Modeling Objectives

- Goal Programming
- Several Objectives
- Minimum Regret Models

Workforce Scheduling

- Some days we will have too many workers
- Excess
- Only concerned with the largest excess
- Minimize the largest Excess

Challenge

- Formulate a Solver Model

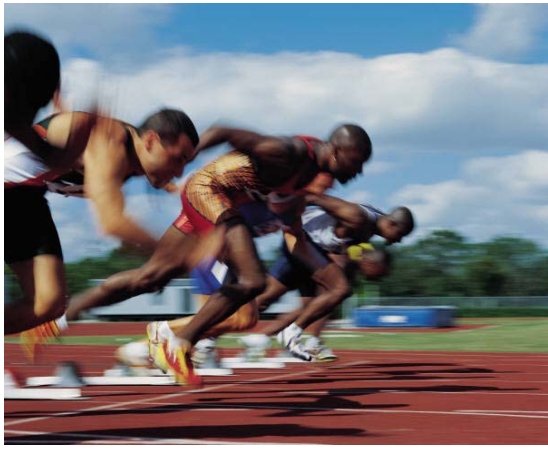
Formulating the LP

Scheduling Postal Workers

Shift	Mon - Fri	Tues - Sat	Wed - Sun	Thurs - Mon	Fri - Tues	Sat - Wed	Sun - Thurs	
Day								**Demand**
Mon	1			1	1	1	1	**17**
Tues	1	1			1	1	1	**13**
Wed	1	1	1			1	1	**15**
Thurs	1	1	1	1			1	**19**
Fri	1	1	1	1	1			**14**
Sat		1	1	1	1	1		**16**
Sun			1	1	1	1	1	**11**

Enhancement

- Given Goals:
  - Have the right number of workers each day
- Cost for exceeding and falling short
  - |Actual Workers – Desired Number|
- Challenge: Formulate a Linear Model



Formulating the LP

Scheduling Postal Workers

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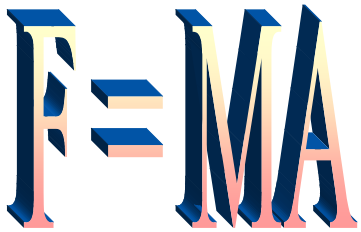
Sun			1	1	1	1	1	**11**

Policy vs Physical Consts

- Physical/Logical Constraints
  - Flow Conservation
  - Capacity
  - ...
- Policy Constraints
  - Meet service levels
  - ...

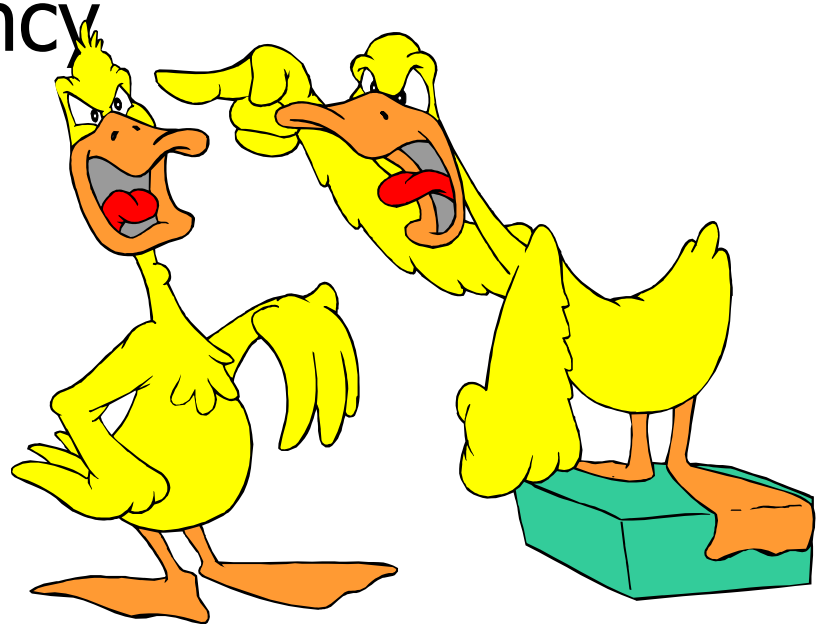


Small Violations

- Can violate policy constraints a little
- Sometimes can violate physical constraints
  - Capacity
  - Raw materials
  - …
- Handle with "deviation from targets"

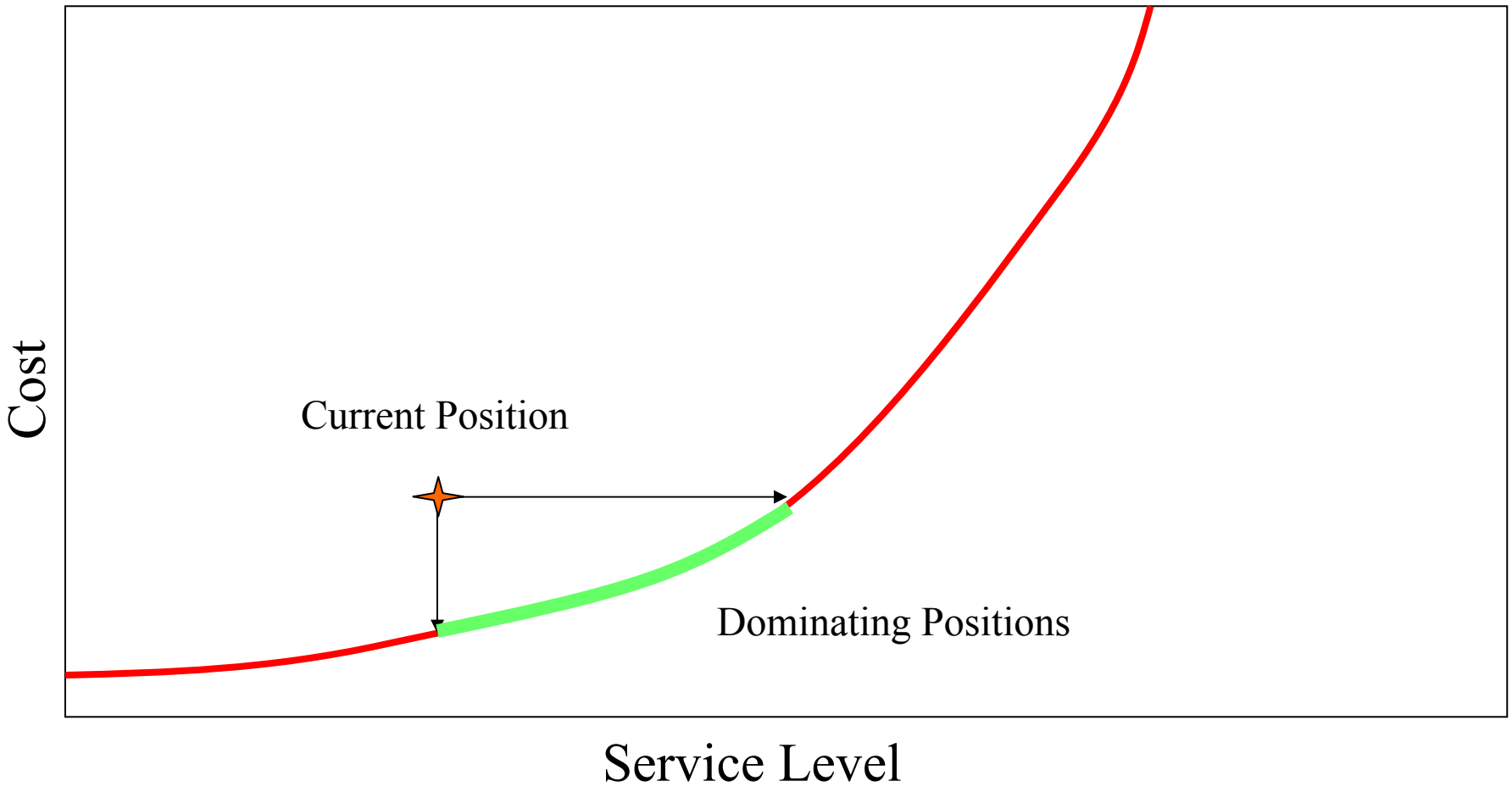
Conflicting Objectives

- Cost vs Service
- Price vs Quality
- Revenue vs Market Share
- Reach vs Frequency
- Risk vs Return
- …



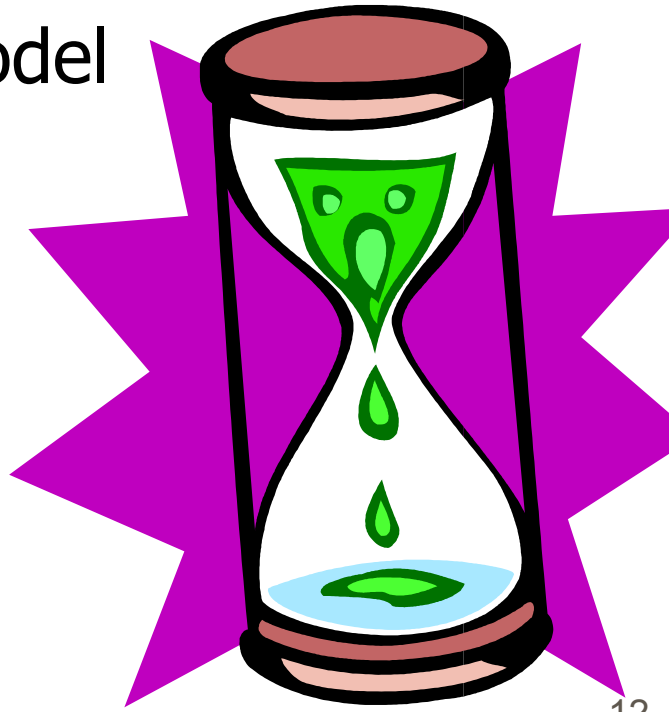
Other Examples

Undominated Optima



Getting to the Frontier

- Associate a value with Service ($/Time)
- Change this value of Time
- Systematic change using Sensitivity Analysis
- Remember the Retail Pricing Model



Reach vs Frequency

- Invest in Marketing
  - Radio Ads
  - TV Ads
  - Newspaper Ads
- Want
  - Reach: Lots of people to see the ads
  - Frequency: People to see the adds lots of times
- Generally conflicting

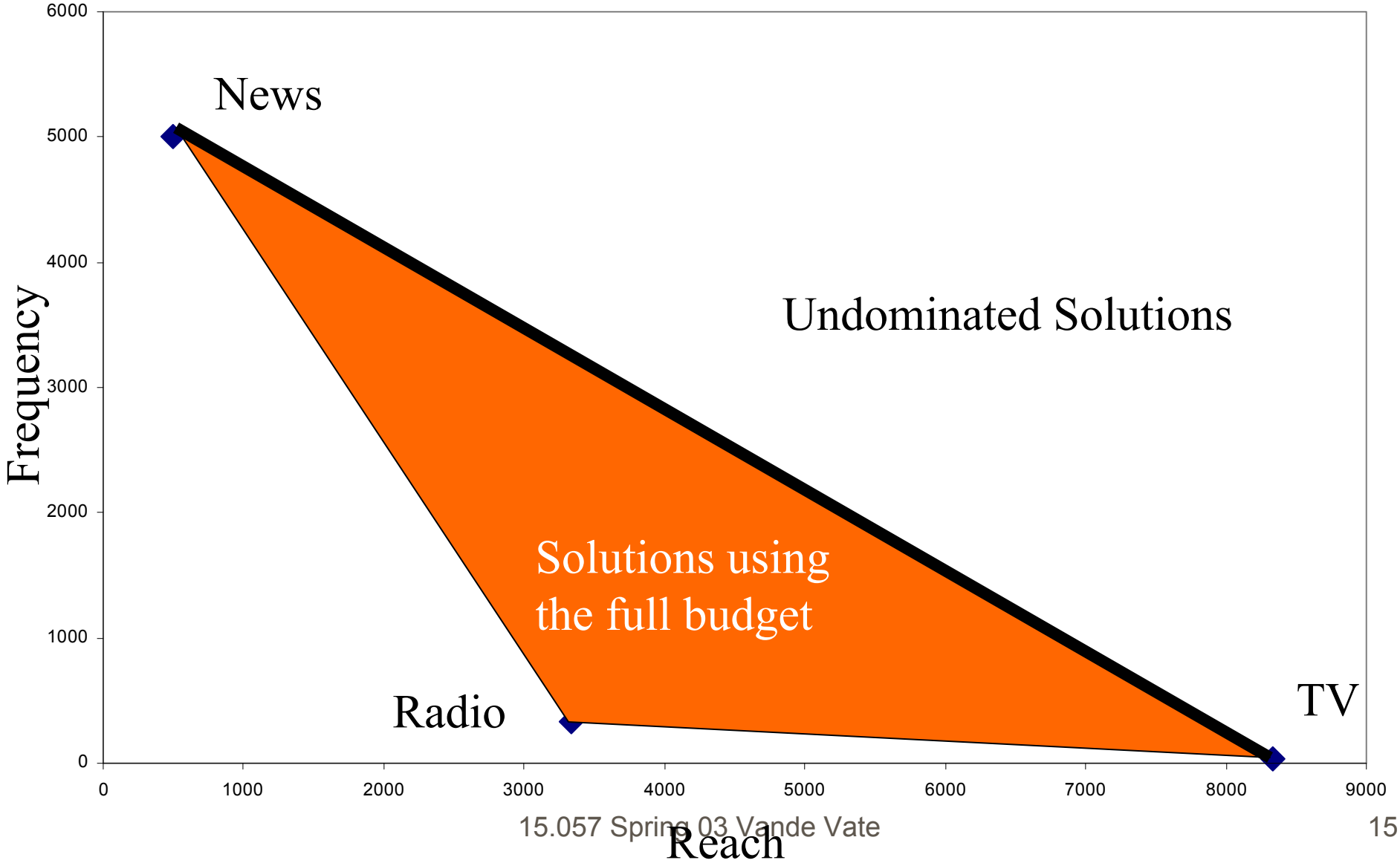
Example

- Budget of $500

Media	Cost	Reach	Frequency
Radio	$15	100	10
TV	$30	500	2
News	$10	10	100

- What are the Frontier Solutions?

The Frontier



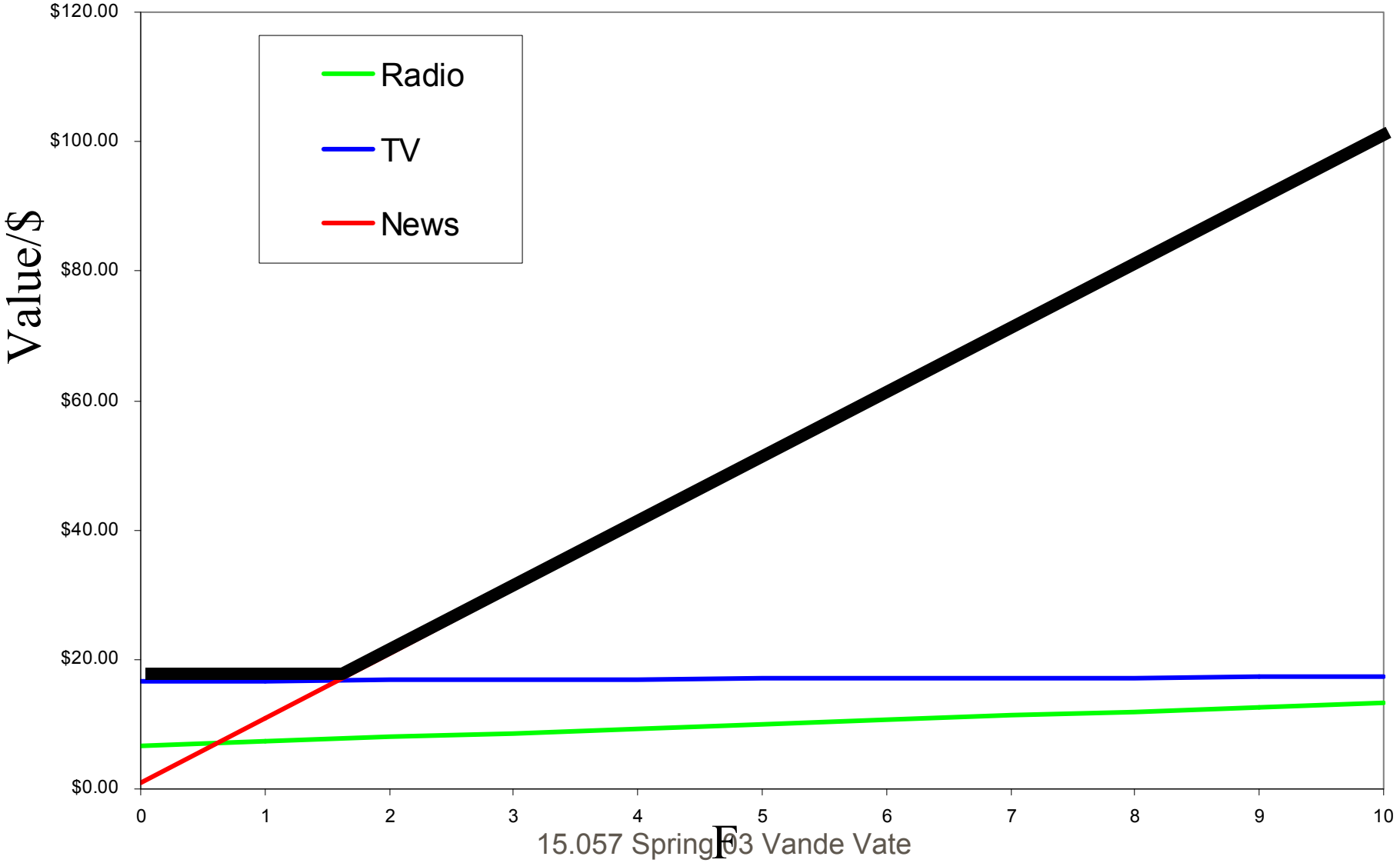
Dollarize

- Reaching one person worth $1 (normalize)
- Frequency worth $f per repetition

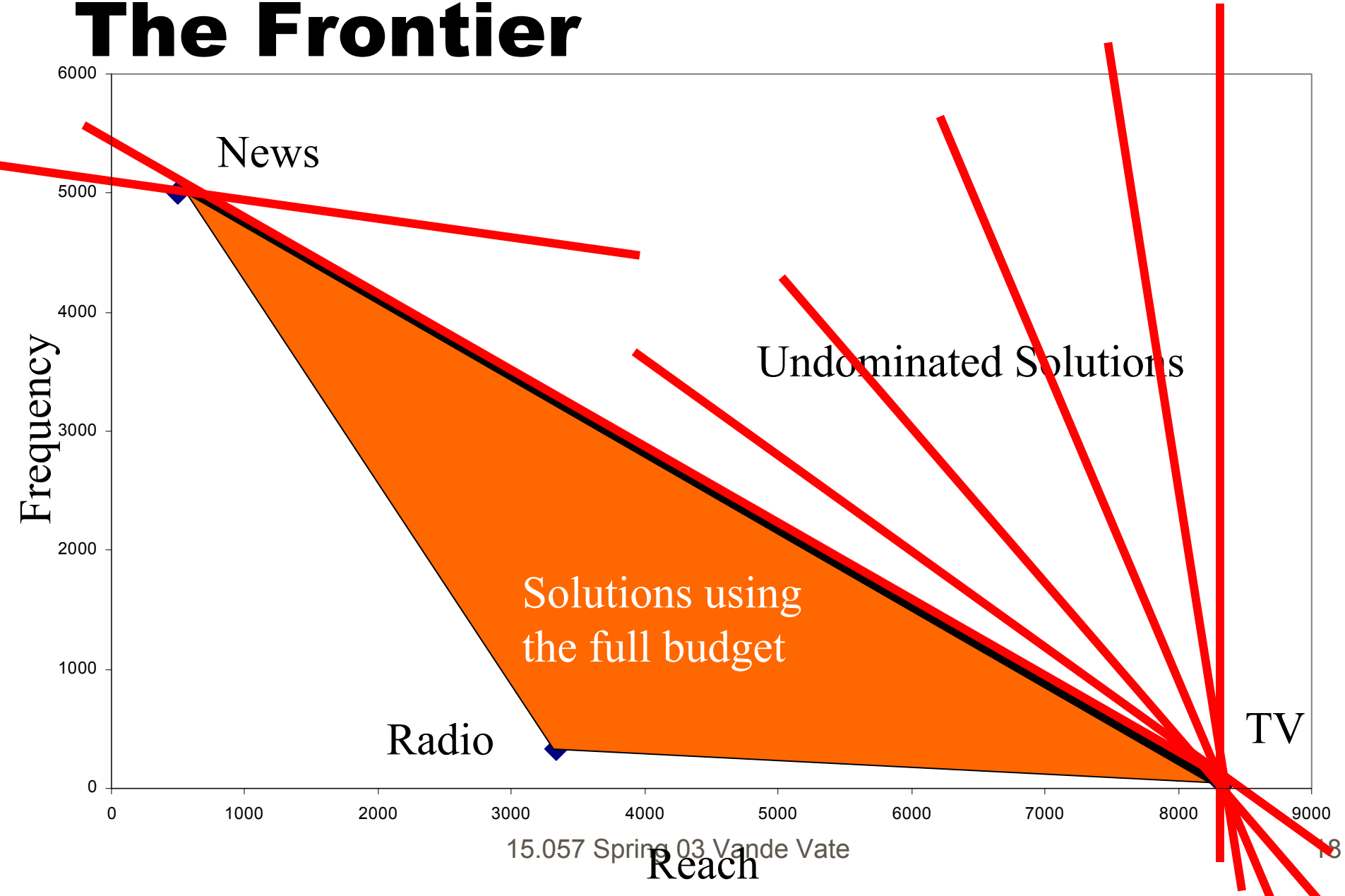
Media	Cost	Reach	Frequency	Value
Radio	$15	100	10	$100 + 10f
TV	$30	500	2	$500 + 2f
News	$10	10	100	$10 + 100f

- What happens to the answer as f increases?

The Frontier



The Frontier



Multiple Scenarios

- Uncertain Future
- Make a decision today
- Live with it tomorrow
- Example
  - Singapore Electric
  - Production plan against FORECASTED orders
  - Uncertain Economic climate
    - HOT (130%) 1/3 chance
    - Temperate (100%) 1/3 chance
    - Cold (70%) 1/3 chance
  - Allow back-ordering

Singapore Electric

Singapore Electric Generator Production

Unit Costs	Jan	Feb	Mar	Apr.	May
Prod.	$ 28.00	$ 27.00	$ 27.80	$ 29.00	
Inventory	$ 0.30	$ 0.30	$ 0.30	$ 0.30	
Prod. Quant.	0	0	0	0	
Prod. Limits	60	62	64	66	
Init. Inv.	15	-43	-79	-113	
Del. Reqmts	58	36	34	59	Minimum
Ending Inv.	(43)	(79)	(113)	(172)	7
Prod. Cost	$ -	$ -	$ -	$ -	
Inv. Cost	$ (4.20)	$ (18.30)	$ (28.80)	$ (42.75)	Total
Total Cost	$ (4.20)	$ (18.30)	$ (28.80)	$ (42.75)	$ (94.05)

Allow Back-Ordering

- Defer demand at a cost of $3/unit/month
- Challenge: Formulate a linear model

With Backordering

Singapore Electric Generator Production

Economic Climate		100%		Unit Revenue	$	50			
Unit Costs	Jan		Feb		Mar		Apr.		May
Prod.	$	28.00	$	27.00	$	27.80	$	29.00	
Inventory	$	0.30	$	0.30	$	0.30	$	0.30	
BackOrder		$3.00		$3.00		$3.00		$3.00	
Prod. Quant.		31		25		24		48	
Inventory		0		0		0		7	
BackOrder		0		0		0		0	
Net Inv.		0		0		0		7	
Prod. Limits		60		62		64		66	
Init. Inv.		10		(17.00)		(28.00)		(38.00)	
Del. Reqmts		58		36		34		59	Minimum
Ending Inv.		(17)		(28)		(38)		(49)	7
Prod. Cost	$	868	$	675	$	667	$	1,392	
Inv. Cost	$	-	$	-	$	-	$	21	Total
Total Cost	$	868	$	675	$	667	$	1,413	$ 3,623
							Net Rev.		5,727

Revised Model

- No constraint on ending inventory
- Charge $50 for any back orders in May
- Maximize Net Revenue
  - $50 for each item sold –
  - Cost –
  - Lost Sales ($50 for back orders in May)

Robust Answers

- Minimum Regret: Max the Min Revenue
  - Regardless of the outcome, we get at least this revenue
- Idea of the model
  - One set of variables for production
  - Inventory variables for each scenario
  - Cost & Revenue calculations for each scenario
  - New Variable for Minimum Revenue
  - Maximize the Minimum Revenue
  - S.t. Minimum Revenue $\leq$ Revenue in Each Scenario

Best Answers

Solution	Net Rev	Jan	Feb	Mar	Apr.
Hot	$ 5,627	60	62	64	47
Temper	$ 4,442	51	62	64	-
Cold	$ 3,604	36	62	41	-
Min Regret	$ 2,221	60	62	64	1

Solution	Hot	Temper	Cold
Hot	$ 5,627	$ 2,802	$ 873
Temper	$ 1,452	$ 4,442	$ 2,513
Cold	$ (1,534)	$ 1,568	$ 3,604
Min Regret	$ 2,221	$ 4,150	$ 2,221

Model (Messy)

				Unit Revenue	$ 50
Unit Costs	Jan	Feb	Mar	Apr.	May
Prod.	$ 28.00	$ 27.00	$ 27.80	$ 29.00	
Inventory	$ 0.30	$ 0.30	$ 0.30	$ 0.30	
BackOrder	$3.00	$3.00	$3.00	$3.00	

					Minimum Net
Prod. Quant.	60	62	64	0.981607	2221.43892

Scenario 1	Economic Climate		80%		
Inventory	24	57	94	47.981607	
BackOrder	0	0	0	0	
Net Inv.	24	57	94	47.981607	
Prod. Limits	60	62	64	66	
Init. Inv.	10	24.00	57.00	94.00	
Del. Reqmts	46	29	27	47	Lost Sales
Ending Inv.	24	57	94	48	$ -
Prod. Cost	$ 1,680	$ 1,674	$ 1,779	$ 28	
Inv. Cost	$ 7	$ 17	$ 28	$ 14	Total
Total Cost	$ 1,687	$ 1,691	$ 1,807	$ 43	$ 5,229
				Net Rev.	2,221

Scenario 2	Economic Climate		100%		
Inventory	12	38	68	9.981607	
BackOrder	0	0	0	0	
Net Inv.	12	38	68	9.981607	
Prod. Limits	60	62	64	66	
Init. Inv.	10	12.00	38.00	68.00	
Del. Reqmts	58	36	34	59	Lost Sales
Ending Inv.	12	38	68	10	$ -
Prod. Cost	$ 1,680	$ 1,674	$ 1,779	$ 28	
Inv. Cost	$ 4	$ 11	$ 20	$ 3	Total
Total Cost	$ 1,684	$ 1,685	$ 1,800	$ 31	$ 5,200
				Net Rev.	4,150

Scenario 3	Economic Climate		130%		
Inventory	0	10	30	0	
BackOrder	5	0	0	46.018393	
Net Inv.	-5	10	30	-46.01839	
Prod. Limits	60	62	64	66	
Init. Inv.	10	(5.00)	10.00	30.00	
Del. Reqmts	75	47	44	77	Lost Sales
Ending Inv.	(5)	10	30	(46)	4,601.84
Prod. Cost	$ 1,680	$ 1,674	$ 1,779	$ 28	
Inv. Cost	$ 15	$ 3	$ 9	$ 138	Total
Total Cost	$ 1,695	$ 1,677	$ 1,788	$ 167	$ 5,327
				Net Rev.	2,221

Summary

- Modeling Goals
- Modeling Conflicting Objectives
  - The Efficient Frontier
- Modeling Scenarios
  - Complicated models
  - Robust Solutions