15.401 Finance Theory

MIT Sloan MBA Program

Andrew W. Lo

Harris & Harris Group Professor, MIT Sloan School

Lectures 15–17: The CAPM and APT

Critical Concepts

- Review of Portfolio Theory
- The Capital Asset Pricing Model
- The Arbitrage Pricing Theory
- Implementing the CAPM
- Does It Work?
- Recent Research
- Key Points

Reading

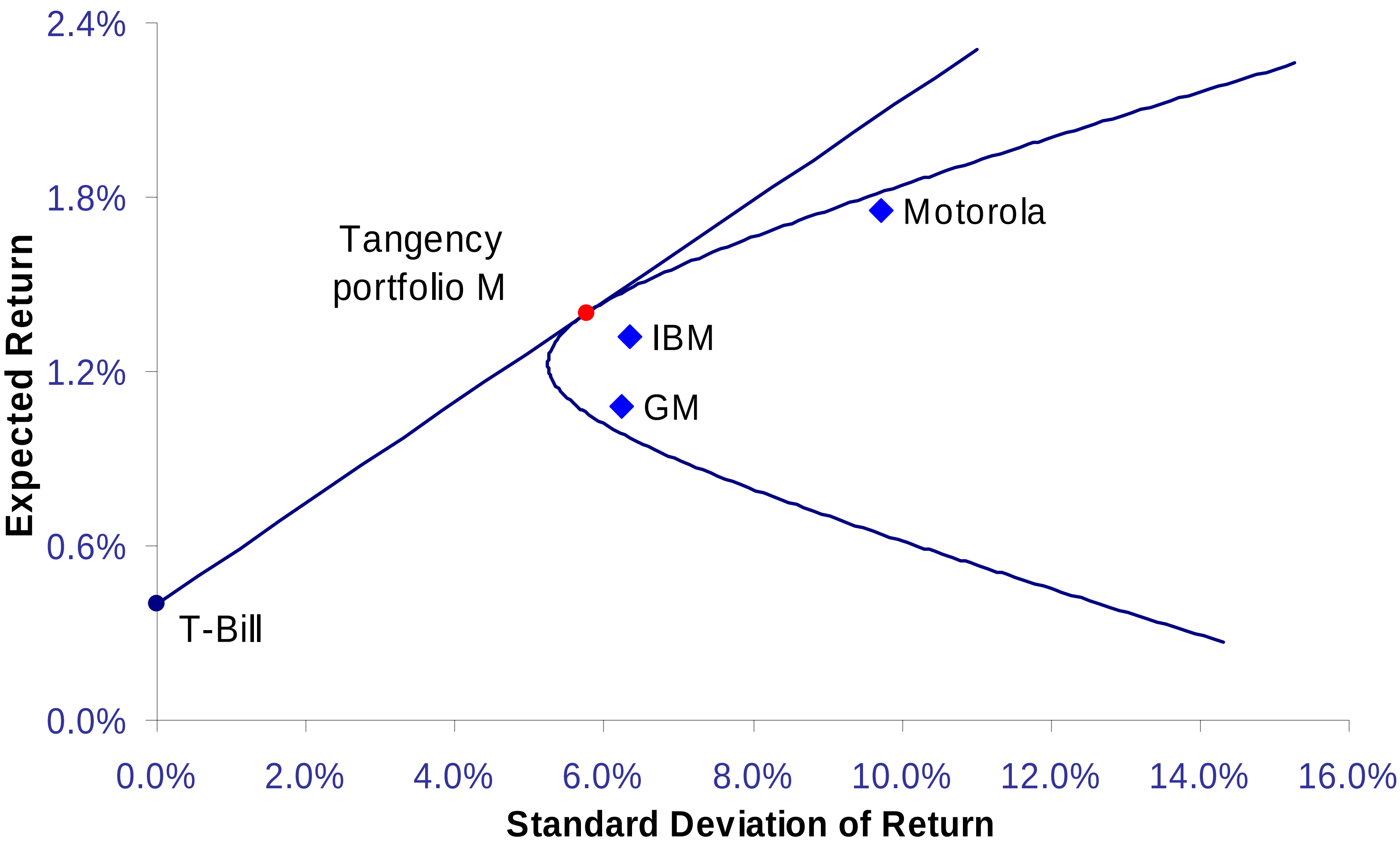
- Brealey and Myers, Chapter 8.2 – 8.3

Review of Portfolio Theory

Risk/Return Trade-Off

- Portfolio risk depends primarily on covariances
  - Not stocks' individual volatilities
- Diversification reduces risk
  - But risk common to all firms cannot be diversified away
- Hold the tangency portfolio M
  - The tangency portfolio has the highest expected return for a given level of risk (i.e., the highest Sharpe ratio)
- Suppose all investors hold the same portfolio M; what must M be?
  - M is the **market portfolio**
- Proxies for the market portfolio: S&P 500, Russell 2000, MSCI, etc.
  - Value-weighted portfolio of broad cross-section of stocks

Review of Portfolio Theory



The Capital Asset Pricing Model

Implications of M as the Market Portfolio

- Efficient portfolios are combinations of the market portfolio and T-Bills
- Expected returns of **efficient portfolios** satisfy:

$$\boxed{\mathsf{E}[R_p] \;=\; R_f \;+\; \frac{\sigma_p}{\sigma_m}(\mathsf{E}[R_m] - R_f)}$$

- This yields the required rate of return or cost of capital for efficient portfolios!
- Trade-off between risk and expected return
- Multiplier is the ratio of portfolio risk to market risk
- What about other (non-efficient) portfolios?

The Capital Asset Pricing Model

Implications of M as the Market Portfolio

- For any asset, define its **market beta** as:

$$\beta_i \equiv \frac{\mathrm{Cov}[R_i, R_m]}{\mathrm{Var}[R_m]}$$

- Then the Sharpe-Lintner CAPM implies that:

$$\boxed{\mathrm{E}[R_i] = R_f + \beta_i \left(\mathrm{E}[R_m] - R_f\right)}$$

- Risk/reward relation is linear!
- Beta is the correct measure of risk, not sigma (except for efficient portfolios); measures sensitivity of stock to market movements

The Capital Asset Pricing Model

The Security Market Line

$$\boxed{\mathrm{E}[R_i] \;=\; R_f \;+\; \beta_i\,(\mathrm{E}[R_m] - R_f)}$$

- Implications:

$$\begin{array}{lllllll} \beta_i & = & 1 & \Rightarrow & \mathrm{E}[R_i] & = & \mathrm{E}[R_m] \\ \beta_i & = & 0 & \Rightarrow & \mathrm{E}[R_i] & = & R_f \\ \beta_i & < & 0 & \Rightarrow & \mathrm{E}[R_i] & < & R_f \quad \text{(Why?)} \end{array}$$

The Capital Asset Pricing Model

What About Arbitrary Portfolios of Stocks?

$$
\begin{aligned}
R_p &= \omega_1 R_1 + \cdots + \omega_n R_n \\
\mathrm{Cov}[R_p, R_m] &= \mathrm{Cov}[\omega_1 R_1 + \cdots + \omega_n R_n, R_m] \\
&= \omega_1 \mathrm{Cov}[R_1, R_m] + \cdots + \omega_n \mathrm{Cov}[R_n, R_m] \\
\frac{\mathrm{Cov}[R_p, R_m]}{\mathrm{Var}[R_m]} &= \omega_1 \frac{\mathrm{Cov}[R_1, R_m]}{\mathrm{Var}[R_m]} + \cdots + \omega_n \frac{\mathrm{Cov}[R_n, R_m]}{\mathrm{Var}[R_m]} \\
\beta_p &= \omega_1 \beta_1 + \cdots + \omega_n \beta_n
\end{aligned}
$$

- Therefore, for any arbitrary portfolio of stocks:

$$
\boxed{\mathrm{E}[R_p] = R_f + \beta_p (\mathrm{E}[R_m] - R_f)}
$$

The Capital Asset Pricing Model

We Now Have An Expression for the:

- Required rate of return
- Opportunity cost of capital
- Risk-adjusted discount rate

$$\boxed{\mathrm{E}[R_p] \;=\; R_f \;+\; \beta_p\left(\mathrm{E}[R_m] - R_f\right)}$$

- Risk adjustment involves the product of beta and market risk premium
- Where does $\mathrm{E}[R_m]$ and R_f come from?

The Capital Asset Pricing Model

Example:

Using monthly returns from 1990 – 2001, you estimate that Microsoft's beta is 1.49 (std err = 0.18) and Gillette's beta is 0.81 (std err = 0.14). If these estimates are a reliable guide going forward, what expected rate of return should you require for holding each stock?

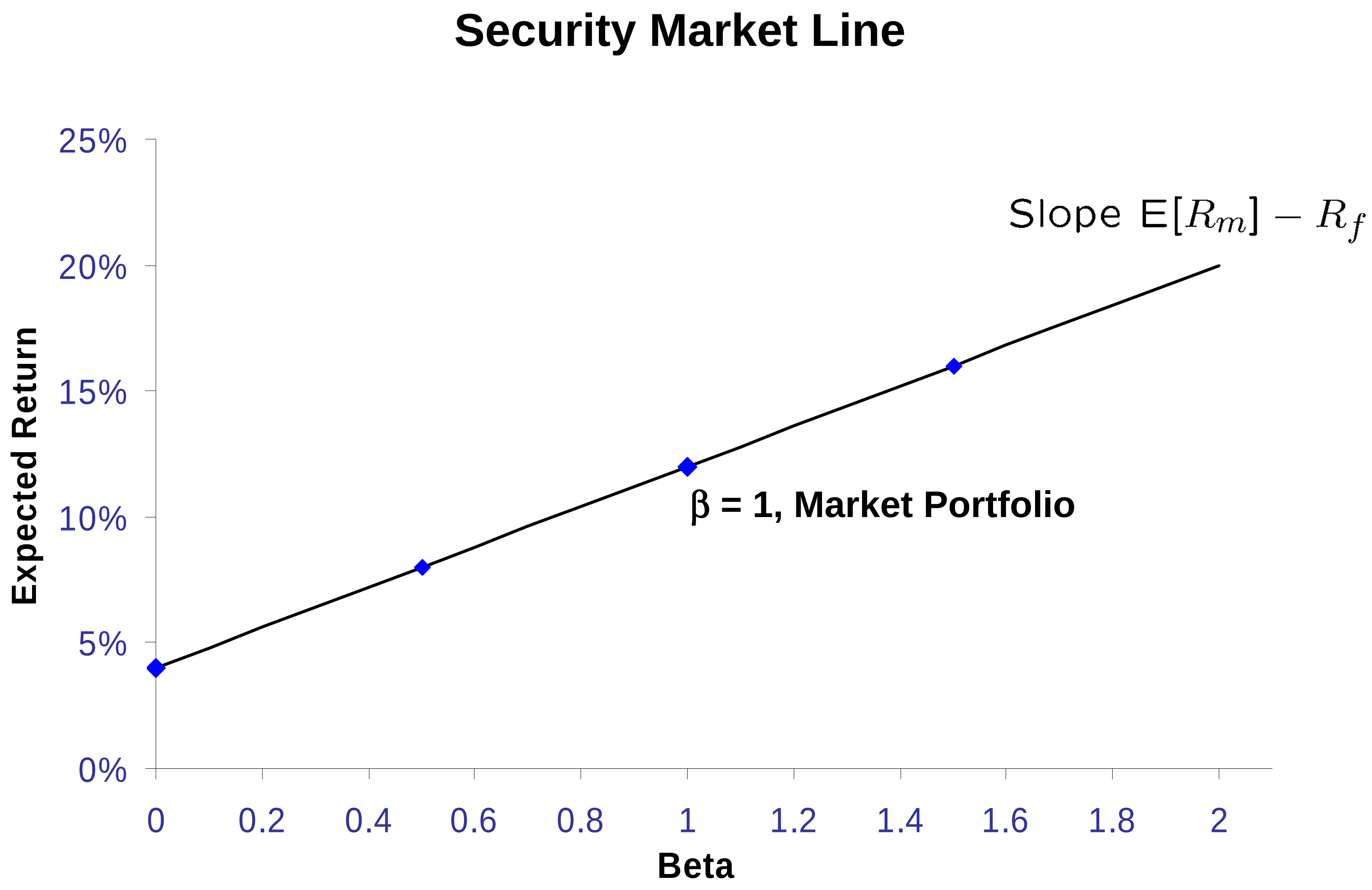
$$\mathsf{E}[R_i] = R_f + \beta_i(\mathsf{E}[R_m] - R_f)$$

$$R_f = 5\% \quad , \quad \mathsf{E}[R_m] - R_f = 6\%$$

$$\mathsf{E}[R_{\mathsf{GS}}] = 0.05 + (0.81 \times 0.06) = 9.86\%$$

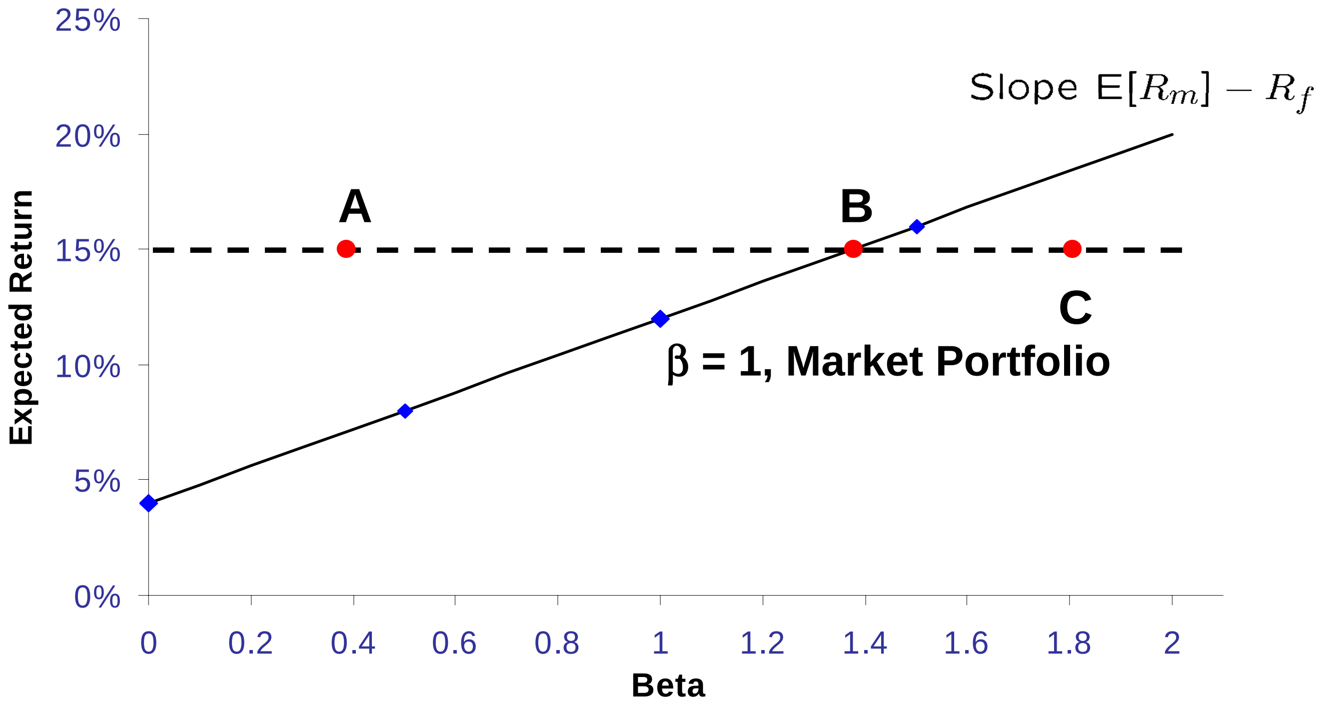
$$\mathsf{E}[R_{\mathsf{MSFT}}] = 0.05 + (1.49 \times 0.06) = 13.94\%$$

The Capital Asset Pricing Model



The Security Market Line Can Be Used To Measure Performance:

- Suppose three mutual funds have the same average return of 15%
- Suppose all three funds have the same volatility of 20%
- Are all three managers equally talented?
- Are all three funds equally attractive?



The Capital Asset Pricing Model

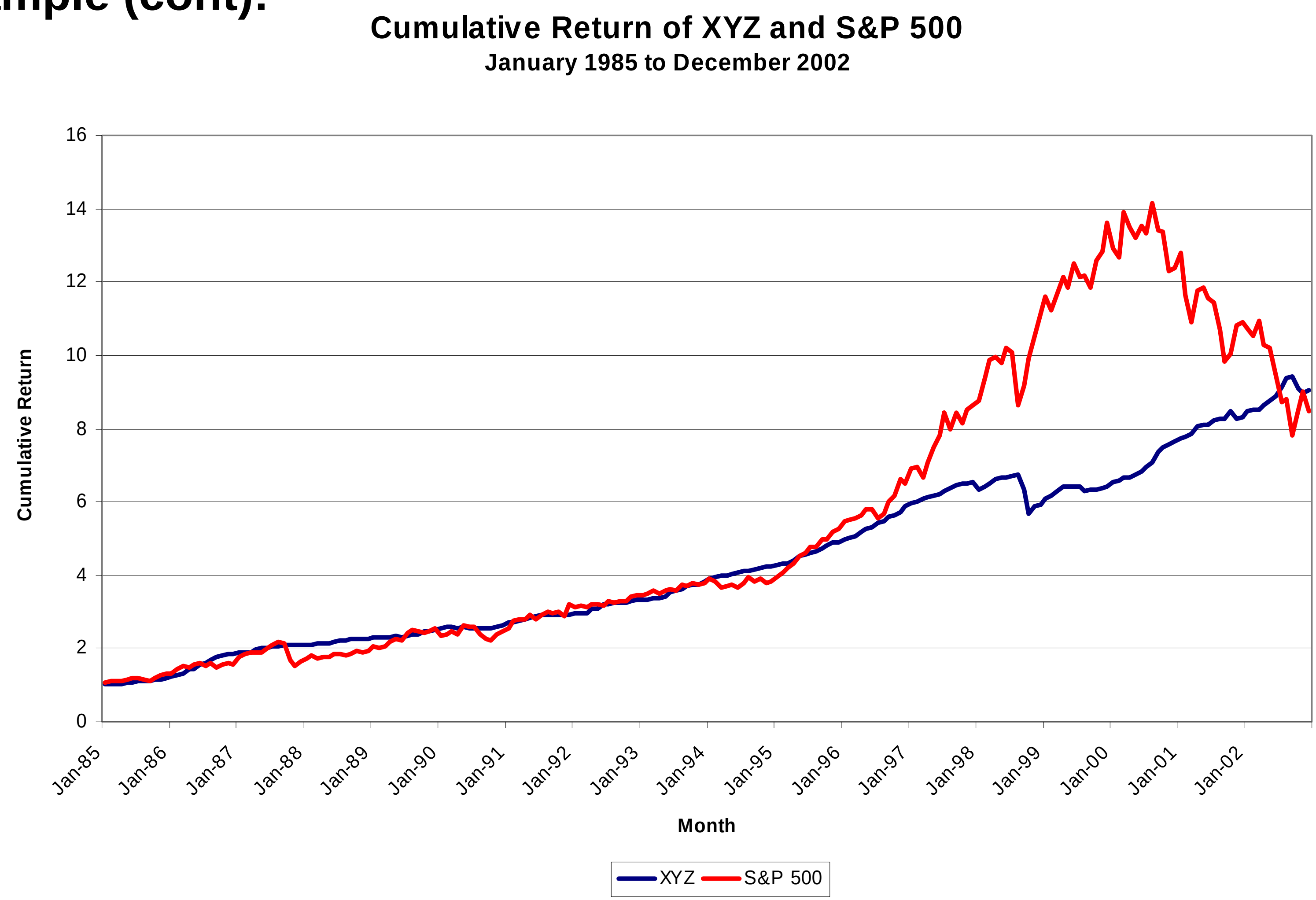
Example:

Hedge fund XYZ had an average annualized return of 12.54% and a return standard deviation of 5.50% from January 1985 to December 2002, and its estimated beta during this period was –0.028. Did the manager exhibit positive performance ability according to the CAPM? If so, what was the manager's alpha?

$$\begin{aligned}
\mathrm{E}[R_i] &= R_f + \beta_i(\mathrm{E}[R_m] - R_f) \\
R_f &= 5\% \quad , \quad \mathrm{E}[R_m] - R_f = 6\% \\
\mathrm{E}[R_{\mathrm{XYZ}}] &= 0.05 + (-0.028 \times 0.06) = 4.83\% \\
\alpha_{\mathrm{XYZ}} &= \mathrm{E}[R_i] - \left\{R_f + \beta_i(\mathrm{E}[R_m] - R_f)\right\} \\
&= 12.54\% - 4.83\% = 7.71\%
\end{aligned}$$

The Capital Asset Pricing Model

Example (cont):



The Arbitrage Pricing Theory

What If There Are Multiple Sources of Systematic Risk?

- Let returns following a multi-factor linear model:

$$R_i - R_f \;=\; \alpha_i \;+\; \beta_{i1} F_1 \;+\; \beta_{i2} F_2 \;+\; \cdots \;+\; \beta_{iK} F_K \;+\; \epsilon$$

$$F_k \;\equiv\; \text{Factor } k \text{ excess return}$$

- Then the APT implies the following relation:

$$\mathrm{E}[R_i] - R_f \;=\; \beta_{i1}\pi_1 \;+\; \beta_{i2}\pi_2 \;+\; \cdots \;+\; \beta_{iK}\pi_K$$

$$\pi_k \;\equiv\; \text{Factor } k \text{ risk premium}$$

- Cost of capital depends on *K* sources of systematic risk

The Arbitrage Pricing Theory

Strengths of the APT

- Derivation does not require market equilibrium (only no-arbitrage)
- Allows for multiple sources of systematic risk, which makes sense

Weaknesses of the APT

- No theory for what the factors should be
- Assumption of linearity is quite restrictive

Implementing the CAPM

Parameter Estimation:

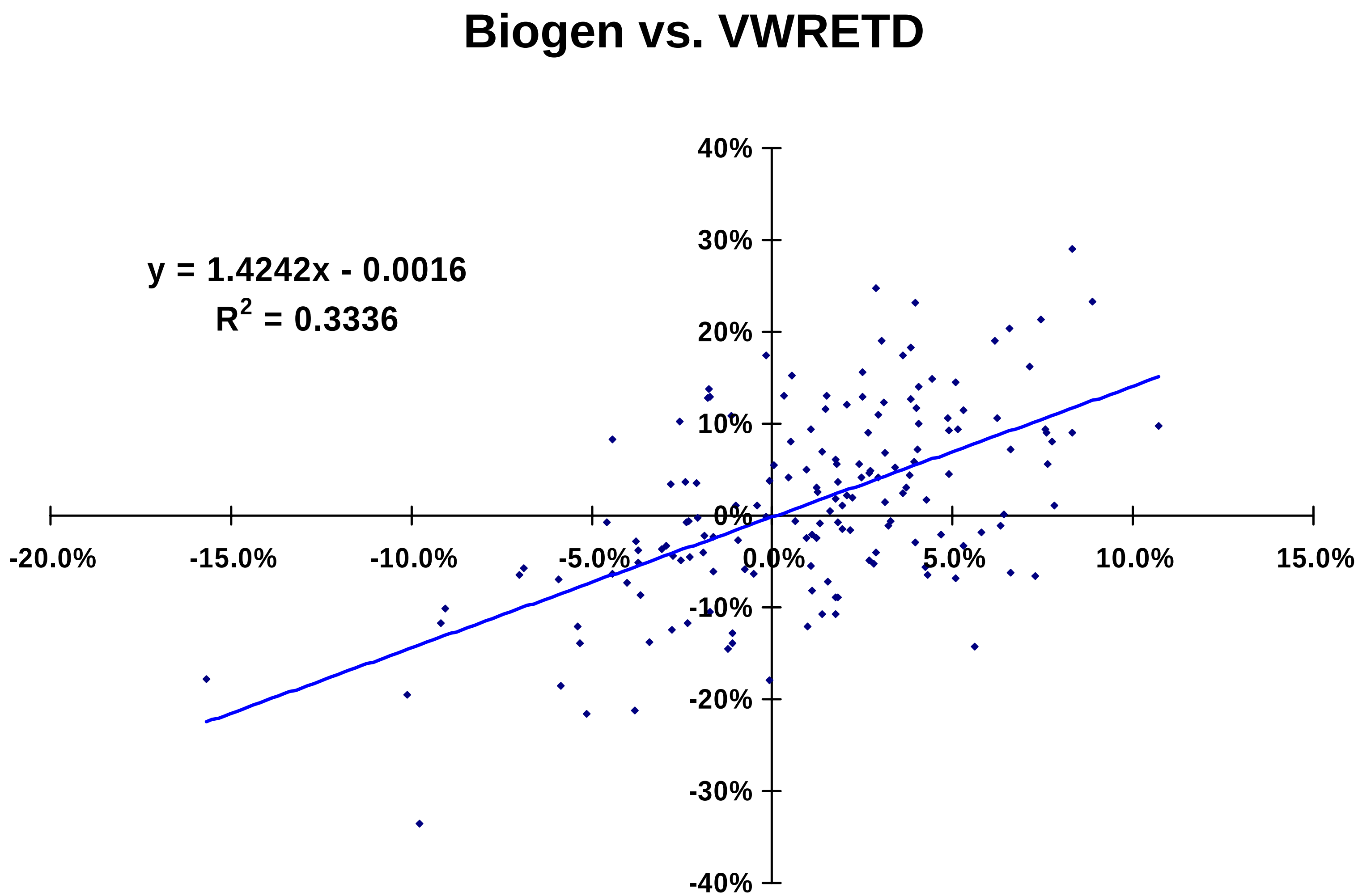
- Security market line must be estimated
- One unknown parameter: β
- Given return history, β can be estimated by linear regression:

$$
\begin{aligned}
\mathrm{E}[R_i] &= R_f + \beta_i(\mathrm{E}[R_m] - R_f) \\
R_i &= R_f + \beta_i(R_m - R_f) + \epsilon \\
R_i - R_f &= \alpha_i + \beta_i(R_m - R_f) + \epsilon \\
\text{CAPM} &\Rightarrow \alpha_i = 0 \\
\text{or} \quad R_i &= \alpha_i + \beta_i R_m + \epsilon \\
\text{CAPM} &\Rightarrow \alpha_i = R_f(1 - \beta_i)
\end{aligned}
$$

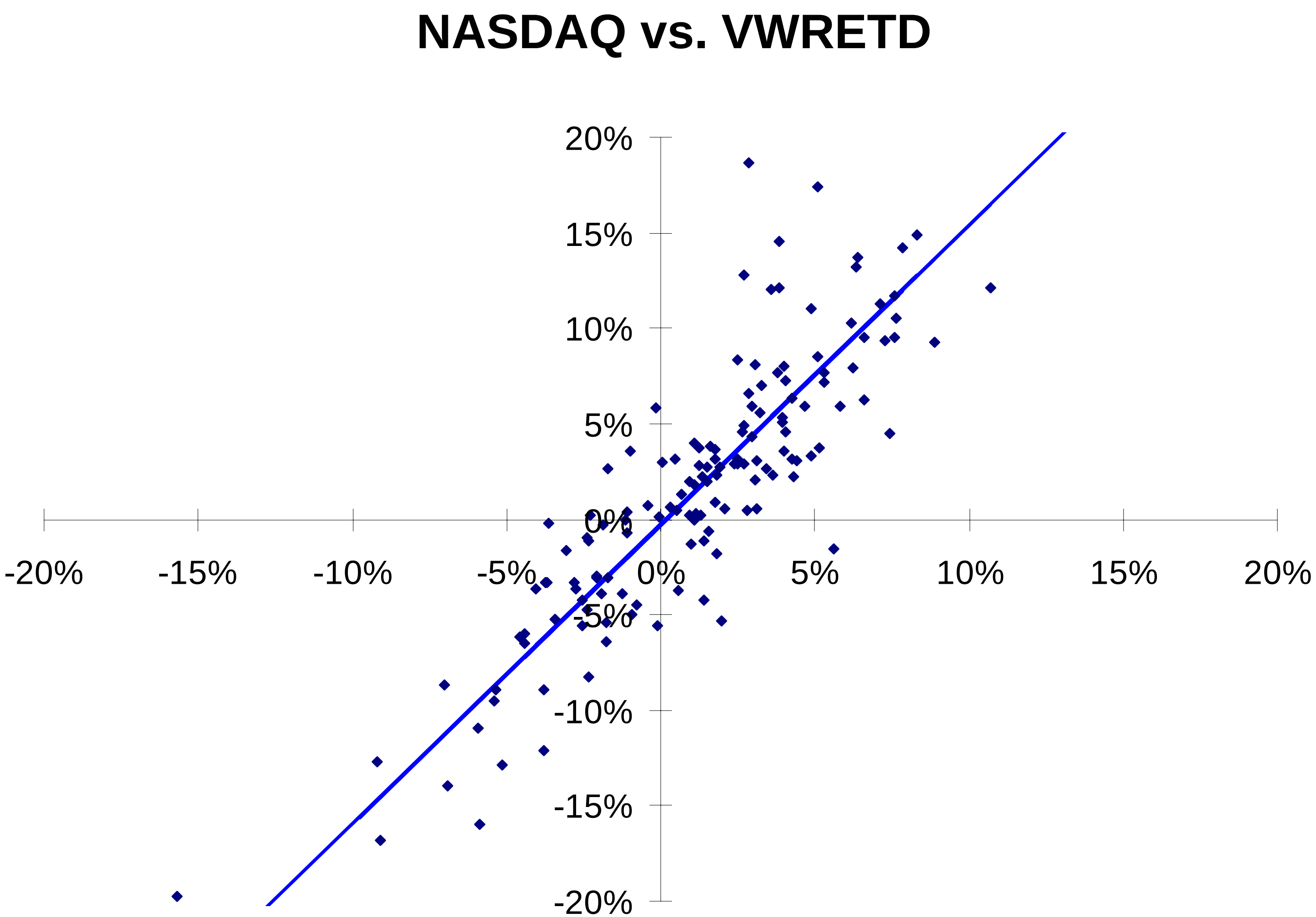
Implementing the CAPM

	A	B	C	D	E	F	G	H	I	J
1	Date	Biogen	Motorola	VWRETD		Biogen Regression				
2	Aug-88	1.9%	-12.5%	-2.8%			beta	intercept	R_f(1-beta)	R_f
3	Sep-88	24.5%	3.1%	3.7%		Estimate	1.43	1.6%	-2.1%	5%
4	Oct-88	1.5%	-10.7%	1.8%		Std Err	0.25	1.1%		
5	Nov-88	-11.9%	-2.3%	-1.6%		R2	17.5%	0.13		
6	Dec-88	-3.4%	12.1%	2.1%			33.7	159		
7	Jan-89	29.8%	7.1%	6.6%			0.6	2.7		
8	Feb-89	1.4%	-6.1%	-1.6%						
9	Mar-89	33.3%	-1.6%	2.2%		Estimated	Monthly	Annual		
10	Apr-89	-2.0%	10.6%	4.9%		alpha:	3.7%	45.0%		
11	May-89	16.3%	23.2%	4.0%						
12	Jun-89	-20.2%	-6.3%	-0.5%		Motorola Regression				
13	Jul-89	7.7%	8.1%	7.8%			beta	intercept	R_f(1-beta)	R_f
14	Aug-89	10.2%	2.0%	2.2%		Estimate	1.42	-0.2%	-2.1%	5%
15	Sep-89	6.5%	-0.1%	-0.2%		Std Err	0.16	0.7%		
16	Oct-89	5.2%	-3.2%	-2.9%		R2	33.4%	0.08		
17	Nov-89	14.9%	5.6%	1.8%			79.6	159		
18	Dec-89	-3.6%	-0.7%	1.8%			0.6	1.1		
19	Jan-90	-10.4%	-6.4%	-7.0%						
20	Feb-90	5.0%	13.0%	1.5%		Estimated	Monthly	Annual		
21	Mar-90	7.9%	5.6%	2.4%		alpha:	2.0%	23.5%		

Does It Work?


Biogen vs. VWRETD
y = 1.4242x - 0.0016
R2 = 0.3336
40%
30%
20%
10%
0%
-10%
-20%
-30%
-40%
-20.0%
-15.0%
-10.0%
-5.0%
0.0%
5.0%
10.0%
15.0%

Does It Work?



Does It Work?

Market-Cap Portfolios:

Over the past 40 years, the smallest firms (1st decile) had an average monthly return of 1.33% and a beta of 1.40. The largest firms (10th decile) had an average return of 0.90% and a beta of 0.94. During the same time period, the Tbill rate averaged 0.47% and the market risk premium was 0.49%. Are the returns consistent with the CAPM?

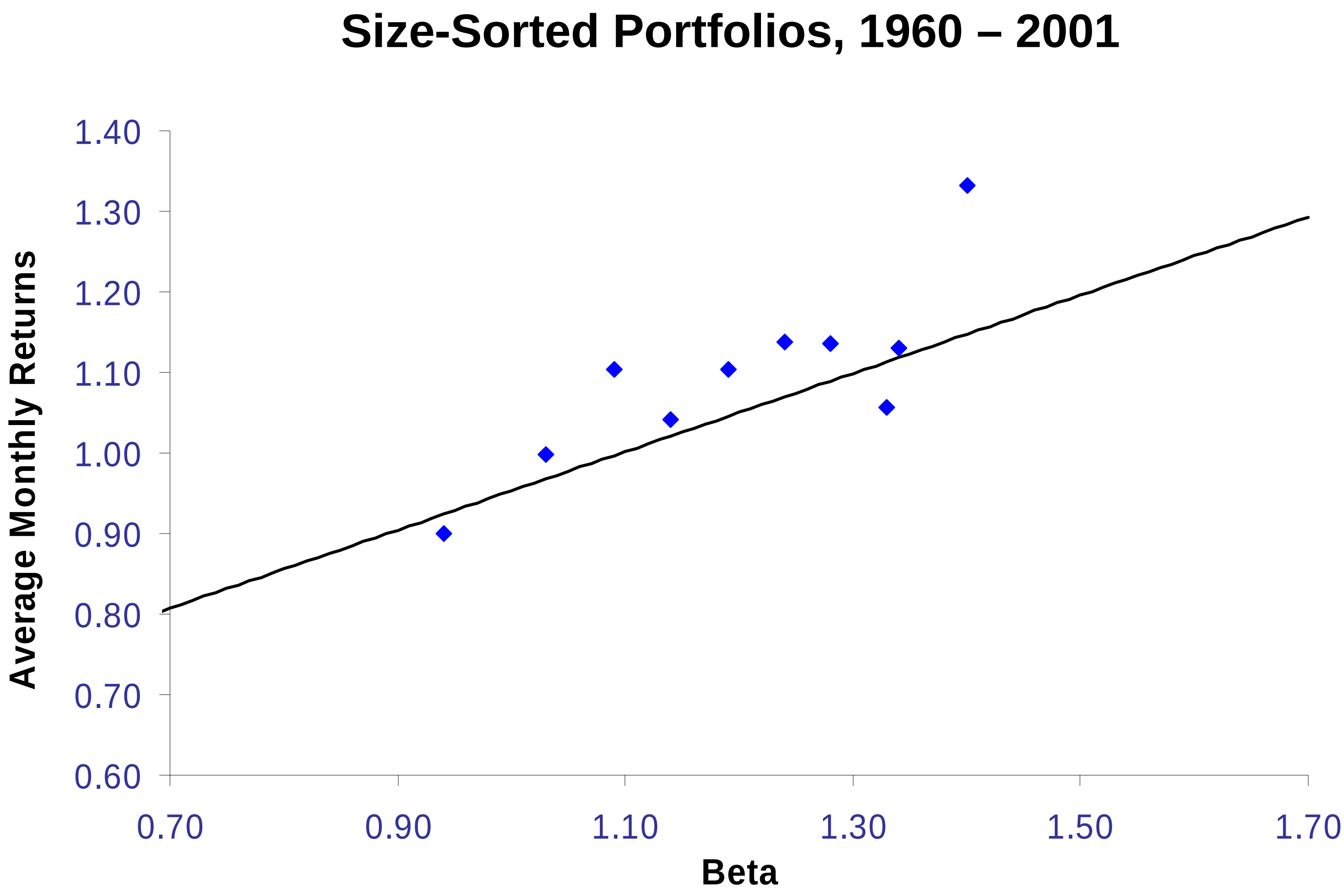
$$\mathsf{E}[R_i] = R_f + \beta_i(\mathsf{E}[R_m] - R_f)$$

$$R_f = 0.47\% \quad , \quad \mathsf{E}[R_m] - R_f = 0.49\%$$

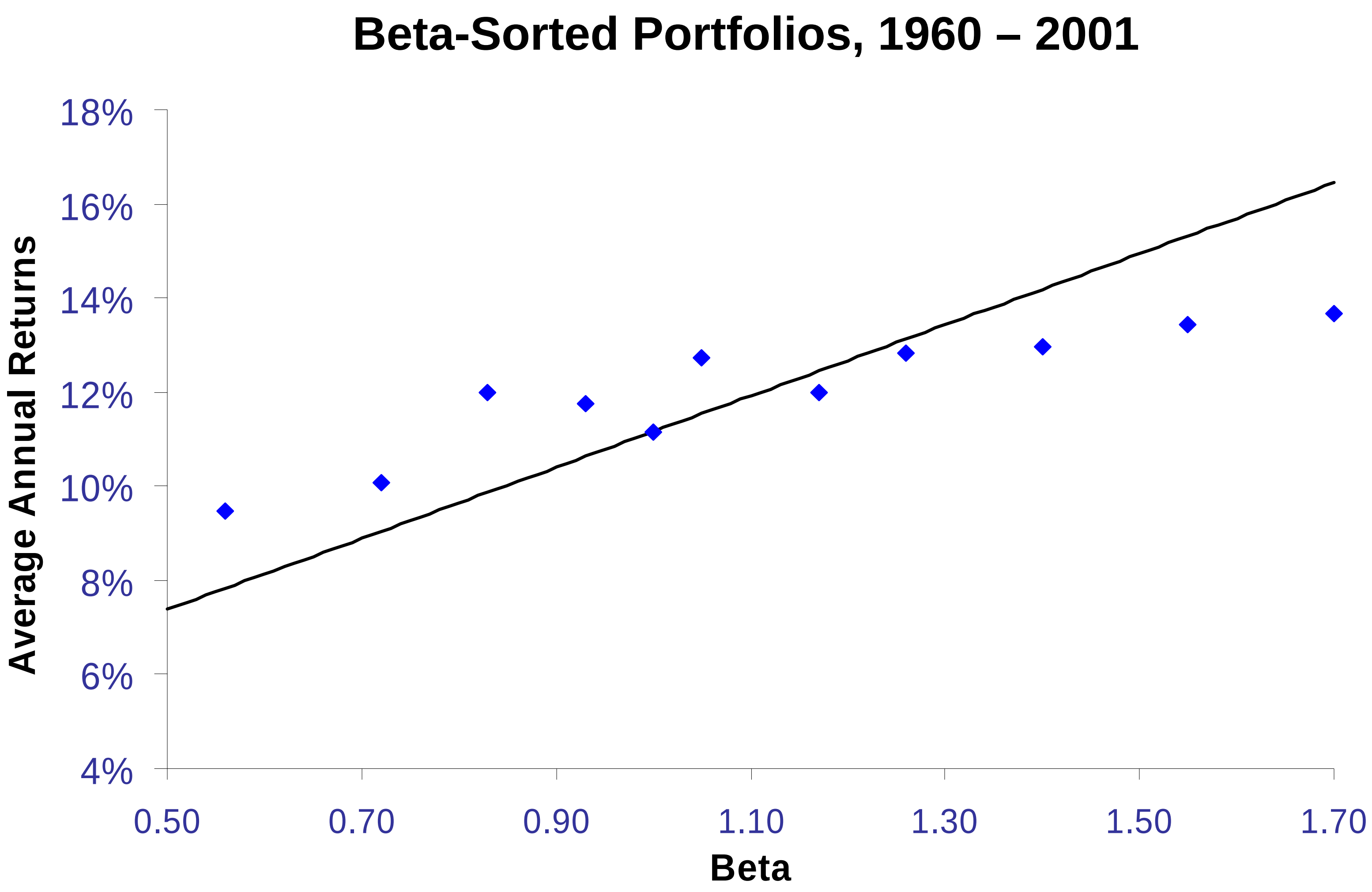
$$\mathsf{E}[R_{\mathsf{Large}}] = 0.0047 + (0.94 \times 0.0049) = 0.93\%$$

$$\mathsf{E}[R_{\mathsf{Small}}] = 0.0047 + (1.40 \times 0.0049) = 1.16\%$$

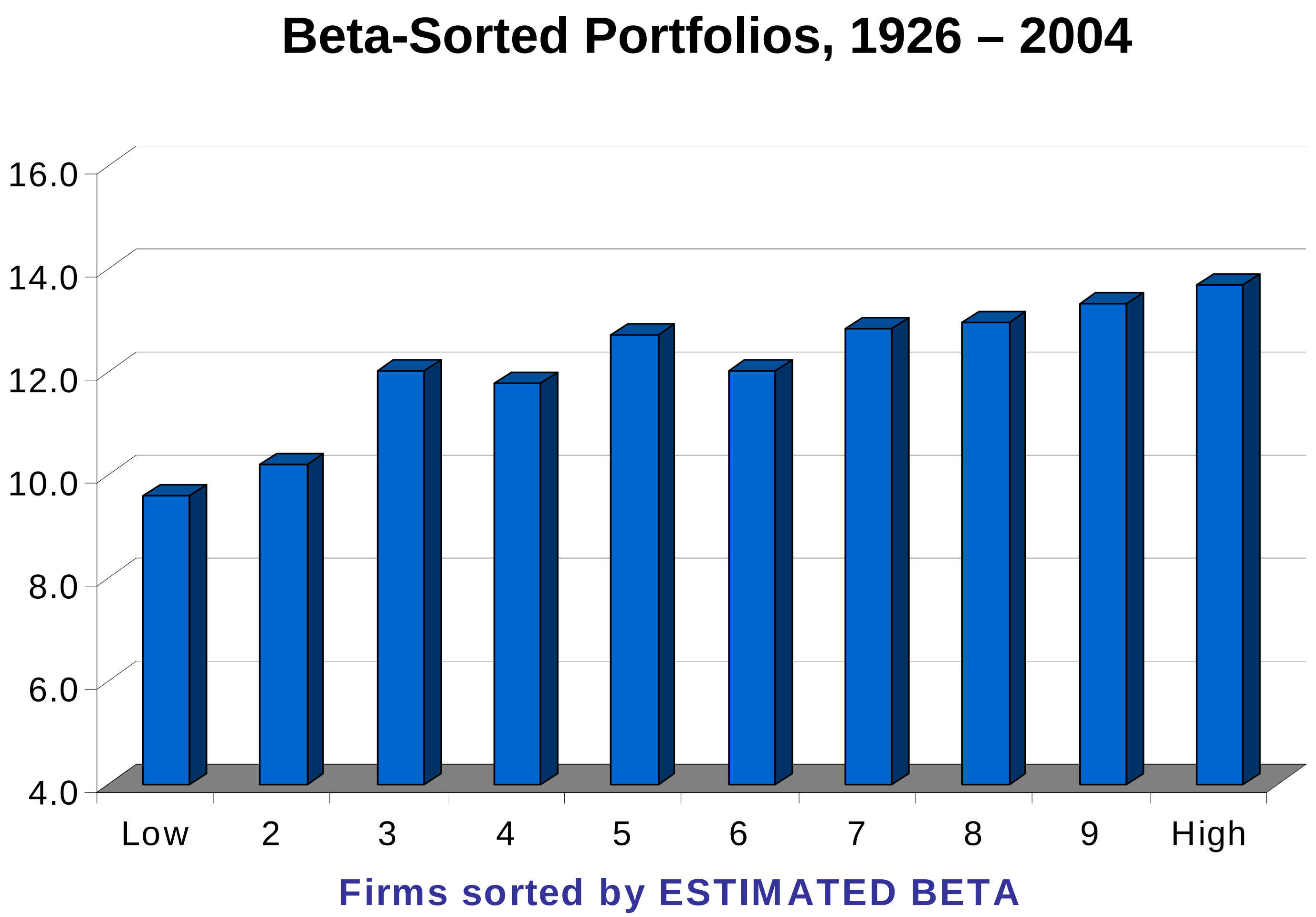
Does It Work?


Size-Sorted Portfolios, 1960 – 2001
Average Monthly Returns
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.70
0.90
1.10
1.30
1.50
1.70
Beta

Does It Work?


Beta-Sorted Portfolios, 1960 – 2001
Average Annual Returns
18%
16%
14%
12%
10%
8%
6%
4%
0.50
0.70
0.90
1.10
1.30
1.50
1.70
Beta

Does It Work?


Beta-Sorted Portfolios, 1926 – 2004
16.0
14.0
12.0
10.0
8.0
6.0
4.0
Low
2
3
4
5
6
7
8
9
High
Firms sorted by ESTIMATED BETA

Does It Work?

Volatility-Sorted Portfolios, 1926 – 2004

Recent Research

Other Factors Seem To Matter

- Book/Market (Fama and French, 1992)
- Liquidity (Chordia, Roll, and Subrahmanyam, 2000)
- Trading Volume (Lo and Wang, 2006)

But CAPM Still Provides Useful Framework For Applications

- Graham and Harvey (2000): 74% of firms use the CAPM to estimate the cost of capital
- Asset management industry uses CAPM for performance attribution
- Pension plan sponsors use CAPM for risk-budgeting and asset allocation

Key Points

- Tangency portfolio is the market portfolio
- This yields the capital market line (efficient portfolios)

$$\mathsf{E}[R_p] \;=\; R_f \;+\; \frac{\sigma_p}{\sigma_m}(\mathsf{E}[R_m] - R_f)$$

- The CAPM generalizes this relationship for any security or portfolio:

$$\mathsf{E}[R_i] \;=\; R_f \;+\; \beta_i\,(\mathsf{E}[R_m] - R_f)$$

- The security market line yields a measure of risk: beta
- This provides a method for estimating a firm's cost of capital
- The CAPM also provides a method for evaluating portfolio managers
  - Alpha is the correct measure of performance, not total return
  - Alpha takes into account the differences in risk among managers
- Empirical research is mixed, but the framework is very useful

Additional References

- Bernstein, 1992, *Capital Ideas.* New York: Free Press.
- Bodie, Z., Kane, A. and A. Marcus, 2005, *Investments*, 6th edition. New York: McGraw-Hill.
- Brennan, T., Lo, A. and T. Nguyen, 2007, *Portfolio Theory: A Review*, to appear in Foundations of Finance.
- Campbell, J., Lo, A. and C. MacKinlay, 1997, *The Econometrics of Financial Markets*. Princeton, NJ: Princeton University Press.
- Chordia, T., Roll, R. and A. Subrahmanyam, 2000, “Commonality in Liquidity”, *Journal of Financial Economics* 56, 3–28.
- Fama, Eugene F., and Kenneth French, 1992, The cross section of expected stock returns", *Journal of Finance* 47, 427–465.
- Grinold, R. and R. Kahn, 2000, *Active Portfolio Management*. New York: McGraw-Hill.
- Lo, A. and J. Wang, 2006, “Trading Volume: Implications of an Intertemporal Capital Asset Pricing Model”, *Journal of Finance* 61, 2805–2840.