Guided Study Program in System Dynamics

System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #29
Saturday, September 4, 1999

Reading Assignment:

Please read the following paper:

- *Second-Order Systems, by Leslie A. Martin (D-4731)*

Also, please read the following:

- Introduction to Computer Simulation,[2] *by Nancy Roberts et al.: Chapter 10*

Exercises:

1. Second-Order Systems

A. Refer to the scenarios that generate asymptotic behavior in Figures 6, 13, and 14 of the paper. Can you think of any real world systems in which a carefully balanced positive feedback loop produces stabilizing behavior?

I wonder if it would be wise to balance a positive feedback loop in a way that stabilizing behavior is produced. The equilibrium reached will be unstable, which may result in exponential growth or decline caused by even the slightest distortion to the equilibrium. If the goal of the system would be to maintain equilibrium, the result of a minor distortion may be disastrous.

An example:
Juan and Luis work in the same big enterprise and hold similar work positions. There is a vacancy at a higher position and both are possible candidates for the position. They have to compete for the vacancy through an oral examination and interview. They are together

[2] Roberts, Nancy, David Andersen, Ralph Deal, Michael Garet, and William Shaffer, 1983. *Introduction to Computer Simulation: A System Dynamics Approach.* Portland, OR: Productivity Press. 562 pp.

in the waiting room on the day of the examination. Each candidate's confidence level depends on the perceived state of the other candidate's confidence level. Thus, if Juan perceives that Luis is nervous, Juan thinks that Luis has low personal confidence in qualifying for the promotion, so Juan gains confidence. Because Juan now shows greater confidence, Luis loses further confidence and gets more nervous, which in turn makes Juan more and more confident in obtaining the position and so on. Juan's confidence increases exponentially while Luis's confidence decreases exponentially.

Any small difference in the expressed personal confidence of Juan and Luis will generate such reinforcing behavior: the exponential growth of personal confidence of the candidate with a greater initial confidence, and the exponential decay of personal confidence of the candidate with a lower initial confidence. But if both candidates have the same initial personal confidence, both Juan's confidence and Luis's confidence will increase or decrease through time to similar values, or simply remain constant.

Assuming that observing the other candidate's confidence, each candidate's confidence level starts dropping: the lower a candidate's personal confidence, the lower the rate of decrease until both candidates reach at the same time a neutral state (neither confidence nor nervousness) and remain at this unstable equilibrium. Even the smallest difference in the perceived personal confidence of Juan and Luis will generate the exponential behavior explained above.

B. The scenarios in this paper vary the sign of the parameters, and the sign and magnitude of the initial values of the stocks. What would happen to the different scenarios if one were to also vary the magnitude of the parameters? Support your answer with examples and graphs of model behavior.

The magnitudes of parameters do not affect the general mode of behavior (as long as the ratio of the magnitudes is constant). If a system exhibits oscillating behavior, changing the magnitudes of the parameters will change the amplitude of oscillation. If a system exhibits exponential or asymptotic behavior, then increasing the magnitudes will increase the rate of change, thereby causing the stock to change more rapidly.

Essentially, note that the magnitude of a parameter controls the value of a flow. Therefore, making a parameter larger will cause the flow to be larger, and thus cause the stock associated with that flow to change more rapidly. Conversely, decreasing a parameter decreases the value of the flow and causes the stock to change more slowly.

2. Modeling Exercise: Waste Disposal

Chapter 10 of Introduction to Computer Simulation leads you through the conceptualization stage with the Solid-Waste Disposal system.

Step 1: Conceptualization

Read Chapter 10 of Introduction to Computer Simulation. In your assignment solutions document, include your answers to Exercises 1-4.

<u>Exercise 1: Perspectives on Solid-Waste Generation</u>

a. A supplier to a car manufacturer who will be able to sell fewer products if parts are reused may be against recycling.

   A Greenpeace volunteer will encourage all initiatives for reducing solid-waste generation and for recycling material.

b. To the car parts supplier, investments made in the past to manufacture products may be of concern. Reduced demand may lead to less than optimal productivity and higher costs, which may reduce the supplier's profitability.

   To the Greenpeace volunteer, it may be important that recycling leads to reduced depletion of resources, less energy consumed by industry, or less air pollution.

<u>Exercise 2: Time Horizon</u>

a. A personal computer has an average lifetime of three years.

b. The reserve index for a resource is the number of years available of that resource, given the present usage rate. The reserve index (RI) equals the present amount of that resource (R) divided by the amount used each year (U): RI = R / U.

<u>Exercise 3: Problem Behavior</u>

a. A problem for a bottle manufacturer may be the time when sales of new bottles drop to a point so low that he might go out of business.

b. Retooling and developing a new marketing strategy for a more expensive, more durable refrigerator.

<u>Exercise 4: Policy Choices</u>

a. Reduce packaging: reduces natural resource depletion
   Prohibit nonreturnable containers: reduces both solid-waste generation (containers are not wasted), and natural resource depletion (fewer new containers need to be produced)
   Impose a high tax on all but the first car owned by a family: reduces natural resource depletion (fewer cars sold, so fewer cars need to be produced), and solid-waste generation (fewer cars will be discarded).

b. Reduce packaging: w
   Prohibit nonreturnable containers: L
   Impose a high tax on all but the first car owned by a family: P

Step 2: Getting Started

To make the exercise more relevant and interesting to each of you, choose a specific product for the model you are going to build. You should have enough background knowledge of the product whose disposal you wish to model to be able to estimate the demand for the product, its lifetime (how long it is used before it gets thrown away), and the time it takes to disintegrate. Identify the key variables to be included in the model.

We will be modeling the disposal of razor blades. We will assume that razor blades are used for an average time of three weeks, and that about 30% of discarded razor blades are recycled. It takes about 6 months for a razor blade to disintegrate.

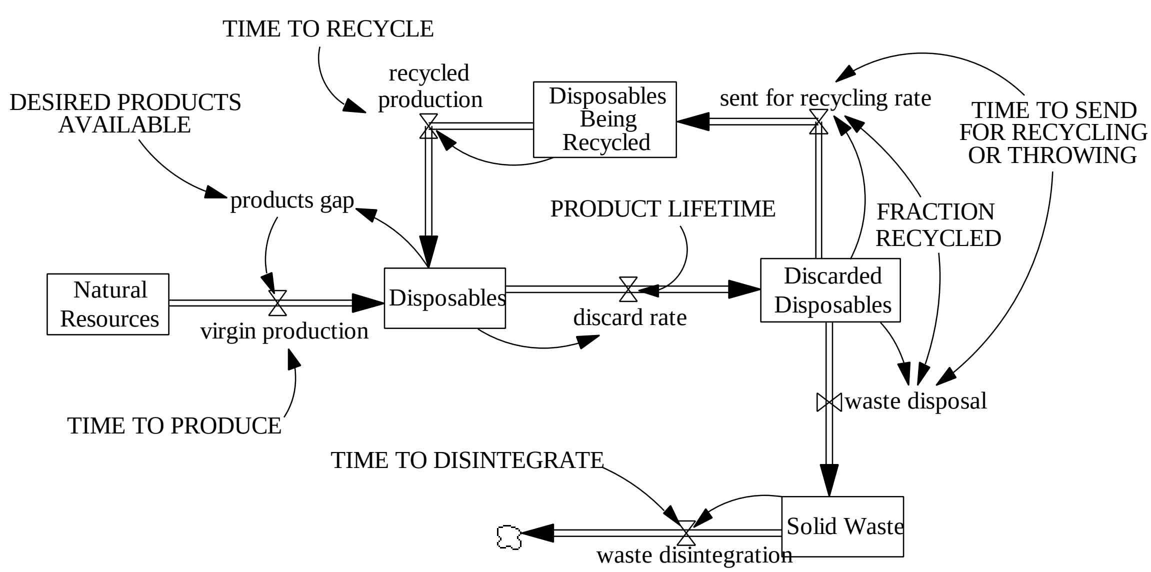
Step 3: Model Structure

A. Start by identifying the stocks in the system. Which variables are stocks, and how are they connected?
Hint: You do not need to build a stock of recycled materials. Instead, recognize that recycling simply returns the product to a state it was previously in.

B. Link the stocks with appropriate flows.

C. Add any relevant auxiliary variables or constants to complete the model structure.
Hint 1: Use the idea of coverage to obtain the desired number of products from the demand for products.
Hint 2: A goal-gap structure determines production.

<u>Model diagram:</u>



Step 4: Equations and Parameter Values

Fill in values and equations for the model components. You will need to estimate many of the parameters and initial values based on your knowledge of the product. In your assignment solutions document, include the model diagram and documented equations.

<u>Model equations:</u>

DESIRED PRODUCTS AVAILABLE = 750000

Units: razor blades

The number of razor blades desired in inventory at any time.

discard rate = Disposables / PRODUCT LIFETIME

Units: razor blades/Month

The number of razor blades discarded every month is determined by the number of razor blades in use at the moment and the average lifetime of a razor blade.

Discarded Disposables = INTEG (discard rate - waste disposal - sent for recycling rate, 0)

Units: razor blades

Used razor blades that have been discarded. Some are sent for recycling, and the rest are thrown away and become solid waste.

Disposables = INTEG (+recycled production + virgin production - discard rate, 0)

Units: razor blades

The number of razor blades in use.

Disposables Being Recycled = INTEG (sent for recycling rate - recycled production, 0)

Units: razor blades

The total number of old razor blades that are being recycled at any time.

FRACTION RECYCLED = 0.3

Units: dmnl

The fraction of used razor blades that are recycled.

Natural Resources = INTEG (-virgin production, 1e+009)

Units: razor blades

The amount of natural resources available expressed in terms of the number of razor blades that can be produced.

PRODUCT LIFETIME = 0.75

Units: Month

On average, a razor blade is replaced after three weeks.

products gap = DESIRED PRODUCTS AVAILABLE-Disposables

Units: razor blades

The number of razor blades that need to be produced to close the gap between the desired and actual number.

recycled production = Disposables Being Recycled/TIME TO RECYCLE
Units: razor blades/Month
The number of old razor blades that are recycled and put back into the market each month.

sent for recycling rate = Discarded Disposables * FRACTION RECYCLED / TIME TO SEND FOR RECYCLING OR THROWING
Units: razor blades/Month
The number of discarded razor blades that are send to the recycling plant.

Solid Waste = INTEG (waste disposal - waste disintegration, 0)
Units: razor blades
Total number of old razor blades that are waiting to be disintegrated.

TIME TO DISINTEGRATE = 6
Units: Month
The time it takes wasted razor blades to disintegrate.

TIME TO PRODUCE = 0.25
Units: Month
The time to produce new razor blades from raw materials.

TIME TO RECYCLE = 0.5
Units: Month
The time it takes to recycle razor blades.

TIME TO SEND FOR RECYCLING OR THROWING = 0.5
Units: Month
The average time it takes to decide whether a particular discarded razor blade is to be thrown or recycled.

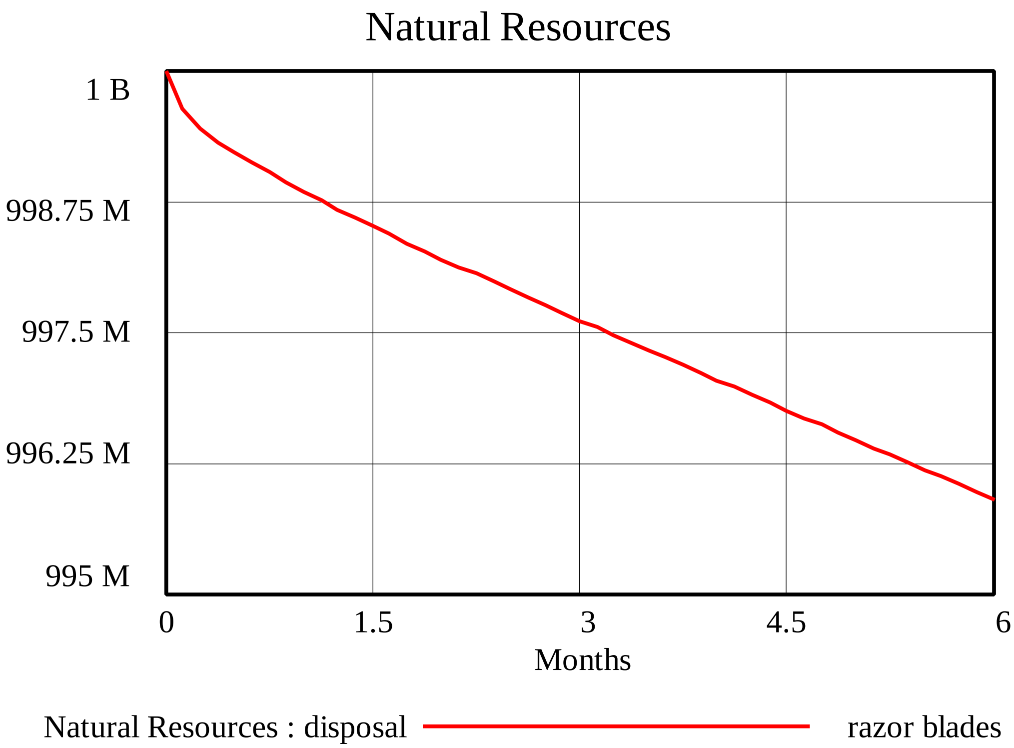
virgin production = products gap / TIME TO PRODUCE
Units: razor blades/Month
The number of razor blades produced every month from new materials.

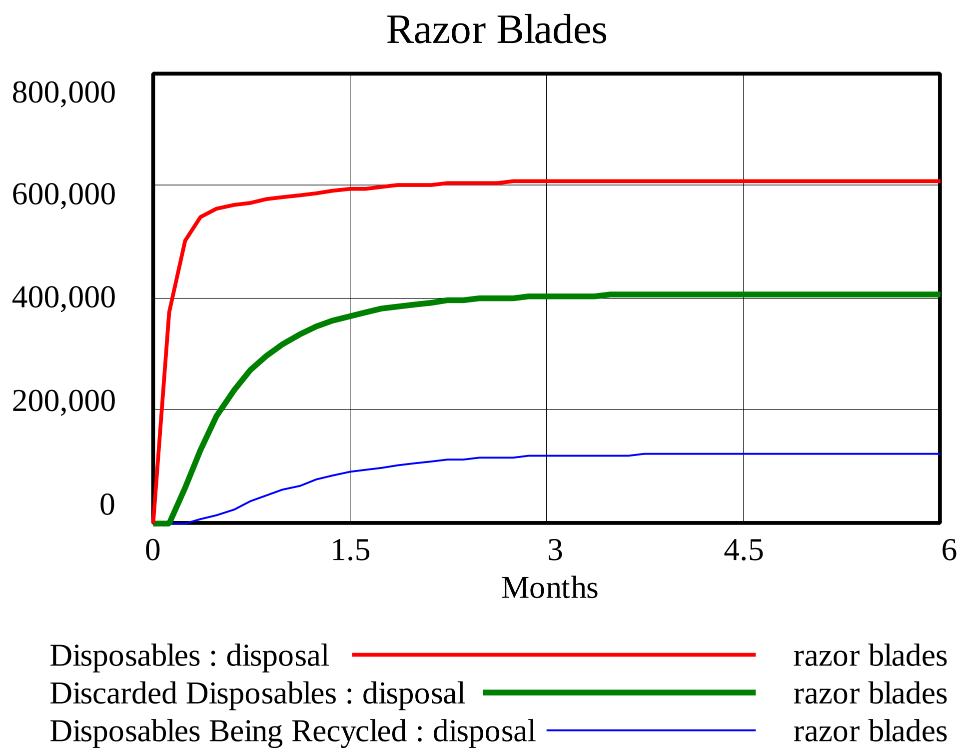
waste disintegration = Solid Waste / TIME TO DISINTEGRATE
Units: razor blades/Month
The rate at which old razor blades are disintegrated.

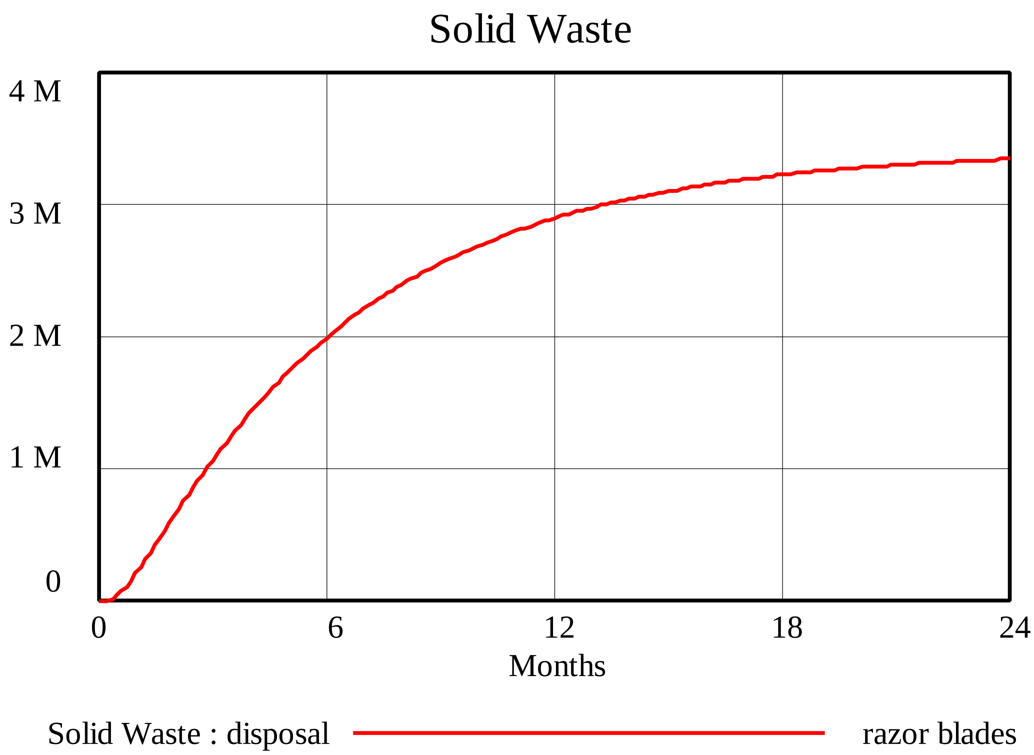
waste disposal = Discarded Disposables * (1 - FRACTION RECYCLED) / TIME TO SEND FOR RECYCLING OR THROWING
Units: razor blades/Month
The razor blades that are not recycled but are sent to the waste disposal plant.

Step 5: Simulating the model

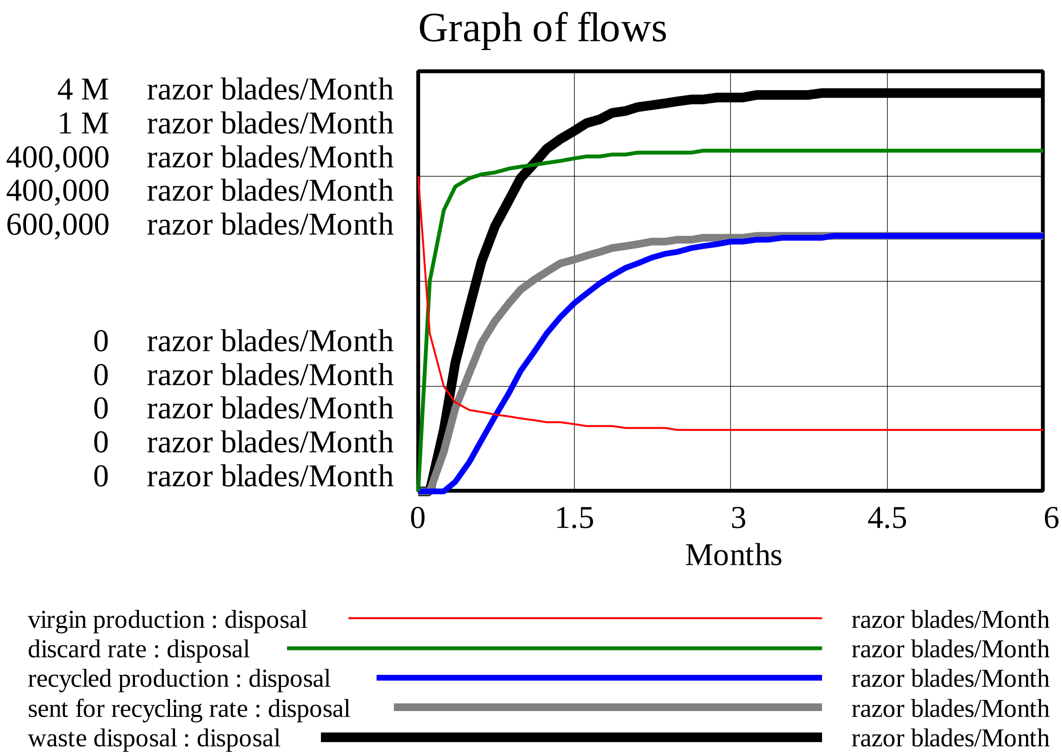
Once the model is complete, make sure that all units are correct. Then simulate the model. In your assignment solutions document, include graphs of the model behavior. Did you observe the behavior that you had predicted in the conceptualization assignment? Why does the model generate the behavior observed?




Razor Blades
800,000
600,000
400,000
200,000
0
0
1.5
3
4.5
6
Months
Disposables : disposal
razor blades
Discarded Disposables : disposal
razor blades
Disposables Being Recycled : disposal
razor blades


Solid Waste
4 M
3 M
2 M
1 M
0
0
6
12
18
24
Months
Solid Waste : disposal
razor blades

Note the different time scale used for this graph. The long time delay involved with the outflow "waste disintegration" from the stock of "Solid Waste" leads to a long time before the stock reaches its equilibrium value.

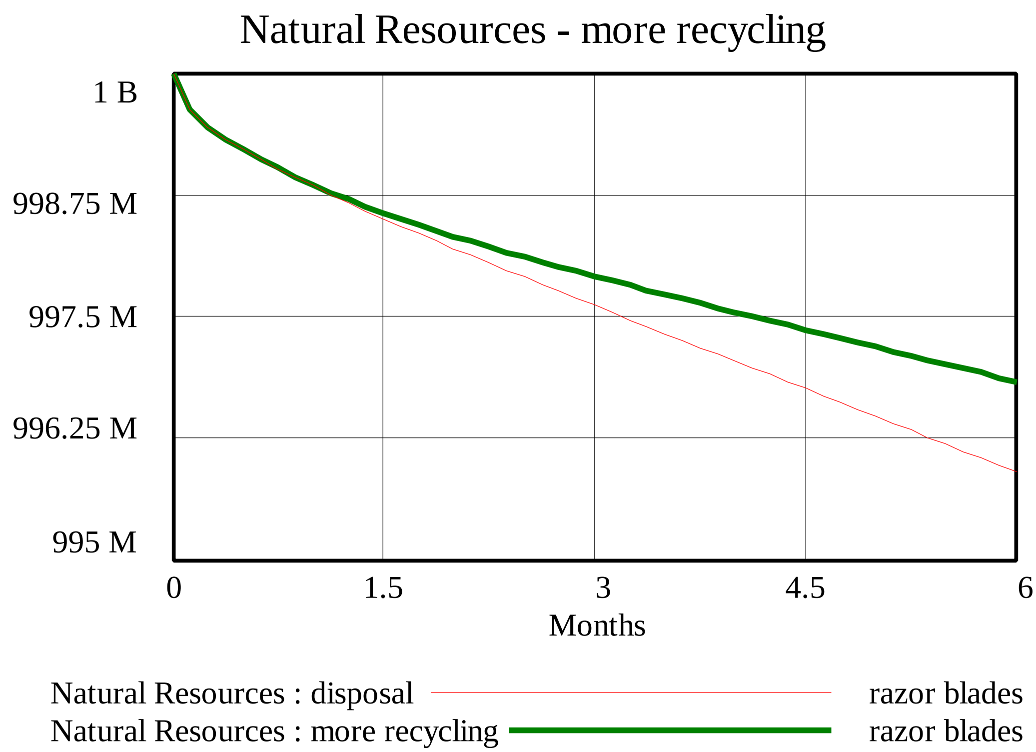


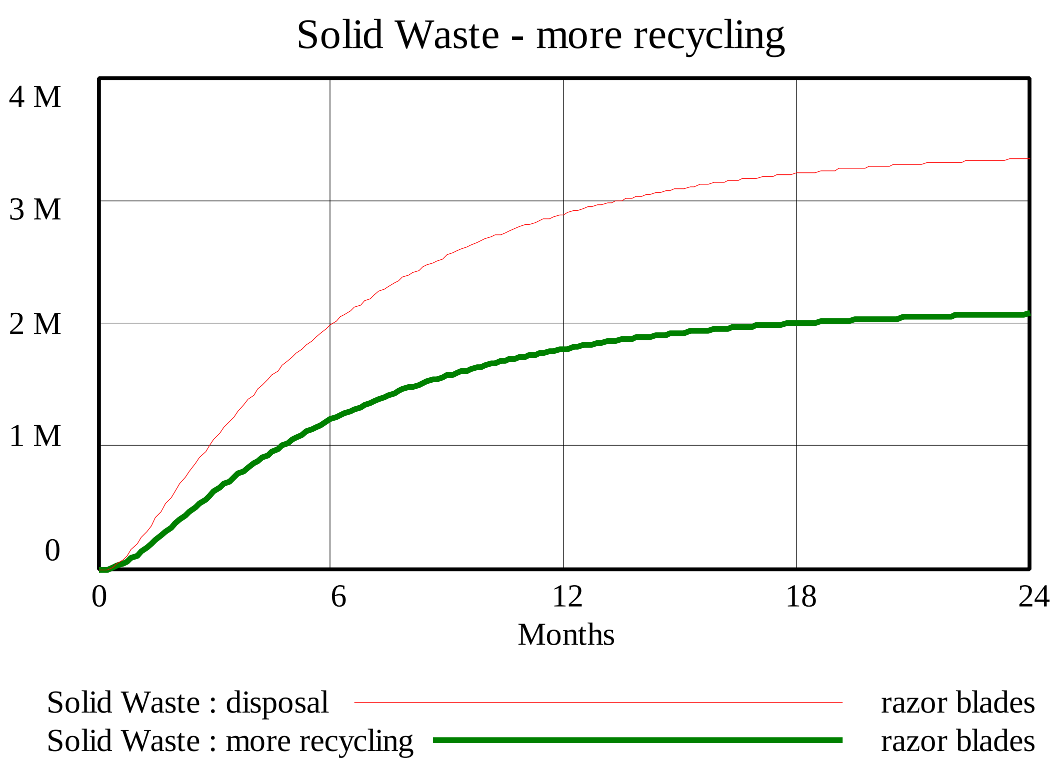
The model generates a behavior similar to that described in the conceptualization. The model reaches its equilibrium. Initially, because there are no "Disposables," the "products gap" is high, so the rate of "virgin production" is high. As "Disposables" increase, however, "products gap" decreases, and so does "virgin production." Hence, the rate of depletion of "Natural Resources" slows down. In addition, increasing "Disposables" increase the "Discarded Disposables," which in turn increases the rate of recycling. Because recycling adds razor blades to the "Disposables" stock, the rate of "virgin production" eventually stabilizes at a low constant value that compensates for the difference between the "waste disposal" rate and "sent for recycling rate."

Step 6: Sensitivity Analysis

A. How would the system behavior change if the government passed legislation offering incentives for people to recycle, raising the percentage of products that is recycled? Simulate the model under these conditions. In your assignment solutions document, include graphs of model behavior, and discuss the effect of more recycling.

When the "FRACTION RECYCLED" is increased to 0.6, the model generates the following behavior:

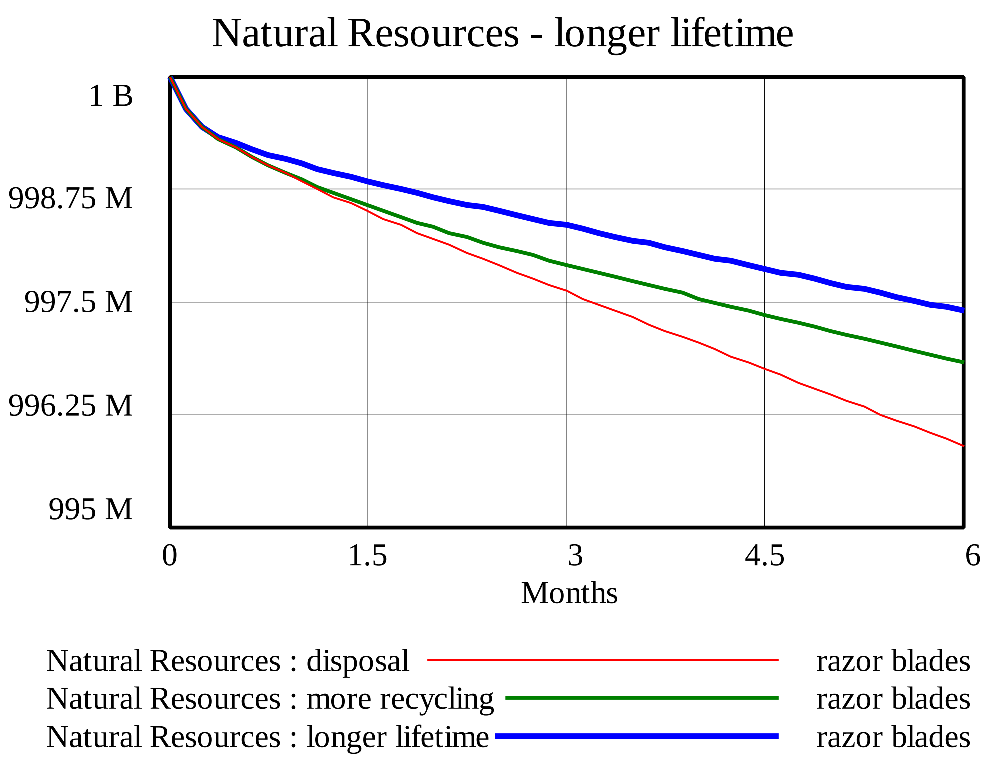

Natural Resources - more recycling
1 B
998.75 M
997.5 M
996.25 M
995 M
0
1.5
3
4.5
6
Months
Natural Resources : disposal
razor blades
Natural Resources : more recycling
razor blades

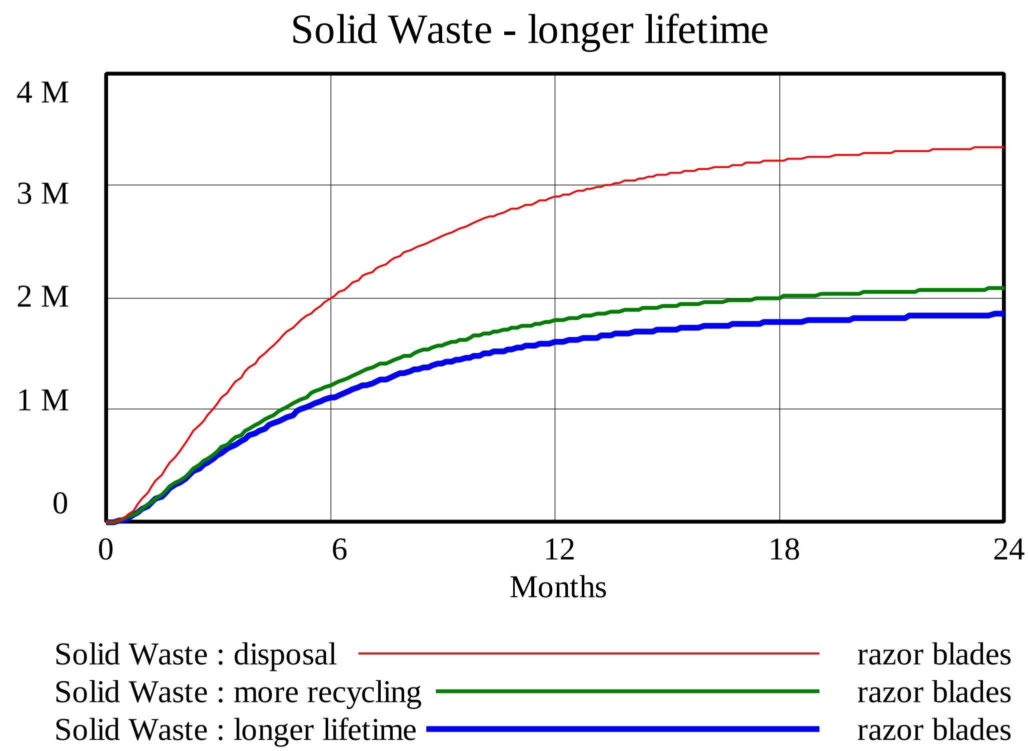

Solid Waste - more recycling
4 M
3 M
2 M
1 M
0
0
6
12
18
24
Months
Solid Waste : disposal
razor blades
Solid Waste : more recycling
razor blades

A higher "FRACTION RECYCLED" increases the "sent for recycling rate," so "Solid Waste" stabilizes at a lower equilibrium value. Also, because most of new demand is met through "recycled production," the rate of "virgin production" is lower, and the stock of "Natural Resources" declines more slowly.

B. Technology improves and the product now lasts much longer. What are the effects of altering the product lifetime? Simulate the model under these conditions. In your assignment solutions document, include graphs of model behavior, and discuss the effect of the technology improvement on the waste disposal system.

When "PRODUCT LIFETIME" is increased to 1.5 months (with "FRACTION RECYCLED" equal to 0.3), the model generates the following behavior:





A longer "PRODUCT LIFETIME" decreases the "discard rate," so "Solid Waste" reaches a lower equilibrium value. Also, because the outflow from "Disposables" is lower, the "virgin production" can also be decreased, and the rate of depletion of "Natural Resources" is lower.

3. Independent Modeling Exercise

"I live on the forth floor of a dormitory that, unfortunately, has no elevators. Before, when I came home at night I always ran out of breath before I reached the top of the stairs. I've decided to exercise more. I found out that after I get into the habit of jogging 8 hours a week—at a leisurely pace, believe me—I am able to climb the 80 stairs that lead up to my room without wheezing.

"When I arrived at school in September, I was only able to make it up 20 stairs before I started to wheeze. At the time I was running 2 hours a week. Upset with my lousy fitness, I decided to start exercising more by running more often. It took me about a week to become motivated to change my exercising habits. My body is even slower to react; after changing my exercising habits, I didn't feel the improvement in climbing stairs for another six weeks. However, I persevered, and now I am finally able to reach my room without coughing and panting. I feel like a new woman."

Leslie, on exercising

A. Use Leslie's testimony to conceptualize and formulate an exercise model to answer the following questions she once had:

- *What will happen to my physical fitness over the next year?*

Hypothesis: "As I increase the number of hours I jog each week, my impending fitness level will improve. I will feel my actual fitness level improve after a time delay in building up muscles of six weeks. Therefore, even when my impending fitness level finally equals my desired fitness level (enough to climb 80 stairs without wheezing), I will continue increasing the number of hours spent exercising each week because my fitness level is less than desired because of the time delay in building up muscles. Eventually, my muscles will build up, and my fitness level will increase and become greater than my desired fitness level. I will then realize that I am exercising more than I need to, and I will start reducing my exercising level. This time, I will end up exercising too little, and I will start wheezing again. So I will increase my exercising level. Thus, my fitness level will oscillate around the desired level, demonstrating damped oscillation. Eventually, I will reach an equilibrium level of exercise each week, enough to enable me to run up 80 stairs."

-- Leslie, on the probable future of exercising and fitness level

Note that had the time delay in building up muscles not been very large relative to the effect of fitness gap on exercising habits, the system would probably show asymptotic growth to the desired fitness level because the delay between attaining an impending fitness level and building up the muscles for the level would be short.

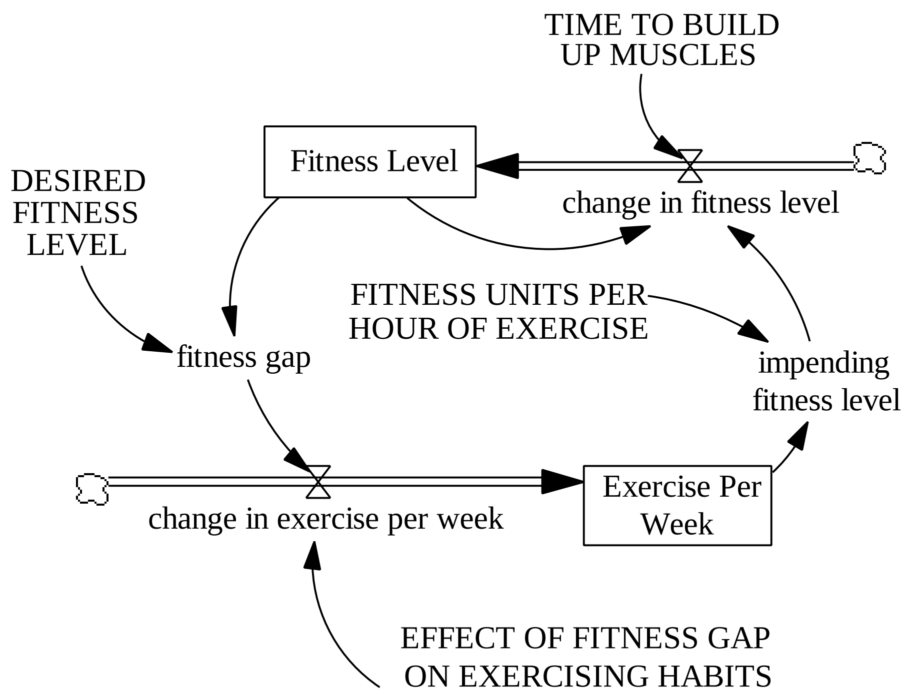
- *What will happen to the number of hours I spend each week exercising?*

Hypothesis: Due to essentially the same dynamics as explained above, the number of hours I spend exercising will also oscillate around 8 hours per week. The amplitude of oscillations will diminish over time and I will reach an equilibrium of exercising 8 hours per week.

In your assignment solutions document, include the model diagram, documented equations, and graphs of the behavior you observe. Explain the dynamic behavior of the model in one or two paragraphs.

Hint: Assume that Leslie's fitness level (measured by the number of steps she can climb without wheezing) improves only by running, not by climbing stairs.

Model diagram:



<u>Model Equations:</u>

change in exercise per week = fitness gap * EFFECT OF FITNESS GAP ON EXERCISING HABITS
Units: hours/(Week * Week)
Leslie compares her fitness to her desired fitness level and accordingly changes the amount of time she spends exercising each week.

change in fitness level = (impending fitness level - Fitness Level) / TIME TO BUILD UP MUSCLES
Units: steps/Week
The change in fitness over time.

DESIRED FITNESS LEVEL = 80
Units: steps
Leslie wishes she could climb 80 steps without wheezing.

EFFECT OF FITNESS GAP ON EXERCISING HABITS = 0.1
Units: (hours/Week)/(steps * Week)
The amount by which Leslie increases her weekly amount of exercising in response to a fitness gap.

Exercise Per Week = INTEG (change in exercise per week, 2)
Units: hours/Week
Number of hours spent exercising each week.

fitness gap = DESIRED FITNESS LEVEL - Fitness Level

Units: steps
The difference between desired and actual fitness levels.

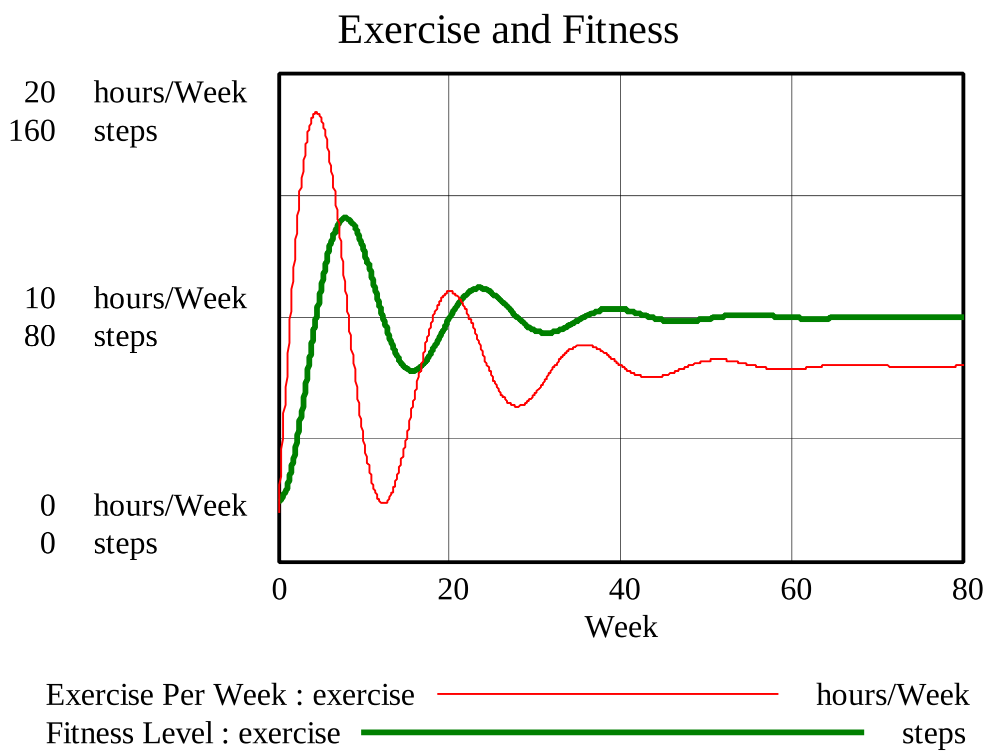
Fitness Level = INTEG (change in fitness level, 20)
Units: steps
The number of steps Leslie can climb without wheezing. This is used as a proxy to measure fitness level.

FITNESS UNITS PER HOUR OF EXERCISE = 10
Units: steps * Week/hour
The number of steps Leslie can climb without wheezing if she exercises one hour per week. Measured in steps per hour per week.

impending fitness level = Exercise Per Week * FITNESS UNITS PER HOUR OF EXERCISE
Units: steps
The fitness level that Leslie can achieve with the current time of weekly exercising.

TIME TO BUILD UP MUSCLES = 6
Units: Week
The time required for Leslie's muscles to build up to the fitness level.

<u>Model behavior:</u>



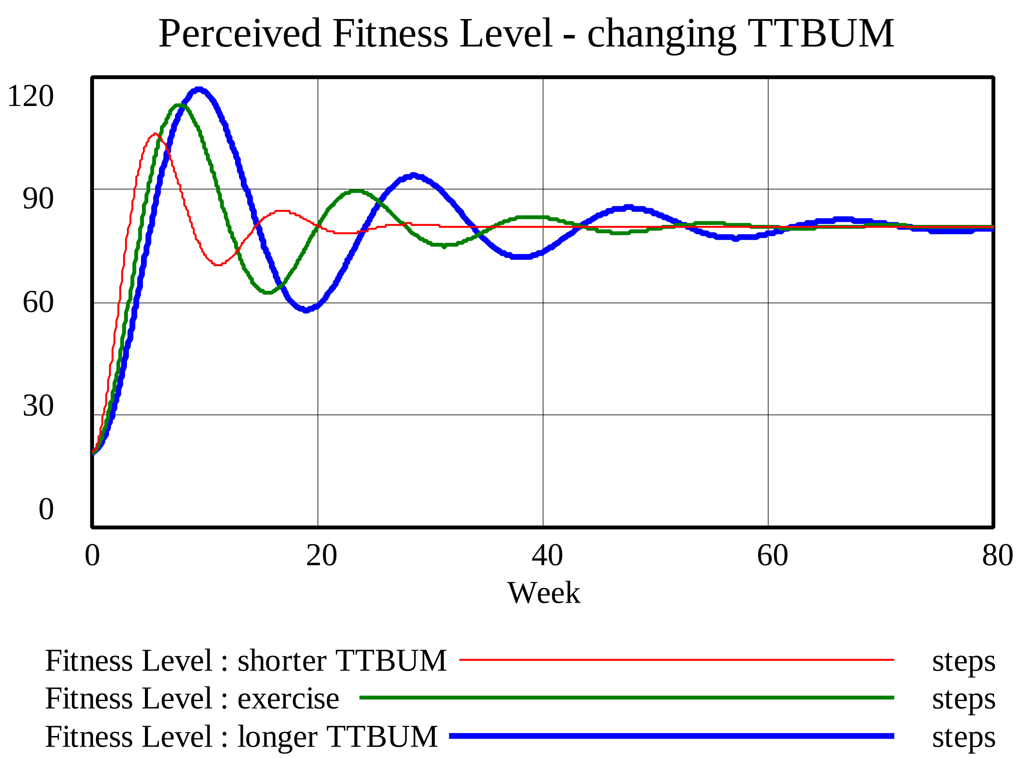
Initially, Leslie’s “Fitness Level” is much below her “DESIRED FITNESS LEVEL,” so she starts exercising more and her amount of “Exercise Per Week” increases. Because it takes her some time to change her exercising habits and because of the time delay in building up muscles to the “impending fitness level,” Leslie ends up exercising more than she needs to, and her “Fitness Level” overshoots its goal. Leslie then stops exercising so much, which eventually causes her “Fitness Level” to drop below the desired level, and she needs to increase her “Exercise Per Week” again. Leslie’s “Fitness Level” then oscillates with decreasing amplitude around the desired level, until equilibrium is established at a “Fitness Level” of 80 steps, and eight hours of “Exercise Per Week.”

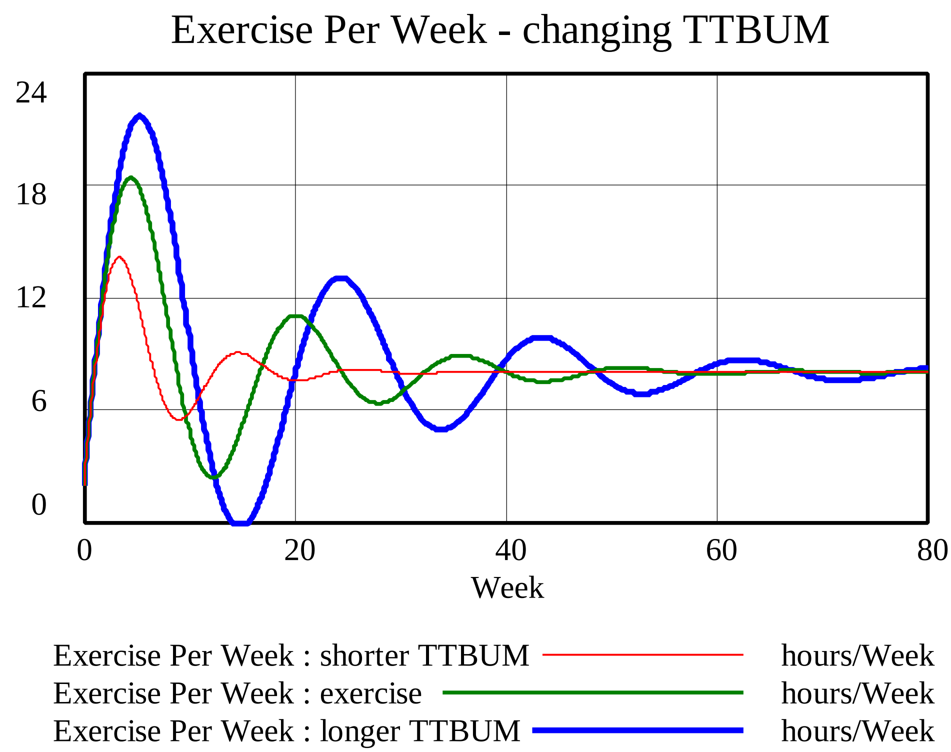
B. How would Leslie’s physical fitness have evolved if she had responded faster to her pathetic condition? How would Leslie’s physical fitness have evolved had she been slower to respond? Why? Justify your answer with graphs of model behavior and an explanation of the dynamics underlying the model.
Hint: Leslie’s response can be measured in two ways. One way is how quickly she changes her exercise habits in response to the realization of her pathetic condition. Another way is how quickly her body responds to the new exercise habits.

Leslie can perceive the effect of exercising faster or slower if the “TIME TO BUILD UP MUSCLES” changed, or she can adjust to the “fitness gap” faster or slower by changing the “EFFECT OF FITNESS GAP ON EXERCISING HABITS.”

As mentioned in the hypothesis earlier, if Leslie's body responds to her exercising faster, that is, if the "TIME TO BUILD UP MUSCLES" decreases, the system should reach equilibrium sooner because the time lag between reaching an impending fitness level and building up the muscles to sustain such fitness level is reduced. The amplitude of oscillations also decreases because Leslie realizes more quickly that she has over or undershot her goal, and starts working towards correcting the error. If Leslie's body responds to her exercising slower, with an increased "TIME TO BUILD UP MUSCLES," the system takes longer to reach equilibrium, and the amplitude of oscillations is greater.

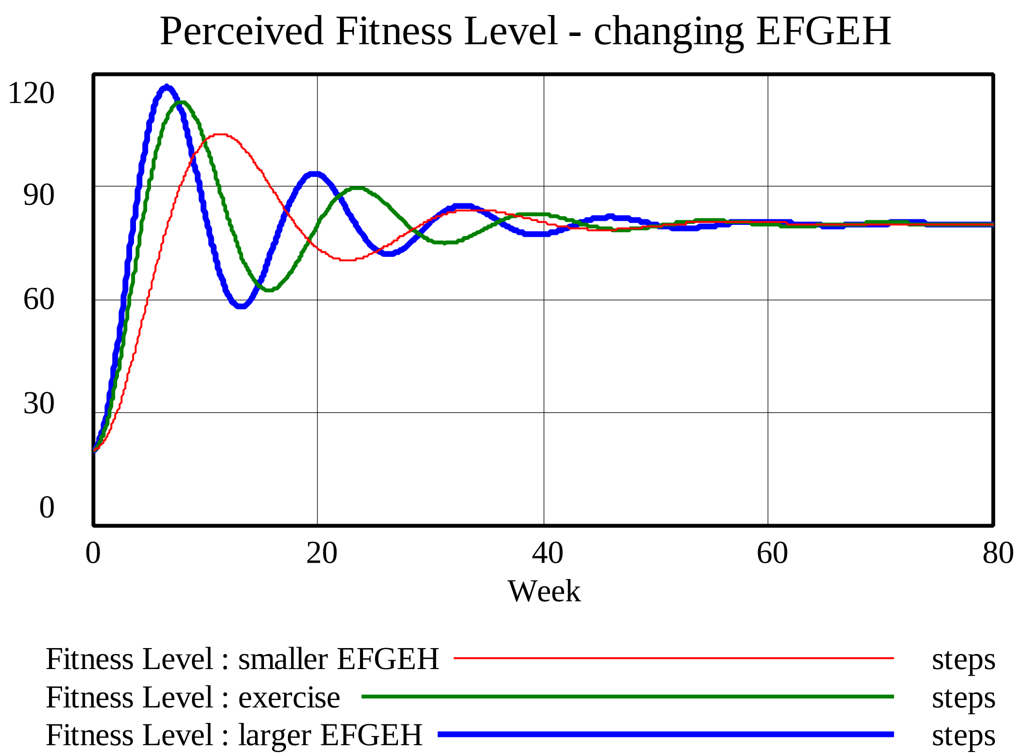
The graphs below show the system behavior with "TIME TO BUILD UP MUSCLES" equal to three weeks ("shorter TTBUM" simulation run) and to nine weeks ("longer TTBUM" simulation run), compared with the base run behavior:

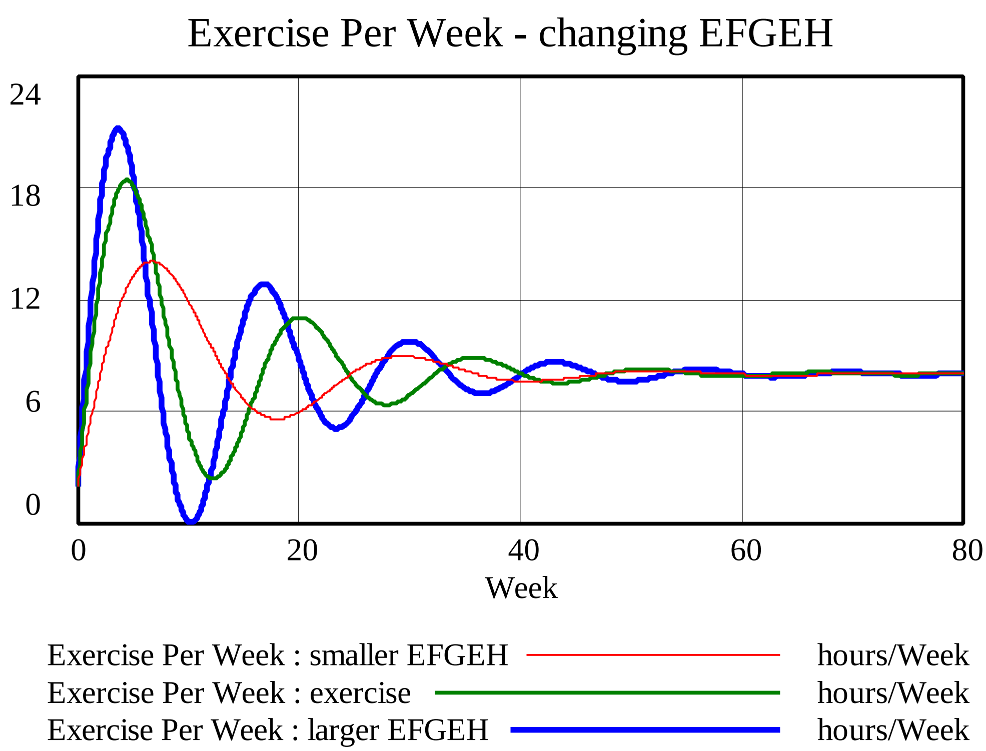




On the other hand, increasing “EFFECT OF FITNESS GAP ON EXERCISING HABITS” means that Leslie reacts very quickly to the “fitness gap,” which makes the system less stable and leads to greater oscillations and a longer time before the system reaches equilibrium. If Leslie reacts more slowly to the “fitness gap,” the time constant increases, and the system becomes more stable.

The following graphs show the behavior of the system with “EFFECT OF FITNESS GAP ON EXERCISING HABITS” equal to 0.05 ((hours/week)/step)/week (“smaller EFGEH” simulation run) and to 0.14 ((hours/week)/step)/week (“larger EFGEH” simulation run), compared with the base run behavior:


Perceived Fitness Level - changing EFGEH
120
90
60
30
0
0
20
40
60
80
Week
Fitness Level : smaller EFGEH
steps
Fitness Level : exercise
steps
Fitness Level : larger EFGEH
steps


Exercise Per Week - changing EFGEH
24
18
12
6
0
0
20
40
60
80
Week
Exercise Per Week : smaller EFGEH
hours/Week
Exercise Per Week : exercise
hours/Week
Exercise Per Week : larger EFGEH
hours/Week

You should also realize that if the time constants are made short enough, the system will simply approach equilibrium asymptotically, without oscillating. This is because Leslie adjusts to the changing fitness level so quickly that she never overshoots the required exercising level. It is the ratio of "TIME TO BUILD UP MUSCLES" to "EFFECT OF FITNESS GAP ON EXERCISING HABITS" that determines the behavior. As this ratio becomes smaller, the amplitudes become smaller, and after a certain value, the behavior is simply asymptotic.

4. Independent Modeling Exercise

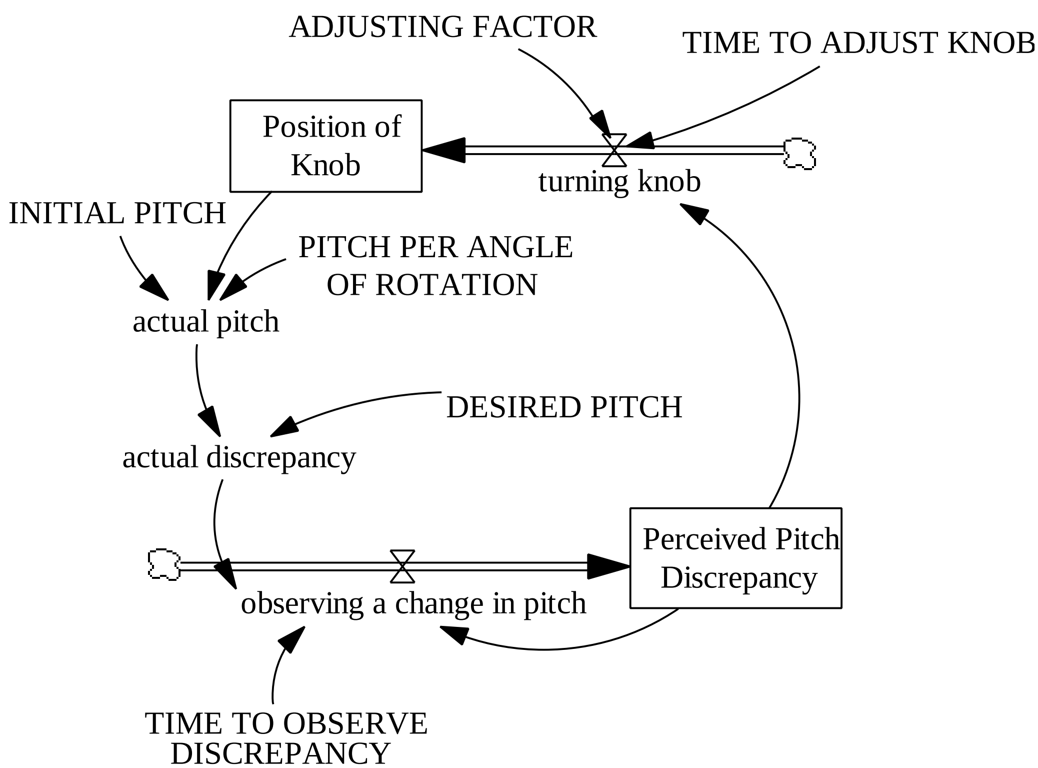
"One of my friends bought me a guitar for Christmas. I have a difficult time tuning the guitar before I sit down to play. I pluck a string, observe the pitch, and then rotate the appropriate nut, thereby changing the tension in the string. Then I listen to the difference between the new pitch and the pitch I want and continue to adjust the nut accordingly. It takes me a second to adjust the knob, but ten seconds to observe the discrepancy in pitch, because I am actually sampling the pitch at discrete intervals.

"Well, the pitch of the note is tabulated in terms of the frequency of the sound wave created by the vibration of the guitar string after I pluck it. A middle G, for example, is 196 hertz (Gs played at higher octaves have higher frequencies). A middle D is approximately 146.8 hertz."

Harriet, on guitars

A. Use the above description to conceptualize and formulate a guitar tuning model that demonstrates how the pitch of Harriet's guitar changes when she attempts to tune a string from a D to a G. In your assignment solutions document, include the model diagram, documented equations, and graphs of the behavior that you observe. Explain the dynamic behavior of the model in one or two paragraphs.

Model diagram:



<u>Model equations:</u>

actual discrepancy = DESIRED PITCH - actual pitch

Units: hertz

The real difference between desired and actual pitch of the guitar.

actual pitch = INITIAL PITCH + Position of Knob * PITCH PER ANGLE OF ROTATION

Units: hertz

The actual pitch of the guitar.

ADJUSTING FACTOR = 0.5

Units: degrees / hertz

The number of degrees by which the knob must be turned to change the frequency by one hertz.

DESIRED PITCH = 196

Units: hertz

This is the desired pitch (middle G).

INITIAL PITCH = 146.8

Units: hertz

This is the initial pitch of the guitar (middle D)

observing a change in pitch = (actual discrepancy - Perceived Pitch Discrepancy) / TIME TO OBSERVE DISCREPANCY
Units: hertz / second
The change in the observed pitch.

Perceived Pitch Discrepancy = INTEG (observing a change in pitch, DESIRED PITCH - INITIAL PITCH)
Units: hertz
The difference in desired and actual pitch, as observed by the tuner. The tuner turns the knob in order to make the value of this stock zero.

PITCH PER ANGLE OF ROTATION = 0.5
Units: hertz / degrees
The number of hertz by which the pitch changes if the knob is turned by one degree.

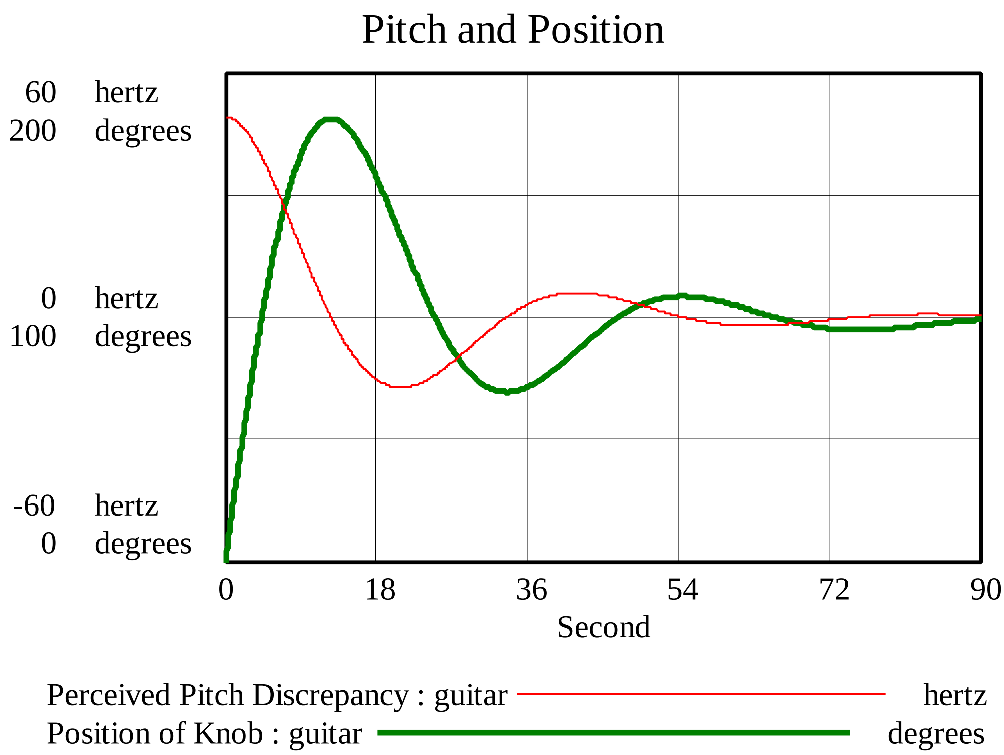
Position of Knob = INTEG (turning knob, 0)
Units: degrees
The current position of the knob.

TIME TO ADJUST KNOB = 1
Units: second
Time taken to adjust the knob when a discrepancy in pitch is noticed.

TIME TO OBSERVE DISCREPANCY = 10
Units: second
This time constant reflects on the sensitivity of the tuner's ear and shows how long it takes the tuner to notice changes in pitch. For a more experienced musician, this number will be small.

turning knob = Perceived Pitch Discrepancy * ADJUSTING FACTOR / TIME TO ADJUST KNOB
Units: degrees / second
The number of degrees by which the knob is turned in each time period to make the perceived pitch equal to the desired pitch.

Model behavior:

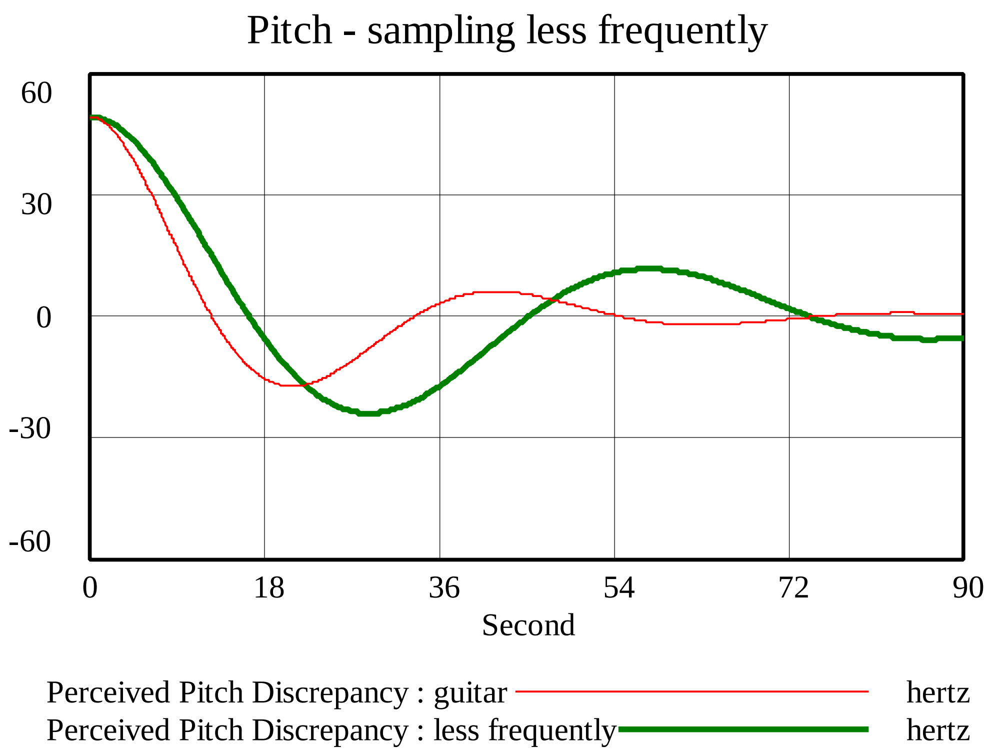


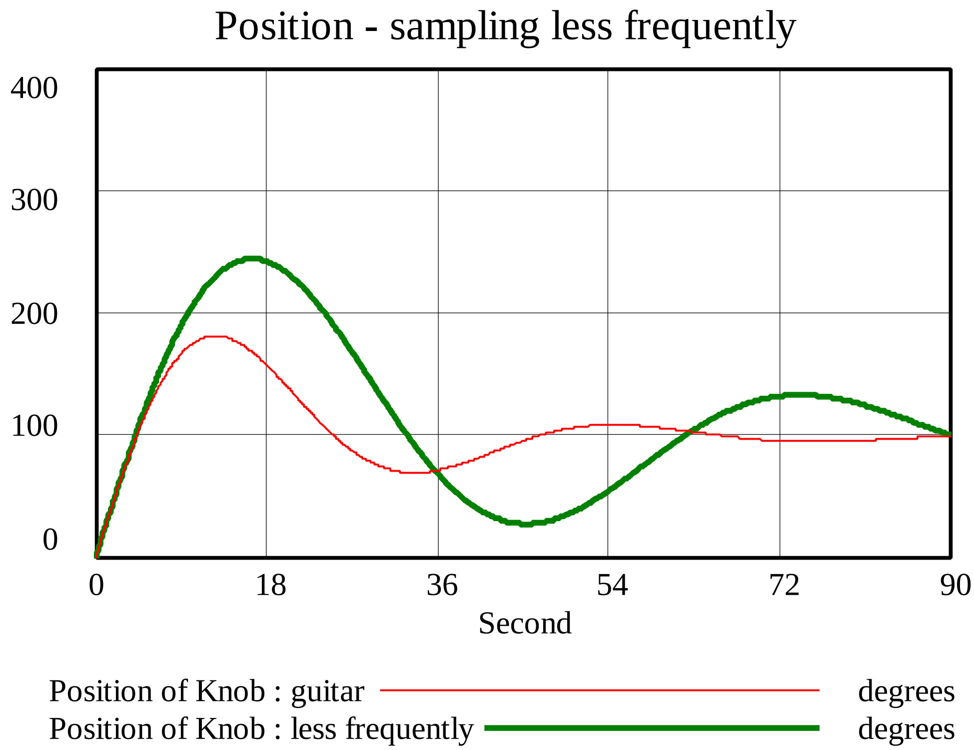
Initially, the "Perceived Pitch Discrepancy" is positive meaning that the "DESIRED PITCH" is greater than the "INITIAL PITCH," so Harriet changes the knob to adjust for this discrepancy. As she turns the knob, she notices the changing frequency. Her perception of the current frequency is delayed because of the large "TIME TO OBSERVE DISCREPANCY," so she turns the knob more than required. After a time lag, she realizes that she has turned the knob too much, and starts to turn it in the opposite direction, again overshooting in the opposite direction because of the time delay in perceiving the true frequency. This produces the oscillating behavior generated by the model.

B. How would the behavior of the model change if Harriet were to sample the pitch less frequently? How would the behavior of the model change if Harriet developed a better ear, shortening the time it takes her to observe the discrepancy in pitch? Why? Justify your answer with graphs of model behavior and an explanation of the dynamics underlying the model.

Sampling the pitch less frequently should increase the amplitude of the oscillations because Harriet will move further away from the desired pitch before she realizes that she has overshot the desired knob position. Similarly, when she starts turning the knob in the opposite direction to correct the overshoot, she will move further from the desired position than she would if she were sampling more frequently. Because of the increased oscillations, the system also takes a longer time to reach equilibrium. Thus, damped oscillations of increased amplitude should be expected. In the system, the time constant

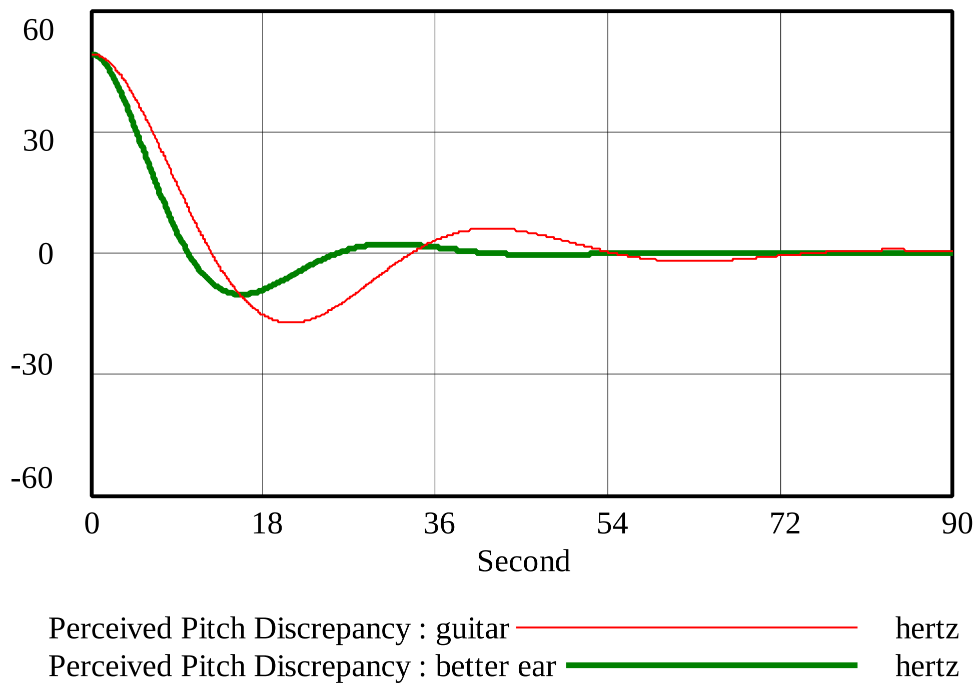
"TIME TO OBSERVE DISCREPANCY" shows how frequently Harriet samples the frequency. Thus, increasing the value of this constant indicates that Harriet samples the pitch less frequently. The graphs below show the behavior of the model with "TIME TO OBSERVE DISCREPANCY" equal to 20 seconds ("less frequently" simulation run), compared with the base run behavior:





Developing a better ear is opposite of the situation described above. With a better ear, Harriet will be able to recognize a discrepancy between the current and desired pitch very quickly, and will start to correct the discrepancy before it is very large. Thus, it can be expected that the amplitude of the oscillations will be smaller, and Harriet will be able to tune the guitar more quickly. The graphs below show the behavior of the model with "TIME TO OBSERVE DISCREPANCY" equal to 5 seconds ("better ear" simulation run), compared with the base run behavior:

Pitch - better ear



Position - better ear