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ANDREW LO: Well, if you remember last time where we left off, we were talking about risk and return. And we said that we were going to make the following simplifying assumption, which is that we're going to assume that investors like expected return, and they do not like risk as measured by volatility. All right?

And so the way that we depict it graphically is to use a graph where the x-axis is standard deviation of your entire portfolio, and the y-axis is the expected return of that portfolio. And the question is, where on this graph can we get to, given the securities that we have access to, that will maximize our level of happiness. Where happiness, again, is assumed to mean higher expected rate of return and lower risk, as measured by variance or standard deviation.

So for example, if you take a look at this simple graph and you ask the question, where on the graph do you want to be, you would like to be always going in the northwest direction. Right? Because north means higher expected return and west means lower risk. So obviously, if we could, we'd love to be on this axis all the way, way up. Right? No risk, lots of return. That's an example of an arbitrage.

And we know that that can't happen very easily because otherwise everybody would be there. And pretty soon it would wipe out that opportunity. So the question from the portfolio construction perspective is now a little bit sharper than it was last week when we started down this path. Now we want to construct a portfolio, we want to take a collection of securities and weight them in order to be as happy as possible. Meaning, we want to be as northwest as possible. So let's see how we go about doing that.

One thing we could do is just pick an individual stock. So if you have these four stocks to pick from, then to go as northwest as possible, you're sort of looking at Merck as, you know, the extreme. But it's not at all clear whether or not that's something that you really want. Because, for example, General Motors, while it has a lower expected return than Merck, it does have a bit of a lower risk. And for some people, that might actually be preferred.

So at this point, we don't have a lot of hard recommendations to provide you with, without any further analysis. So we're going to do some further analysis today. And the analysis is to ask the question, all right, what are the properties of mean and variance for a given portfolio, not

just for an individual security. So it turns out that there's a relatively straightforward way of answering this. And let me just go through the calculations and then we can see what that implies for where we want to be in that mean-standard deviation graph.

So the mean of a particular stock I'm going to write as expectation of R_i , or μ_i , for short. The variance of a particular stock I'm going to write as σ_i^2 , or the standard deviation is then just σ_i . OK? And it turns out, you can show this rigorously. I won't do that, but you can take a look at that if you are unconvinced.

It turns out that if you construct a weighted average of stocks so that the return of the portfolio is given by that top line, R_p , then when you take the expected value of that top line, what you get is the middle line. In other words, the expected return of a portfolio is just equal to the exact same weighted averages of the expected rates of return of the individual components.

So you understand the difference between the second and the first line in that red box? That's a very important distinction. The top line is basically an accounting identity. It says that when you want to compute the actual realized return in your portfolio, you just take a weighted average of what you did on each stock, what your return is on each stock. That's an accounting identity.

The second line is not an accounting identity. It comes from an accounting identity. But what it says is that on average, the rate of return of your portfolio is equal to a weighted average of the average rates of return on each of the components of your portfolio. OK? That's a very important principle. Any questions about that before we move on?

OK. So I'm going to write that as a shorthand for the portfolio, μ_p . So μ_p is just equal to this whole expression right here. All right. So what we've now deduced is that the mean of my portfolio is simply the weighted average of the means of each of the securities in the portfolio. What I'm going to turn to next is a much more complicated calculation, which is, what is the variance of my portfolio. It turns out that the variance of the portfolio is not a simple weighted average of the variances of my individual securities. This is where it gets complicated, and also where it gets really interesting and valuable from the investor's perspective.

OK. Let's do the calculation. You're going to have to dredge up your old DMD knowledge here of how to compute variances of sums of random variables. The variance of my portfolio return, R_p , is simply equal to the expected value of the excess return of that portfolio, in excess

of its mean, right, squared, and then take the expectation of that. And remember that the return of the portfolio is just a weighted average of the returns of the individual securities. And the mean of the portfolio is just the same weighted average of the means of the securities.

So when you plug those relationships in, what you get is that the variance is simply equal to the expected value of the square of this long weighted average. So you've got a weighted average, a bunch of terms, right? And then you square that and then you take the expected value. Well, if you've got n terms in that weighted average, and you square that n terms, how many terms comes out of that square? Anybody? n terms, and you square those n terms, so those n terms multiplied by itself, how many terms do you get when you do that? Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, why not just n squared? You're thinking about the unique elements, maybe, the off diagonal. I'm talking about all of them. So with n terms, when you square it, n terms multiplied by n terms, you get n times n terms, n squared terms. And so these n squared terms all look like this. They all look like ω_i times ω_j multiplied by the excess return of i times the excess return of j . Sometimes i equals j . And when that happens, you get ω_i squared times the variance of security i . But when i does not equal to j , then you're going to get ω_i times ω_j times the covariance between the return on i and j .

And another way of writing that covariance is equal to the correlation between i and j multiplied by the standard deviation of i and the standard deviation of j . The point is that when we look at the variance of a portfolio, it's not just the simple weighted average of the variances of the component stocks. It's actually a weighted average of all the cross products, where the weights are also the cross products of the weights. So it's these. It's all of these. And how many of these are there? Well, i goes from 1 to n , j goes from 1 to n , n times n , you get n squared of these.

So this actually has a nice representation. It actually comes out of a table. Right? So you can think of all of the portfolio weights multiplied by the excess returns on one dimension of the table, the columns, and the exact same entries in the rows. You've got n columns, n rows, n times n , or n squared elements that make up the variance of your portfolio. So this is where it gets really complicated. But this is why we have computers and spreadsheets and things like that.

So in order to figure out the variance of your portfolio, you've got to basically add all of the

elements in this table. You've got n squared things to add up to get the variance of your portfolio. And the insight of modern finance theory-- the reason that Harry Markowitz won the Nobel Prize in economics for this idea-- is that when you add up all of these different elements, you can get results that are very different from just looking at one or two stocks. Because, in particular, there are some cases where these cross products are either small or maybe even negative.

And in that case, it actually helps you to reduce the overall riskiness of your portfolio. This is the intuition. This is the mathematics underlying, don't put all your eggs in one basket. When you don't put all your eggs in one basket, what you're getting, what you're benefiting from, are these cross products that can either be small or negative, and will ultimately reduce the fluctuations of your portfolio.

Another way of thinking about it is that some stocks go up, some stocks go down. And if they don't go up and down in the exact same way, if they are unrelated or less related, then when you put it all together in a portfolio it dampens the riskiness of your overall holdings. OK. So there are n squared elements that you have to add up in order to get the portfolio variance. It's literally the sum of all of the entries in this table. And the point of the next few bullet points is that there are a lot more covariances than there are variances.

The variances are along the diagonal of that table, right? That's the variance of the first stock, the variance of the second stock, dot, dot, dot, the variance of the n stock. Those are how volatile the individual stocks are. But there are only n of those. There are n squared minus n of the other stuff. That's where you were thinking about n minus 1, right? It's n squared minus n , or n times n minus 1. Other non-diagonal entries. So what this suggests is that the covariances are a lot more important in determining the riskiness of your portfolio than the riskiness of the individual stocks.

You know, Intel looks like a scary stock because it's really volatile. But when you've got n Intels in your portfolio, even though each of the individual stocks is scary, what you have to keep your eye on is how they are correlated. Because the correlations in a portfolio of n stocks is actually more important than the individual variances. The individual variances matter, but they don't matter nearly as much as the covariances. OK?

So now I'm going to do an example. It's hard to get intuition for n by n matrices, unless you're Rain Man, or some incredible genius. The way that I think about this is, let's look at two assets.

OK? If I can understand two assets, I can sort generalize and think about n assets. So I gave you the general result just to tell you that this is how you do it. But now to understand it, really, let's focus just on two assets. OK?

Suppose we only have two stocks, a and b . And I'm going to calculate the expected return and standard deviation of a portfolio with just those two stocks. So the portfolio weights are going to be ω_a for stock a , and ω_b for stock b . And ω_a plus ω_b adds up to 1. So just keep that in mind. I don't put it there just because I want to save a little space on the slide. But these are the weights for the two stocks. And those are the only two things you hold. All right?

The expected value is as we said. It's a weighted average of your holdings of a and b , right, multiplied by the mean of a and the mean of b . Now the variance, the variance of these two securities when they are put into a portfolio is going to be simply ω_a^2 times the variance of a plus ω_b^2 times the variance of b . And now you only have one cross product to worry about because you've only got two stocks. Right? And that is 2 times ω_a times ω_b times the covariance between a and b .

Now you can do this in a 2 by 2 table, just like the n by n table. And with a 2 by 2 table, you know that you've got two diagonal elements and two off diagonal elements. And that's where you get that number 2 for the 2 times ω_a , ω_b times the covariance. OK? And I rewrite it in terms of correlations because it's easier to think in terms of correlations. Why? Because correlations we know have to be between minus 1 and 1.

And so if I tell you a stock has a correlation with another of 30%, you can actually get your arms around that. You can get intuition for that. All right? So it's a little easier to interpret. But either way is fine. So we now have an expression for the mean and the variance of a portfolio of two assets. Any questions about this? Everybody understand this and how I got it? No tricks up my sleeve. It's really meant to be relatively straightforward. But the implications will be dramatic, as I'm going to show you in a minute. OK.

So let me go through at least one numerical example. And then I'm going to show you why this is actually so important. From 1946 to 2001, Motorola had an average monthly return of 1.75%, and a standard deviation of 9.73%. General Motors had an average return of 1.08%, and a standard deviation of 6.23%. That has obviously since gone up a great deal, the standard deviation, that is. And their correlation is 0.37, or 37%.

How would a portfolio of the two stocks perform? Well, it depends. What are the weights? Right? If you change the weights, you're going to change the performance. So let's just take a look. If you put all your weight on General Motors and nothing on Motorola, not surprisingly, you're going to get General Motors' return. At the other end of the extreme, if you put all your weight on Motorola and nothing on General Motors, you're going to get Motorola's characteristics. All right? Nothing mysterious about that.

However, if you weight the two, so if you put 50-50 on General Motors and Motorola, you're going to get a return that is 50-50 of each of those returns. But you're going to get a risk that is not 50-50. Right? Because the risks, the risks don't aggregate in a linear way. The risks actually aggregate non-linearly because of those covariances or correlations. OK.

So what you see here is that by taking a weighted average of the two, what you can do is you can boost your return to 1.42%, but the extra risk you're taking is from 6.23% to 6.68% per month. That's not a big increase in risk, but that is a pretty significant increase in return. Now again, I'm not telling you that's what you all should do, because it depends on your risk preferences.

And for the next lecture or two, I'm going to be saying that over and over again. I'm going to be saying it depends upon your risk preferences. But at the end of the next two lectures, I'm going to be able to tell you something that doesn't depend on your risk preference. I'm going to be able to tell you something that all of you should be willing to do if you're rational. And if you're not, I will trade with you personally to help you learn rationality. OK? But for now, I don't have a story to tell you about which of these rows you ought to take. There is no ought to here. It's just a matter of what you prefer, risk versus return. Right?

Now interestingly, I put here one more case which lies outside of the band of 0, 1. This is a case where you've put 125% of your wealth into Motorola. And you've put negative 25% of your wealth into General Motors. Remember, we talked about this in the very beginning. You're using short selling to short sell General Motors, which you might think is not a bad idea nowadays, given how troubled they are. You're going to short General Motors, take the proceeds, and put that plus the 100% that you started with, you're going to put that in Motorola.

So you really want to make a big bet on communications and microprocessors. And you're going to leverage that bet by putting a negative bet on the auto industry. That's what that last

row implies. And what you get is a much, much higher rate of return than anything else in the other examples. Right? Much higher rate of return. But look at the risk. Now the risk is close to double what you would have gotten had you put all your money in General Motors. Are you willing to do that?

Well, you can easily imagine there are people in this class that would be delighted to do that, and there are others of you that would be scared to death. That's way too much volatility. These are monthly numbers, by the way. So if you want to calculate the annual equivalent, how would you do that? How would you get an annual standard deviation from a monthly standard deviation? Any idea? Well, let's do the return first. How do you annualize the monthly return to get an annual return? Let's forget about compounding. What would you do?

AUDIENCE: [INAUDIBLE].

ANDREW LO: What? Multiply by 12. That's right. If you forget about confounding, you multiply by 12. What if you didn't forget about compounding? Then what do you do?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Add 1, raise it to the 12th power, then subtract 1. That's right. OK. What about risk? Ah, see? This is tricky. What's the variance of a 12-month return in relationship to the variance of a one-month return? We haven't talked about that. What is it? Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: No. No. It doesn't compound in that way. Variance is actually a little simpler, under certain assumptions. Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Right. There's no correlation between one month to the next.

AUDIENCE: [INAUDIBLE]

ANDREW LO: That's right. Agreed. So let's assume that away. Then it's constant. But the monthly variance is constant. But what's the annual variance? What's the riskiness of a 12-month return if you know what the riskiness of a one-month return is? Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, it can, depending on your assumptions. But I want a simple set of assumptions. Andy?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Variance, yes. Multiply by 12. Now why is that? It's because-- and you know all this. You know this from your DMD. At least I think you do. You may not think you do, but you do. The variance of a plus b is equal to what? Who can tell me? What's the variance of a plus b? Two random variables. Variance of a plus variance of b plus 2 times the covariance of a and b. If the covariance is 0, then it is the variance of a plus the variance of b. Absolutely, that's right.

So if we assume there are no cycles, there are no predictabilities, there's no regularities, there's no correlation, every month is independent of every other month, then the variance of a 12-month return is literally just 12 times the variance of a one-month return, assuming the variances stay constant throughout the month. The monthly variance doesn't go up or down. OK?

Now, that's the variance. What about the standard deviation? The standard deviation is the square root of the variance. So that means that the standard deviation on an annualized basis is equal to square root of 12 times the standard deviation of a monthly. OK. So finally, we're almost there. What's the square root of 12? Something between 3 and 4, right? So I don't know, call it 3 and 1/2, whatever. We have to take this number and multiply it by about 3 and 1/2. And that will get us an approximate annual standard deviation.

So a 12% volatility multiplied by 3 and 1/2, you're talking about some crazy volatile stock, or portfolio. But actually, nowadays, you know, we're used to that kind of volatility. That's no big deal. But relative to the individual stock, it's quite a bit more volatile. OK? So this is an example of how you compute the riskiness and the return of a portfolio of two stocks. And when you do this, you get a whole bunch of possibilities. And the idea behind modern portfolio theory is you pick the one that you like the best. And that's optimal for you.

Now let me graph this in that mean-standard deviation graph, and you'll see something really interesting. So here's General Motors and there is Motorola. And what I've graphed with the red dots is just the different combinations of 25/75, 50/50, 75/25, and so on. And so the red dots that are strictly between 0 and 1, where the weights are between 0 and 1, those are the dots contained in that arc between GM and Motorola.

So the first point I want to make is that when you graph the risk/reward trade-off, as I told you,

it's not linear. It's non-linear. In fact, it's curved. It looks like a bullet. OK? That tells you right away that there's something more subtle about portfolio theory than just taking weighted averages. The means are weighted averages, but the variances are not because of the covariance between them. And then when you take the square root to get the standard deviation, it looks a little bit more complicated, yet again. It looks like this.

Now that red dot that goes beyond Motorola, that red dot is the case where you're shorting General Motors and you're using the proceeds to take an extra large position in Motorola. OK? So you're going beyond the Motorola risk/reward point. So I need you to spend a little bit of time now twisting your brain in a way that makes this more understandable. What I mean by that is that you have to think about the weights of the various different portfolios. But the weights don't show up on this graph.

But I want you to keep those weights in the back of your head. So while you're looking at this graph, you have to remember that as you move along, that bullet, the omegas, the weights are changing. So at the General Motors dot, your weight for General Motors is a 100% and your weight for Motorola is 0.

At the Motorola dot, your weight for General Motors is 0 and your weight for Motorola is 100%. And as you vary the weights, you trace this arc, this curve. And as you go beyond either General Motors or Motorola-- in other words, as you short sell one to invest more than 100% in the other, you then go outside of the two points in either direction. Right? Any questions about this graph? Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: OK. So how do you calculate the correlation? That's a good question. Historically, the way you would do it is exactly the way that these correlations look like they're defined. So in other words, you would calculate the covariance by essentially taking a historical average of these cross products. So going back to the first slide in this particular lecture, I gave you a formula that shows you how to do that. Let me go back and take a look at it so that you refresh your memory.

Let's see. Oh, it was not in this lecture. It was in the previous lecture. In a previous lecture, I gave you a formula that shows you how to calculate the weighted average of this, or the historical average. It's basically just taking historical data and then estimating this quantity right here, this particular cross product. The omega weights come out, because that's what you get

to decide.

But the inside stuff, how to calculate that is simply to take, historically, your actual return minus the mean of i , R_{jt} minus the mean of j , and then take 1 over t , and that's your estimator for the σ hat ij . So you get historical data and you would estimate the mean, estimate the mean, take the cross products over time. This t goes from 1 to t , and that gets you an estimator.

Is that the best estimator? Not necessarily, because things change over time. So you have to worry about that. And that's what you learn in 15433, how to actually implement a lot of these ideas using data. Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: OK.

AUDIENCE: [INAUDIBLE]

ANDREW LO: OK. So the question is, is there any recommended length of time for which you would go about estimating the data, estimating these quantities using the data? The answer is no. There's no recommended time, simply because you have to trade off two things. The longer time you have, the more accurate your estimate is going to be. In the limit, as t goes to infinity, you'll get a perfect estimator. OK?

However, that's under the assumption that nothing ever changes. And I'm pretty sure that beyond some t , over let's say 150 or 200 years, you know, we didn't exist in terms of stock markets and data. I don't think that, for example, General Motors goes back 200 years. So beyond some t , this is going to look very boring. But the other thing you have to trade off is the fact that market conditions change. So General Motors today is not what General Motors was, even 10 years ago.

So if you use lots of data, you're going to build into your results what are called non-stationarities. And the bottom line is you have to balance off the non-stationarities against the error that you introduce by not using enough data. And that really is where 15433 comes in. So there are methods that we have developed for balancing those two. But the bottom line is that there isn't one single answer. So it depends on how unstable the markets are and how volatile the underlying estimates are, given the data. Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, sure. You could try to include a full cycle. But then you're left with the question, the full cycle of what? For example, there are these things called Kondratiev cycles that claim to be something like 50 year periods. So if you want to include one of those, you've got to include 50 years of data. But do you really think that General Motors is the same company over those 50 years?

You're building in a lot of bias in terms of the non-stationarity of one aspect, while incorporating the stationarities of another aspect. So how do you balance off those two, right? This is where it gets more complex. And you have to take a stand on the kind of non-stationarities that are in the data. Other questions? Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, first of all, we're not trying to predict the stock market. Remember, we're not trying to predict where markets are going to go. What we're suggesting is that there are underlying parameters of the data that are stable over time. So in other words, the stocks can go up or down, right? But on average, they have some level of return that people expect, given their risk. That's what we're assuming is stable.

Similarly, covariances. Stocks can go up or down in relationship to other stocks going down or up. We don't know what's going to happen day to day. But over a period of time, it seems like there's a pattern where most stocks on the New York and American stock exchanges and NASDAQ, they seem to go up together and they seem to go down together, on average. So that's what, really, what we're trying to distill from the data.

It's very different than trying to forecast what's going to happen with the stock market next week. And this, I told you, is the fundamental difference between academic finance and Warren Buffett. Warren Buffett is all about prediction. He's not about creating a good portfolio that will be worth something reasonable over a period of time.

What he wants to do is he wants to beat the market. He wants to find undervalued stocks, invest in them, and then sell them when they actually reach equilibrium or become overvalued. Right? That's a very different approach and perspective than what we're doing here. So that's a good question, and it highlights that distinction. Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yes, there is. And there have been many studies that have been done on it. And that is beyond the scope of this course. So I'm going to refer you, again, to 15433. But I will tell you, at the end of this, once I get through this and make sure everybody is with me, I will tell you how this works in practice. OK? So I'll talk a bit about applications. But first I want to get through the theory to make sure we all understand it. All right.

So we now have this bullet. And the bullet obviously depends on all the parameters. But in particular, it depends upon the correlation. The reason that there is a bullet shape is because there is a correlation between these two stocks. And by the way, this correlation and this bullet shape is really important. OK? I'll tell you why it's really important.

Take a look at a vertical slice, a vertical line going through the GM dot. That vertical line is the riskiness of GM. Right? It's the standard deviation of GM. One of the things that you'll notice about this graph is that if you only had two stocks available to you, GM and Motorola, the least amount of risk that you could possibly create for yourself is if you put 100% in General Motors. In other words, if all you cared about was going west, the most extreme west you can go other than putting your money in T-bills over here, the most extreme west you can go is General Motors. You can't get any less risky than that. Yeah? Question?

AUDIENCE: [INAUDIBLE]

ANDREW LO: What I'm saying is, if you are trying to get less risky as possible, and you only had General Motors or Motorola, then the most west you can go is General Motors. Right? That's all. However, however, this is the important point. If now I let you take weighted averages of the two, if I give you the right to form a portfolio, then you can get a dot, which is this dot right here. That dot, everybody in this room should prefer that dot to General Motors.

Why? Because it's less risk than General Motors, but it's also higher return. There's no downside, at least from the perspective of mean and standard deviation. There may be other reasons you don't like that dot. But if all you care about is mean and standard deviation, that dot is strictly preferred. So I just made all of you better off with this piece of knowledge. Just by telling you how to weight these two stocks and the fact that they've got some kind of correlation, I've given you a mechanism of reducing your risk and, and increasing your expected rate of return, both.

OK? So if you were really risk averse, if you said to yourself, you know what? I don't want all this fancy stock picking. Just give me the least risky stock of the two. Then you'd be at General

Motors. And then if I came to you and said, you know what? I can do even better than that. I can get even less risk than General Motors. And at the same time, I'm going to give you higher return. So right there, the value of portfolio theory is pretty clear. It gives you options you did not have. Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: That's right. That's right. When you have correlations that are unstable over time and you didn't account for them, you can get into trouble. OK? Again, let's come back to that after I go through all of this. All right? So, just if this is a clarifying question, I'll answer. But extensions, let's wait until we actually go through this. OK.

So let me talk about the General Motors and Motorola example, but now where I am going to tell you that correlations are changing over time. So let's do the example that Lewis wants us to do. Let's actually assume the correlation is, I don't know, 0 or 1 or minus 1. Let's go through all three cases and see what happens. You'll see some remarkable things coming out of this.

If the correlation is 0, then it's pretty easy for you to just plug in 0 for that correlation in that second equation. And all you get are the first two terms of that variance expression. Right? So when you assume that the-- when you assume that the correlation is 0, when this thing is 0, what happens is that this entire term goes away and all you have are the first two terms. That's this column right over here. OK? So I've just produced the previous table but with the assumption of 0 correlation, and I get that.

Now I'm not going to graph it for you yet. I want to just show you what happens when you choose different assumptions. So that's one assumption. On the other hand, if I choose a different assumption, if I choose the assumption that the correlation is, let's say 1-- so in other words, they are perfectly correlated-- then I'm going to get, I'm going to get this. I'm going to get this becoming 1, and then I'll get another simplification of this expression.

And similarly, if I assume that the correlation is minus 1, then I'm going to get yet another simplification of this expression. It's an algebra that I'll show you in a minute. But this is what I'm doing in these three columns. I'm assuming different values of correlation between

General Motors and Motorola, and then computing the standard deviation of the portfolio using this formula. OK? So you can all do this at home, do this in a spreadsheet. It's going to be very easy to check my work.

But I want to show you the graph because that's really, I think, the insightful intuition here. Look at the graphs. These are the graphs of the three different cases of zero correlation, perfect correlation, and perfect negative correlation. I want to go to each one of these with you.

So the case of perfect positive correlation, it turns out that the risk/reward trade off is actually just a straight line. It gets really, really simple. It's just a straight line. So in particular, there is no non-linearity. Because if they're perfectly correlated, you know what? You basically have virtually the same stock. The only difference is that they're different scales of each other. Right? But if they're perfectly correlated, that means that there's a linear relationship between those two stocks.

And so when you do this mean variance analysis, in mean variance space, you basically get no nonlinear-- you don't get that little bump, the little bullet that we saw before. Right? So there's no way you can get less risk than General Motors unless you end up shorting Motorola and putting it in General Motors. And then your expected return goes down, as well as your risk.

All right. Now let's take the case where you've got no correlation. It turns out that if you have no correlation, you get the bullet. But the bullet is even more wedge shaped. What that means is that you can reduce even more risk without getting rid of return. Remember the red dot that we saw in the previous slide? This red dot? Well, in the case where you've got 0 correlation-- this is the case where the correlation is 0.37, 37%. Right? That's what the data tells us.

But if you assume that the correlation was zero, you would get even more of a savings of volatility for a given level of expected return. OK? So it would look like this. You see how this bullet sticks out a lot more than the previous red dot that was somewhere over here? OK? Now let's continue. Suppose the correlation is minus 50%. Then you get an even wider bullet, that sticks out even more, that saves you more standard deviation. You're getting to the west even more.

And finally, and here is where you get a really remarkable result. If the correlation is minus 100%, you know what you get? You get a piecewise linear trade off. You get this and that. It

doesn't turn into a wedge. It turns into a triangle that actually hits the x-axis. And the reason this is such a startling result is it tells us that there exists a way to construct a portfolio that gives us a rate of return of like 1.39% with no risk. Zero risk. 1.39%. Let's just call it 1.3% to be conservative. Multiply that by 12 and you're going to get something like what? 16% a year?

You tell me if you know of any investment opportunities that gives you a return of 16% a year with no risk, and I'll examine that for you carefully. OK? It doesn't exist, of course. The reason it doesn't exist is because you can't find two assets that have perfect negative correlation. If you could, there are wondrous things you could achieve with that combination. And portfolio theory basically tells us how, right? This is a recipe book for how to exploit correlation. All right?

Now again, I can't tell you where you should be on this curve other than if it's really minus 1, then I would be here. OK? Lots of return, no risk. That's an arbitrage. We know that that can't possibly happen. There's no free lunch. There's no arbitrage, on average, over circumstances. However, if the bullet is minus 0.5, then you've got lots of opportunity to create really attractive portfolios that doesn't require Warren Buffett's skills. There's no forecasting here. Right? We're not trying to pick stocks. We're not trying to see how the market's going to do next month. Who knows?

All we're assuming is that means and variances are stable over time, and the correlation is stable over time. Those are nontrivial assumptions, I grant you. But if you believe in those assumptions more than you believe in your ability to forecast what's going to happen to General Motors six months from now, then this might be a good way to construct a portfolio.

But I haven't told you where on this curve you ought to be. I just told you how to construct that curve. All right? It's your job to look at the curve and say, I like this point, or I like that point, based upon your own personal preferences for risk and reward. So we're not there yet where I can tell you how to behave. I will get there in about a lecture and a half. But we've got to build up the infrastructure to be able to get us there. OK.

So what this tells us is that we need to know what the correlation is in order to figure out where we are going to be on these different curves, which curve is going to apply. When you have lots of correlation, a correlation of 1, there really isn't much of a risk savings per unit return. We can't get a lower risk for a given level of return. But we can if there is less correlation than perfect. And these are the different curves that illustrate that. OK.

So there are some other examples that I'd like you to work through on your own. This is another portfolio calculation. Just go through the same calculations that we did here. And you know, you'll graph the different risk/reward trade off between these two, General Dynamics and Motorola. And you can get exactly the same analysis with those two stocks, General Dynamics and Motorola. OK?

Now what about if you got a risk free rate? So suppose that the two assets that I want to look at is not General Motors and Motorola, but rather the stock market and treasury bills. Then what does your risk/reward trade off look like? Well, it turns out that in the case where treasury bills are in question, the volatility of treasury bills is virtually zero. It's not exactly zero because there may be some kind of randomness in the underlying rates of return because of inflationary expectations. But as an approximation, if it's a risk free rate and you know that you're going to get that risk free rate, then the volatility of that return is in fact 0 over that period of time.

So in that case, the expected rate of return of a portfolio between the stock market and T-bills, that's the weighted average. But the variance is going to be very simple because it's going to be the variance of 1, which is 0, plus the variance of the other, plus 2 times the weighted average times the covariance. But there is no covariance. Right? There's no correlation because one of the things is non-random.

And so when you work out the weights for the two and you graph them, you get this. This is a beautiful thing, nice and simple, no weird curves or any kind of bullet shape. You've got T-bills here. You've got the stock market here. And the weights, as you vary them, will bring you anywhere along this line or possibly up over here.

If you're in the middle of this line-- so literally, if you have the same distance between here and here and here and here-- that actually gives you a 50-50 weighting on those portfolio weights. So geometrically, this actually corresponds to a 50-50 weighting of T-bills and the S&P. OK? Now question, what happens when you are over here? Let's suppose you're at this point. At this point, what would your portfolio weights look like? How would you characterize that? Yeah, Brian?

AUDIENCE: Short T-bills to buy into the stock market?

ANDREW LO: That's right. Short T-bills. What does it mean to short T-bills? What are you doing? You're

borrowing. You're borrowing money. You're leveraging. When you're shorting T-bills, you're basically borrowing and getting cash upfront you're to pay back later with interest. So shorting T-bills is just borrowing. If you're borrowing money and you're putting it into the stock market in addition to 100% of your own wealth-- you've borrowed additional money to put it in the stock market-- then you're going to be way up here.

Higher return, much higher up here than down here, but you're going to get higher risk, as well. So leverage, this idea of borrowing and putting your money in the stock market, that increases your expected rate of return. But it also increases your risk. OK? Leverage increases your risk. And now getting back to the question that Lewis asked us, is this where we got into trouble with the current crisis? Yes, in a nutshell it is. But it's more complicated because the underlying securities are more complex.

But the basic idea is if you leverage up, if you leverage up, way up here-- you're up maybe out there-- and all of a sudden there's a bump in the road and what you are leveraging, this thing that you're investing in is not nearly as smooth and as riskless as you thought it was, it could wipe you out. And one of those elements that could cause such a wipeout is if you somehow forgot about the fact that correlations can change. So you thought that you were, I don't know, somewhere here, and all of a sudden correlations go to 1 and now you're actually over here.

You see how risk can change really quickly? Correlations don't have to be stable over time. And that's the lesson that most people in industry, who don't have a finance background, who've never taken this course, they won't know. These are physicists or mathematicians or computer scientists. They estimate the correlation. It's a parameter, like the gravitational constant or Avogadro's number. Let's plug it in. Hey, 9.08×10^{23} , that's what it should be. And nobody ever told them that it could change.

And when it changes, bad things can happen really quickly. So we're going to come back to that. But let's get the standard theory down first, and then we'll talk a bit about application.

Anon?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Right here? Of course. Because you might want to get more return and you're willing to take on risk. Let's take a look at this. This bullet point here says that you're going to be at approximately, something-- if the correlation-- let's be realistic about it. OK? The correlation is not going to be minus 0.5. It's going to be more like 0.37. So this bullet here is going to be like 1-- I don't know, let's call it 1.3% just to make it easy. And you've got a standard deviation of about 6%. OK?

So on an annualized basis, that's giving you a return, 1.3 times 12 is something like what, 16%? 16% return, but the risk on an annualized basis multiplied by 3 and 1/2 is about, let's say 20%. So 20%, 22% annual standard deviation for a 16% rate of return. Now some of you might like that. But I have a few friends here for whom that return is just boring. That's just not going to get their attention.

What they want is they want to be up here, you know, like at 2% rate of return. 2% rate of return per month is about 24% a year. So if you're a hedge fund manager, 24% is when you start to begin to feel alive. You know? That's when things really start to happen for you. And at 24% annual return, you can't have volatility of like 15% or 20% unless you're doing something, you know, really different than the standard market portfolio.

So a hedge fund manager is going to say, Anon, give me a break. You know, this is going to put me to sleep. I want to be up here. But you know, you've got a family, three kids, you have to worry about making mortgage payments. That kind of lifestyle is not for you. So in both cases, though, in both cases, we can agree that where we want to be is on this curve, right? In other words, you would never want to be here, at this point.

Why? Because if you were here, you can either, for the same level of risk, increase your return. Or for the same level of return, decrease your risk. So what we can agree on, even though we don't agree on where we want to be on that curve, we can all agree we want to be on that curve as opposed to inside the curve. Right?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Exactly. Great point. Why would we want to be on the bottom half? Nobody would want to do that. Because if we were on the bottom half, you can just as easily move up to the top, and you

have a higher expected rate of return for the same level of risk. So you're exactly right. That bottom half of the curve, you can just throw it away. All right? The only people that are going to be down there are knuckleheads. All right? So we don't want to be down there.

So in fact, the only part of the curve that really matters is this point, going all the way up. And we're going to give a name to that in a minute. That's going to be called the efficient frontier because you would never want to do anything else. Actually, that's not quite true. What everybody would like to do is they'd like to be up here.

Unfortunately, we can't get there. We can't get there with just General Motors and Motorola. In a few minutes, I'm going to show you how you might be able to get there with other stocks. If you introduce other assets, then you might be able to get there. But you can't get there right now. OK.

So here we are with stocks and T-bills. And we know that we'll get the linear combination here. What I'd like to do next is to make this yet more complicated. All right? And before I do, let me just summarize what we've done so far. What we've done is to show that with two assets, we can do all of this analytically, and illustrate graphically, all of the intuition that holds for the more general case.

With two assets, the expected rate of return is just a simple weighted average. But the variance is not just a simple weighted average. It's more complicated and it depends, in particular, on the correlation. For different correlations, we get different shapes of trade offs in mean-variance space. And you have to understand what those trade offs are.

If it turns out that one of the two assets is T-bills, for example, then the trade off is really straight line. OK? But if both assets are risky, then you get the bullet shape until you either get minus 1 or 1 in terms of correlations. And then you get straight lines of different stripes in those two different cases. OK. Now we're ready to talk about the general case.

The general case works exactly the same as the two asset case. You've got your means. You've got your variances. And you've got your covariances. And you add up all of these different covariances to get the total variance of the portfolio. And if you consider a couple of simple cases, like for example, an equal weighted portfolio-- so you've got n assets and you put each of your-- for each of your assets, you put $1/n$ of your wealth in them.

Then you can show that the variance of your entire portfolio is equal to the average variance

plus n times n minus 1 times the average covariance. When n gets large, then it turns out that the average variance doesn't matter. What is driving the risk of your portfolio has nothing to do with the variance of the individual components. What it has to do with is what the average covariance is. That's what's driving the risk of your portfolio. Because you've got a lot more covariances than you do variances. OK?

So this is kind of a neat insight because it says that it's really important how things are related. And as those relationships change, the risk of your portfolio is going to change. So getting back to Lewis's point about what's going on in current markets and what caught a lot of portfolio managers by surprise with the subprime markets, if the risk of your portfolio is approximately given by the average covariance and you've been assuming all along that you've got this big pool of mortgages, and the mortgages are all uncorrelated, you've essentially assumed that you've got virtually no risk.

Because the average covariance by assumption and by historical analysis is close to zero. But when the real state market goes down nationally, then everybody starts to default. And foreclosures become very highly correlated. And so you can see how overnight, literally overnight, your risks can shoot up. And you're not prepared for that unless you know that this is what's going on in your portfolio. Yup?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Great question. Why should things change in terms of correlation? Well, you know what? I'll give you an example. I'll give you a personal example? Correlation is a function of human behavior, right? I mean, prices are being formed by you and I, investors. And so correlations simply means that all of us end up doing the same thing around to same time. Right?

I'm going to be heading to the airport later on this evening. And I don't expect that there's going to be much of a big deal getting on my flight. I'll probably get to the airport about half an hour, 45 minutes ahead of time. It's just a shuttle. So I'm heading to Washington, so it's not a big deal. What's going to happen in two weeks from today? Anything? Anything going on two weeks from today that you can think of? Thanksgiving.

So if I went to the airport two weeks from today and tried to get on that shuttle half an hour before, you think I can get on the flight? Why not? Because everybody else is going to do that. Well, why should everything be correlated on that day? Isn't that a Wednesday like every other Wednesday? Well, it's not. It's because somehow we've all decided that we're going to take off

at the same time on the same day in that year. Right?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, first of all, I don't think Thanksgiving is that low a probability. OK? But to your point, is that the correlation changing? Well, it is. Correlation is a statistical measure of two objects. And what we're trying to capture is when they move up or down at the same time. Right? So what I'm trying to get at is that there are certain periods of time where human behavior, all of a sudden, becomes very highly correlated.

And there are many reasons for that. In particular, Thanksgiving is a reason we all decide that Thursday is this national holiday. And therefore, on that Wednesday, we actually have to travel in order to get to where we need to go by Thursday. Right? But any arbitrary Wednesday is not going to be necessarily highly correlated. So when I go to the airport on a typical Wednesday, I don't expect it to be so bad. But on Wednesday before Thanksgiving, it's going to be a madhouse. Right? That's an example of a changing correlation because of a particular coordination that we've all agreed upon.

Now that's a relatively artificial example. But I did it to make a point. Now let me give you a real answer to your question about why things may be correlated. When we are all scared about the value of our investments, when our fear circuitry gets triggered, what's the natural instinct for all of us because it's hardwired into our brains? It's to get to safety. It's to get out of those bad assets and get into the good assets. Right?

You saw that three month T-bill yield at 10 basis points, or 5 basis points? That's a sign that we're all scared to death and we want to get to safety. That's an example of a correlation. Why? Because everybody is going to be selling at the same time. In a crowded theater, if you smell smoke and somebody shouts fire, what are you going to do? It's not rocket science to predict that the four exits that are out there will be a bit crowded after that. Right?

So correlation is not a physical quantity. That's the problem with physics and biology. Physics has parameters that don't change over time. I wish we could have that in finance. We don't. We have parameters that are not parameters. They're random variables. They depend on a lot of things. Correlation is one of them. And until we start recognizing that correlations are really part and partial of human interactions, we're going to continue to make mistakes like we've made over the last 10 years. And that's a simple example of that. Other question? Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yes.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yeah. That's true. That's possible. So you have to decide whether or not what you're looking at is an aberration, or whether it's something that's systematic. So for example, if I didn't know anything about Thanksgiving and I happened to travel on Wednesdays on a regular basis to Washington, then you know, in two weeks, I'll get-- it's really crowded. And then it won't be crowded, won't be crowded, won't be crowded. And pretty soon, after a while, I'll say, well, that was just a 1% event.

Of course, next year it will happen again. And then I'll say, gee, you know, well, that's another 1%, kind of. But pretty soon I'm going to realize that, gee, it seems like there's a pattern here. And that 1%, it's not just so simple as a 1% error. Statistics, remember, is a mathematical quantification of our stupidity. Right? I mean, what we don't know. Well, we don't know why things happen the way they happen, so we put a distribution on it. We say that it's normal. And we say that there's 5% this way, 5% that way.

But that just is a representation of our ignorance. The more we know, the less we have to rely on statistics. So I don't need statistics to tell me that two weeks from now, it's going to be really crowded at Logan. But if I didn't know about Thanksgiving, if I came from Mars and I was doing a study of airport congestion, it would take me a while to get enough data to figure out that once a year on the Wednesday before the Thursday in, you know, November, it gets crowded.

So the challenge for all of you is how much intelligence can you bring to the analysis. What I'm showing you is very simple mathematics. Mathematics is not enough. If you just had the mathematics, you would be losing money, you know, continuously. Because there's just much more to financial markets than just the math. OK? The math is trivial. This is high school algebra. But the real key is to put together the framework of economic and financial analysis with the mathematics.

So let me continue doing that. This is another example of computing the average variance, and then looking at the volatility of your particular holdings, given different correlations. So I want you to do lots of exercises where you look at different correlations. I don't ever want you

to take correlation as a parameter that is fixed over time, when you apply the stuff. When you're doing problem sets in final exams, that's fine. You know, the correlation is whatever it is. But recognize that in practice, these things change a lot over time. OK?

Now the idea behind correlation is that they help you reduce the ups and the downs of the variance of your portfolio. But it turns out that there's a limit to how much of the risk you can reduce. And this graph basically shows that as you add more and more securities-- even though the correlations are bouncing around-- as you add more and more securities, there ends up being some kind of a steady state limit to what the variance of your overall portfolio is.

After 20, 30, 40, 50, 100 stocks, you'll notice that the variance of your portfolio doesn't go down anymore. And it turns out that that limit, whatever that is, is what we consider to be the systematic risk that is implicit in the economy. In other words, that's the risk that no matter how well diversified you are, you've got to bear that risk. Everybody, all of us, we have to, we can't get any less risky than that unless we start putting our money in the mattress or in T-bills. OK?

That limit is known as the systematic, or market risk, of a portfolio. So, see here is the graph of your portfolio variance as you increase more and more securities. But at some point, it asymptotes to this level, which is what I'm going to be focusing on, as undiversifiable risk. It's the risk that's left over after all of the various correlations have done what they can to dampen the ups and the downs of your collection of securities.

And obviously, there's going to be some calculations you'll need to do to figure out what that number is. But I want to give you the intuition first, and then we'll do the calculations. And then we're going to study the properties of this hard limit. OK?

Now, I'm about to proceed to the next stage of our analysis where we start asking the question, how do we pick the very best possible portfolio. But before I do, I want to make sure that everybody is comfortable with the analytics we've developed. If you have any questions, now would be a good time to ask. Because from this point on, I'm going to assume that you understand how portfolio theory works, the mechanics of portfolio weights, and how to compute means, variances, covariances, and what they imply for the portfolio. Question, yeah?

AUDIENCE:

Earlier, you said that we start from this, ensuring that the markets are pretty much stable. But I'm wondering, how reasonable is that assumption? And why would nobody ever pick a lower part of on the curve, for example, you know, if there was a big scandal tomorrow, like a

corruption, like I'm thinking about Enron or [INAUDIBLE] in Italy. Why would I not want to short sell, you know, the higher return stock and buy the lower return stock, and take the lower part of the curve?

ANDREW LO: You might, if there are other elements that are in this analysis. So as I said, this analysis assumes there are only two dimensions, mean and standard deviation. Right? You've now introduced a third, which is corporate responsibility or corporate governance or fraud or something like that. Then you would need three dimensions. And there'd be something sticking outside the screen.

And then you'd have to choose among the three possibilities. So it's possible that you want to be down here because this company has a better reputation than this one up here. But then what you're telling me is that reputation matters to you beyond standard deviation and expected return. What's that?

AUDIENCE: Market conditions.

ANDREW LO: Stable means that these parameters are likely to stay the same over time.

AUDIENCE: There's a crisis every three years. There's an industry crisis every two years.

ANDREW LO: Right. Well, the argument that a financial economist would make-- and I'm not saying that I believe in this. I'm going to tell you what I believe at the end of this class. But I'm telling you now what the party line is, and what's in your textbook. What's in your textbook and what the party line is is that when you use historical amounts of data, those kind of crisis periods are in there already.

So if I estimate the correlations, including October '87, including March 2000, the bursting of the internet bubble, August 1998, LTCM, if I include all of those in my data, then it's captured in there. Right? It's not captured today. In other words, this point here for General Dynamics doesn't reflect that there's a crisis today. What it reflects is, on average, over a long period of time-- some of which includes crises-- that's what the expected return looks like. And so I've already incorporated that into the parameters.

And if it so happens that today is a bad day for General Dynamics, that shouldn't influence whether you buy it or not. Because what you should be thinking about is over the next 15 or 20 years, how will my portfolio do. So that's another difference in perspective from us versus

Warren Buffett. Although actually, not as much as you think. Warren Buffett typically takes a long term perspective, right?

His investment in Goldman Sachs? We all thought it was a great deal. He's lost money in it since he put that money in a few weeks ago. But is he worried about it? I don't think so. Because he's invested in it for the next 15 to 20 years. And I believe, as he does, that he got a great deal over that time period. So if you care about what's going to happen tomorrow, then all of the things that I'm telling you, you should not take that as seriously. Because these kind of parameters aren't made for day to day forecasting.

Then you need to get hedge fund models. And for that, you need to take not only 433, but a number of other courses, and courses that aren't even offered yet. Because frankly, if I told you how to do it, I'd have to kill you. Right? Highly proprietary. But this is the approach where we're not trying to forecast markets. And so I'm acknowledging that you're right that there are instabilities.

But as long as I'm using a long enough amount of data, that it's in there. It's captured in there, in that set of parameters. OK? If it's not, if you know of something that's not in here, obviously you can't expect this to give you the right answer. And you have to adjust your use of this technology, accordingly. Other questions? Yeah?

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yeah. That's right. Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Well, you have to know how to build those other variables into this kind of a framework. That's another way of putting it. So in other words, this framework is still useful in the case where you've got other variables. But you just need to know how those other variables will impact the parameters for this analysis. If you can figure that out, then you're a hedge fund manager. Right? That's what hedge funds try to do. They try to figure out how all of these relations impact on how to construct a good portfolio. All right?

AUDIENCE: [INAUDIBLE]

ANDREW LO: That's right. For example. That would be one way of looking at it. But there are other ways. It doesn't have to be quantitative. There are a lot of talented hedge fund managers that don't know how to use a calculator, make a lot of money. OK? So don't think that it's all about quantitative analysis. That's what we think. That's how I think. That's how MIT may think. But that's not the only way to make money out there. You know, Warren Buffett, I don't think knows how to calculate these covariances. But he's done OK.

So but I think what we can do with this framework is to analyze how he does what he does, and understand it in the context of this kind of a framework. All right. So let's now go to the next topic, which is, all right, how do we choose a good portfolio, given what we now know about this framework? Well, we said before-- Ryan pointed out-- that we never want to be on the lower part of that frontier. So that's one thing we know. Right? We don't want to be on the bottom part. So we want to be on the efficient frontier.

Now this entire bullet can be viewed as what's called the minimum variance boundary. Meaning, for any given investment over here, we have the absolute minimum variance that has the same level of expected return. So when I go, when I go horizontally, I'm looking at the smallest amount of risk I can take for that same level of expected rate of return. That's what this bullet can be viewed as giving you.

And it turns out that because we like expected return, and we don't like risk, we want to be on the upper part of that bullet. So this upper part is known as the efficient frontier. So that's the one thing we can tell about portfolio theory. It's that we want to be on the upper branch. But where on the upper branch depends upon our risk preferences. OK?

So here's a concrete example. Suppose you can invest in any combination of General Motors, IBM, and Motorola. What portfolio would you choose? So these are the data that you would start with. OK? The means and the standard deviations, and then of course, the covariances. You've got to have the covariances to be able to calculate that bullet. So you can invest in any combination. What would you choose?

Let's calculate the expected rate of return and the variance of that portfolio. And you want to ask the question, what looks good to you? OK? Well, it turns out that when you calculate this kind of a bullet, you find out something that's, yet again, amazing. What you find is that the bullet actually is better than any of the three stocks that you started with.

So look at where the three stocks are-- General Motors, IBM, and Motorola-- and the bullet, look at where the bullet is. The bullet is strictly to the northwest of these stocks. Right? In other words, you can do better than any one of these, either by going here or by going here. For the same level of risk, you can get a higher return. For the same level of expected return, you can get lower risk. OK?

So the first point is that portfolio theory, for multiple stocks, is even more compelling. Because now, with those two stocks, at least the two stocks were on that minimum variance boundary. Now, with three stocks and more, it's possible that none of the stocks are going to be on the minimum variance boundary. Yeah?

AUDIENCE: What I don't understand is this. S This is backward looking. So if people between companies and so on all make this graph and they all decide to strategize based on this graph, the price of these stocks next day are going to be correlated in a different way than you expect because it's going to effect the--

ANDREW LO: Very good point. But it turns out your conclusion is false. But that's a good question to ask. And I'm going to answer that, not today, but on Monday. When we go over the question, what happens when everybody does it, I'm going to deal with that question head on. OK? That's going to be a very important point. And this is going to be one of the very few instances where when everybody does what I tell you they're going to do, that's actually going to give you an equilibrium. And everything will work out in just the right way. That's the magic of the CAPM.

But you're right that Warren Buffett could not answer that criticism. If you talk to Warren Buffett and said, Warren, if everybody did what you did, then it wouldn't work. And he would say, you know what? That's right. That's why nobody can ever do what I do. I'm just smarter than everybody else. And so, I'm sorry. That's the way it goes.

Here, I'm not appealing to everybody being smarter than everybody else. Because frankly, we're not. What I'm appealing to, what I will appeal to, is if everybody does the right thing-- by the right thing, I'm about to tell you what that right thing is-- everybody is on that upper branch and maximizes their risk/reward trade off, a very special thing happens. All right? I'm going to keep that as a surprise for Monday. OK, question? A question? Yeah.

AUDIENCE: [INAUDIBLE]

ANDREW LO: Yeah. Well, first of all, it's not clear that Warren Buffett is number one. All right? He's got a long track record and he's got the biggest pie. But in terms of actual track record, he doesn't have the best track record. There are people that you've never heard of that have a better track record than Warren Buffett. For example, there's a fellow by the name of James Simons-- who I may have mentioned earlier on as a hedge fund manager-- who also happens to be a first rate mathematician, who started up a hedge fund called Renaissance Technologies.

That is probably the single best track record of any manager in the history of investments. And he does it completely quantitatively, completely automated. He hires something like 100 PhDs that work on nothing else but how to forecast the next minute, as well as the next hour, the next year. It's an extraordinary track record. So there are a number of folks like that. Let me not dwell on that, but I'll come back to that in a few lectures when we talk about performance attribution. OK.

So I'm going to leave you with one final thought since we're out of time. There is going to be a very special role played by a portfolio called the Tangency Portfolio. And I want you to think about how your risk/reward trade off would look when you mix your T-bill risk-free asset with arbitrary portfolios on this bullet. Think about what that trade off would look like. And ask yourself the question, does there exist a special portfolio on that efficient frontier that everybody in this classroom is going to want to have?

All right? I want you to identify them. And I'm going to ask you that question on Monday, and I expect an answer. All right? I'll see you on Monday.