- Know shipments from
  - Amsterdam
  - Antwerp
- And they provide...
  - Leipzig 200
  - Nancy 600
  - Liege 0
  - Tilburg 400

That means…

- We can arbitrarily set the (relative) value of one constraint to 0. (the one we throw away)
- Set the shadow price or marginal value of supply in Amsterdam to 0, then the shadow price of supply in Antwerp is -$67.5.
- Why negative?
- If we had extra supply, where would we want it? Amsterdam or Antwerp?

Internally Consistent

- Given the Shadow Prices

Constraint	Shadow Price
Amsterdam	-
Antwerp	-67.5
The Hague	-17.5
Leipzig	120
Nancy	107.5
Liege	41
Tilburg	59.5

Example: Antwerp-Tilburg

Reduced Cost = Cost - Origin - Dest.

$$118 = 110 - (-67.5) - 59.5$$
$$= 110 + 8$$

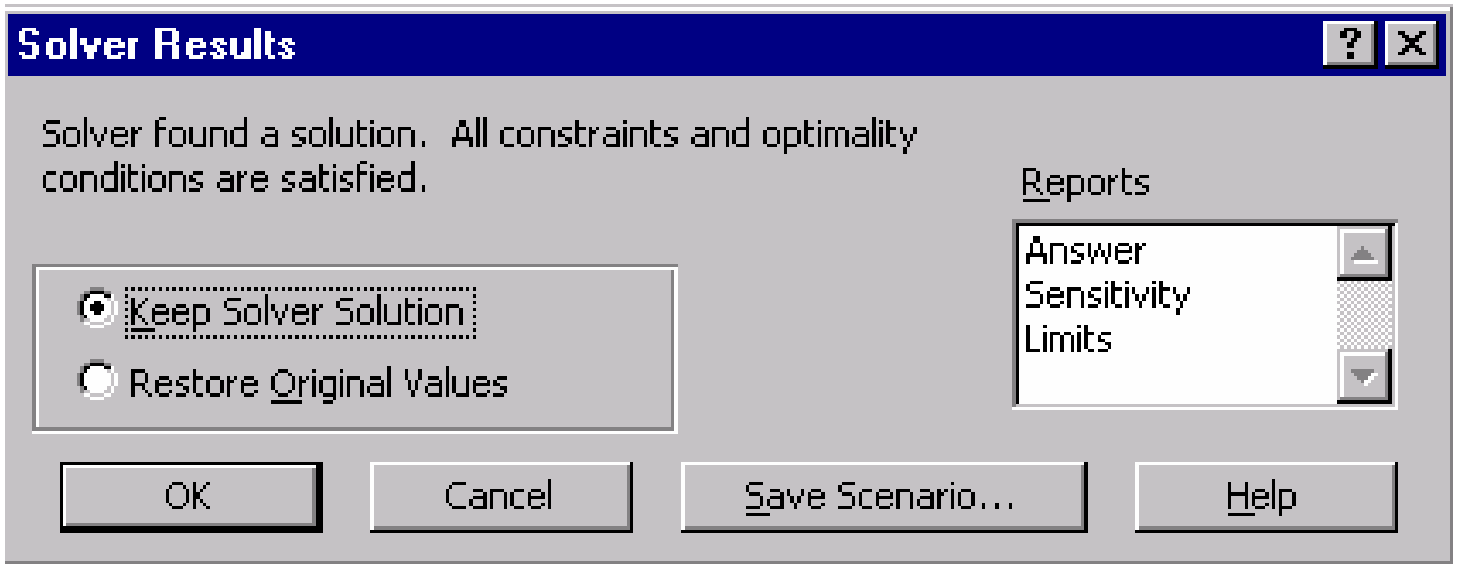
- We can calculate the Reduced Costs

Edge	Value	Reduced Cost	Cost
Amsterdam Leipzig	300	-	120.0
Amsterdam Nancy	-	22.5	130.0
Amsterdam Liege	200	-	41.0
Amsterdam Tilburg	-	-	59.5
Antwerp Leipzig	-	8.5	61.0
Antwerp Nancy	700	-	40.0
Antwerp Liege	-	126.5	100.0
Antwerp Tilburg	-	118.0	110.0
The Hague Leipzig	100	-	102.5
The Hague Nancy	200	-	90.0
The Hague Liege	-	98.5	122.0

Summary

- Solver can tell us at what price a non-basic (inactive) variable will be attractive through the Reduced Cost.
- Solver can tell us how changes in the price of a basic variable affect the solution
- Solver can tell us the value of a resource via the Shadow Price or Marginal Value

Sensitivity Info From Solver



Sensitivity Report: Price

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$9	Amsterdam Leipzig	300	-	120	0	17.5
$D$9	Amsterdam Nancy	-	22.5	130	1E+30	22.49999998
$E$9	Amsterdam Liege	200	-	41	98.49999999	41
$F$9	Amsterdam Tilburg	-	-	59.5	1E+30	0
$C$10	Antwerp Leipzig	-	8.5	60.99999999	1E+30	8.499999988
$D$10	Antwerp Nancy	700	-	40	8.499999988	1E+30
$E$10	Antwerp Liege	-	126.5	99.99999998	1E+30	126.5
$F$10	Antwerp Tilburg	-	**118.0**	110	1E+30	118
$C$11	The Hague Leipzig	100	-	102.5	8.499999988	0
$D$11	The Hague Nancy	200	-	90	22.49999998	8.499999988
$E$11	The Hague Liege	-	98.5	122	1E+30	98.49999999
$F$11	The Hague Tilburg	500	-	42	0	59.5

Sensitivity Report: Resource

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$G$9	Amsterdam Total	500	-	500	1E+30	0
$G$10	Antwerp Total	700	(67.5)	700	200	0
$G$11	The Hague Total	800	(17.5)	800	300	0
$C$12	Total Leipzig	400	120.0	400	0	300
$D$12	Total Nancy	900	107.5	900	0	200
$E$12	Total Liege	200	41.0	200	0	200
$F$12	Total Tilburg	500	59.5	500	0	300

Value

- If our proposal comes up non-basic, reduced cost tells us how much harder we have to work to make it attractive.
- If we are unsure of prices, price sensitivity can tell us whether it is worth refining our estimates of the values
- Marginal values can help us target investments in capacity

Caveats

- Sensitivity Analysis is pretty nerdy stuff
- Technical difficulties
- Only meaningful for changes to a single value
- Only meaningful for *small* changes
- Doesn't work for Integer Programming
- Can always just change the values and re-solve, but...

Bad Example

Autopower Transportation Model

Unit Cost

From/To	Leipzig	Nancy	Liege	Tilburg
Amsterdam	$ 120.0	$ 120.0	$ 120.0	$ -
Antwerp	$ -	$ -	$ -	$ 120.0
The Hague	$ 120.0	$ -	$ 120.0	$ 120.0

Shipments

From/To	Leipzig	Nancy	Liege	Tilburg	Total	Available
Amsterdam	-	-	-	500	500	500
Antwerp	400	100	200	-	700	700
The Hague	-	800	-	-	800	800
Total	400	900	200	500	-	
Required	400	900	200	500		

Total Cost

From/To	Leipzig	Nancy	Liege	Tilburg	Total
Amsterdam	$ -	$ -	$ -	$ -	$ -
Antwerp	$ -	$ -	$ -	$ -	$ -
The Hague	$ -	$ -	$ -	$ -	$ -
Total	$ -	$ -	$ -	$ -	$ -

Sensitivity

- Moving one unit from Liege to Tilburg should save $120

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$12	Total Leipzig	400	120	400	0	0
$D$12	Total Nancy	900	120	900	0	0
$E$12	**Total Liege**	**200**	**120**	**200**	**0**	**0**
$F$12	Total Tilburg	500	-	500	0	500
$G$9	Amsterdam Total	500	-	500	1E+30	0
$G$10	Antwerp Total	700	(120)	700	0	0
$G$11	The Hague Total	800	(120)	800	0	0

- Solver says the price is not good for 1 unit

Try It!

Autopower Transportation Model

Unit Cost

From/To	Leipzig	Nancy	Liege	Tilburg
Amsterdam	$ 120.0	$ 120.0	$ 120.0	$ -
Antwerp	$ -	$ -	$ -	$ 120.0
The Hague	$ 120.0	$ -	$ 120.0	$ 120.0

Shipments

From/To	Leipzig	Nancy	Liege	Tilburg	Total	Available
Amsterdam	-	-	-	500	500	500
Antwerp	400	100	200	-	700	700
The Hague	-	800	-	-	800	800
Total	400	900	200	500	-	
Required	400	900	200	500		

Total Cost

From/To	Leipzig	Nancy	Liege	Tilburg	Total
Amsterdam	$ -	$ -	$ -	$ -	$ -
Antwerp	$ -	$ -	$ -	$ -	$ -
The Hague	$ -	$ -	$ -	$ -	$ -
Total	$ -	$ -	$ -	$ -	$ -

Thursday

- Spreadsheet Models
  - 03ShortestPathModel.xls
  - 04TransshipmentModel.xls
  - 05SingaporeElectricGeneratorModel.xls
- Download the free student version of the AMPL/CPLEX 8.0 System from www.ampl.com
- http://www.ampl.com/cm/cs/what/ampl/DOWNLOADS/cplex71.html#new
- AMPL Example Model
  - TransportationModel.mod
- Access Database
  - TransportationData.mdb