

# **Graphical Integration Exercises**

## **Part Five: Qualitative Graphical Integration**

Prepared for the  
MIT System Dynamics in Education Project  
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September 19, 1997

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## 1. ABSTRACT

Stocks accumulate the difference between the inflow and outflow from the stock over time. The value of a stock at any time can be calculated by integrating (adding continuously) the flows affecting the stock. Graphical integration is the process of estimating the behavior of a stock by looking at graphs of the flows into and out of the stock.

This paper studies various positive and negative flows and the effect of the flows on stocks: positive flows (inflows to a stock) increase the value of the stock, while negative flows (outflows from a stock) decrease the value of the stock. The slope of the flow graph determines how rapidly the value of the stock changes. Thus, flows that are increasing in magnitude and flows that are decreasing in magnitude affect stocks in different ways. This paper helps the reader to develop an intuitive understanding of how stocks are affected by flows. The focus of this paper is to estimate the general behaviors of stocks and not to calculate the exact value of a stock at a particular point in time.

## 2. INTRODUCTION

Graphical integration is the process of estimating the behavior or value of a stock by studying the flows into and out of the stock. Previous Road Maps chapters introduced graphical integration of linear flows in a quantitative manner. After completing Graphical Integration Exercises Part 1 through 3,<sup>1</sup> the reader should be able to perform simple arithmetic operations to calculate the value of the stock at any time from a graph of the linear flow behavior. If more than one flow affect a stock, the behavior of the stock depends on the sum of the effects of each of the flows: the effect of the net flow. Precise quantitative measurements of flows, however, may not be readily available. A system dynamicist should be able to simply look at the shape of a flow graph and qualitatively determine the behavior of the stock. Often, the behavior pattern of a stock is more important in determining the behavior of the system than the exact value of the stock. Readers should also learn to estimate the flows from looking at the stock diagrams. The process of reverse graphical integration, introduced in Graphical Integration Exercises Part Four: Reverse Graphical Integration,<sup>2</sup> is helpful in understanding the behavior of systems.

This paper generalizes some of the behaviors of linear flows seen in the first three papers in the Graphical Integration series. The reader will better understand the fundamental concepts of graphical integration and will be able to estimate the behavior of stocks easily and without precise calculations. This paper includes a discussion of fundamental properties of linear flows, followed by a set of exercises with detailed solutions.

## 3. GRAPHICAL INTEGRATION OF LINEAR FLOWS

The linear flows discussed so far in Road Maps have been functions that can be graphed as straight lines. Among the topics covered in this paper are constant, linearly

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<sup>1</sup> Alice Oh, 1995. *Graphical Integration Exercises Part One: Exogenous Rates* (D-4547), December 1, 19 pp;

Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571), May 29, 25 pp;

Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part Three: Combining Flows* (D-4596), March 14, 32 pp.

All papers written for the System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

<sup>2</sup> Laughton Stanley, 1996. *Graphical Integration Exercises Part Four: Reverse Graphical Integration* (D-4603), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 20, 25 pp.

increasing, and linearly decreasing flows. A general set of steps for performing graphical integration qualitatively is presented.

The reader should be able to combine flows to find the net flow. The net flow is simply the arithmetic sum of all the individual flows over time. The reader should understand certain mathematical terms such as slope, magnitude, and value. The concept of the slope of a line and methods to calculate the slope are discussed in Graphical Integration Exercises Part 2: Ramp Functions.<sup>3</sup> The slope of a line measures how rapidly the y-axis value of the line changes with respect to the x-axis value. All graphs in this paper have “time” on the x-axis; hence, the slope measures how rapidly the y-axis value of the line changes over time. The magnitude of a number is also called the absolute value of the number. The magnitude measures the numeric part of a number, ignoring the sign of the number. For example, the magnitude of “-6” is “6,” the magnitude of “-17” is “17,” and the magnitude of “+3” is “3.” The value of a number essentially shows where the number appears on the number line. The value is defined by the magnitude and the sign of the number. For example, the value of “-10” is “-10,” and the value of “+14” is “+14.” Thus, for a positive function, the value and magnitude are the same, so an increase in the magnitude also means an increase in value: the function moves away from zero. For negative functions, however, increasing value means that the function is moving closer to zero, and thus has decreasing magnitude. Thus, a negative function with increasing value has decreasing magnitude. A negative function with decreasing value grows more and more negative with time (moves away from zero) and thus has increasing magnitude.

### 3.1 Types of Linear Flows

Flows can be positive, zero, or negative. A positive flow is an inflow to a stock, while a negative flow is an outflow from the stock. A flow with a value of zero means that nothing flows into or out of the stock. A biflow may be positive and negative over different time periods, but at any given time, a biflow has either a positive or negative value.

This paper studies linear flows that can be graphed as straight lines. Such flows may be constant, increasing, or decreasing in magnitude. A constant flow means that in each time period, a fixed amount is added to or removed from the stock. Linearly increasing or decreasing flows are called ramp functions.<sup>4</sup> The value of the flow changes

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<sup>3</sup> Kevin Agatstein and Lucia Breierova, D-4571.

<sup>4</sup> System dynamics modeling software applications use “RAMP” in the list of built-in functions to plot linearly increasing and decreasing functions.

by a fixed amount during each time interval. Thus, the amount by which the stock changes in each time period is not constant over time because the value of the flow is changing.

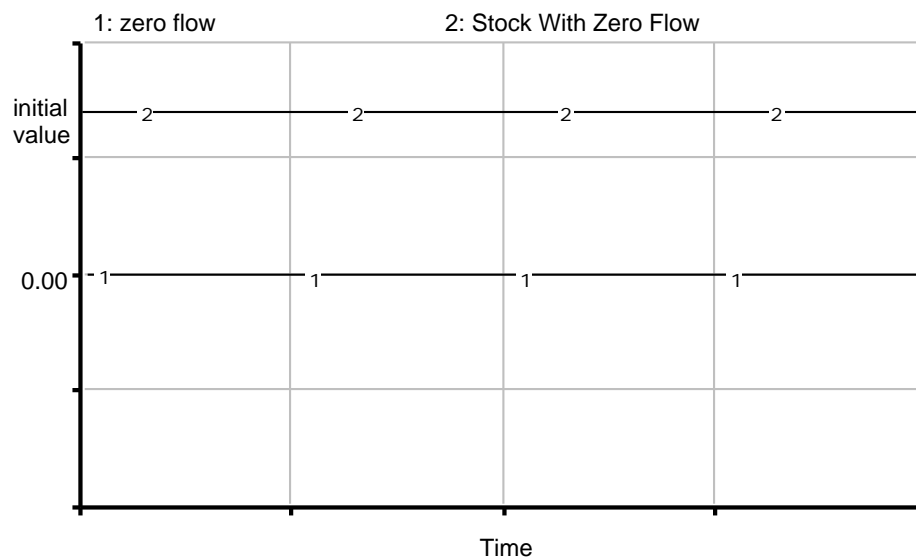
Therefore, based on whether the flow is positive, zero, or negative, and whether the magnitude of the flow is constant, increasing, or decreasing, linear flows studied in this paper can be classified into seven types:

1. zero
2. positive and constant
3. positive and increasing in magnitude
4. positive and decreasing in magnitude
5. negative and constant
6. negative and increasing in magnitude
7. negative and decreasing in magnitude

The paper studies the effect of each type of flow on the stock.

### 3.2 Zero Flow

A zero flow means that the sum of all the flows into the stock equals the sum of all the flows out of the stock. Thus, a zero net flow causes no change in the value of the stock, and the stock remains at the initial value, irrespective of whether the initial value is positive, zero, or negative. Figure 1 shows a zero flow and the stock the flow affects. The stock has an initial value equal to “initial value.”



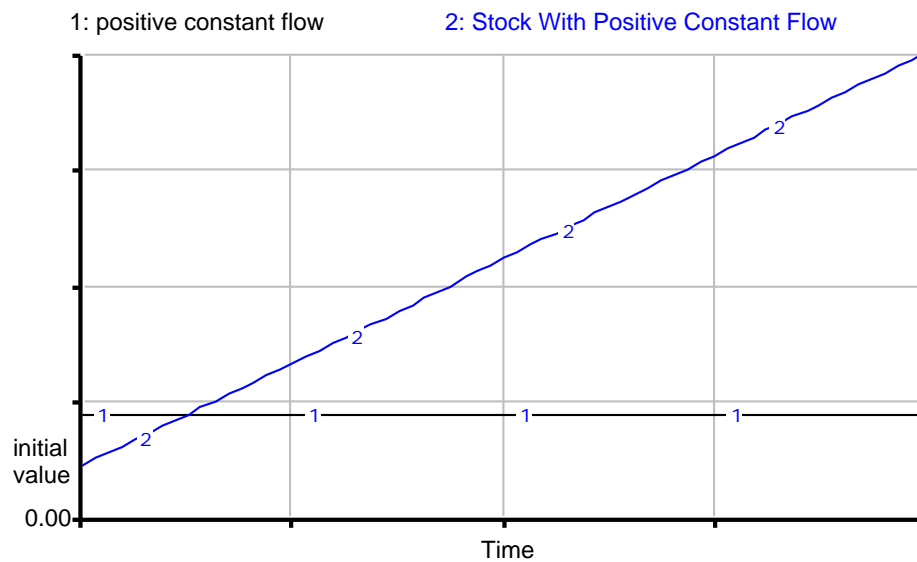
**Figure 1: Net zero flow and stock affected by a net zero flow. The stock remains at the initial value equal to “initial value.”**

### 3.3 Positive Flows

Positive flows correspond to flows into a stock. A positive flow increases the value of the stock. A positive flow may be constant, increasing, or decreasing in magnitude.

#### 3.3.1 Constant Positive Flows

The graph in Figure 2 shows a constant positive flow and the stock affected by the flow. The initial value of the stock is “initial value.” The value of the stock increases by a constant amount each time period because the slope of the stock graph, which is equal to the value of the flow, is constant. The reader will be unable to calculate the slope of the stock graph because the value of the stock at any time is not given.



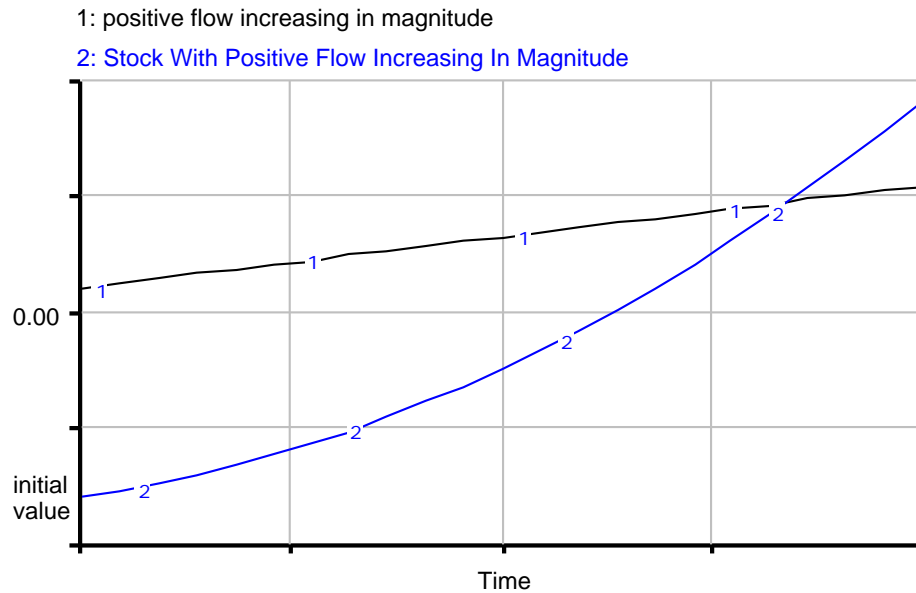
**Figure 2: Positive constant flow and the stock affected by the flow**



### 3.3.2 Positive Flows Increasing in Magnitude

A positive flow increasing in magnitude adds a larger amount to the stock in each successive time period. Thus, the value of the stock grows at an increasing rate over time.

Figure 3 shows a positive linearly increasing flow and the stock affected by the flow. The stock has a negative initial value equal to “initial value.”



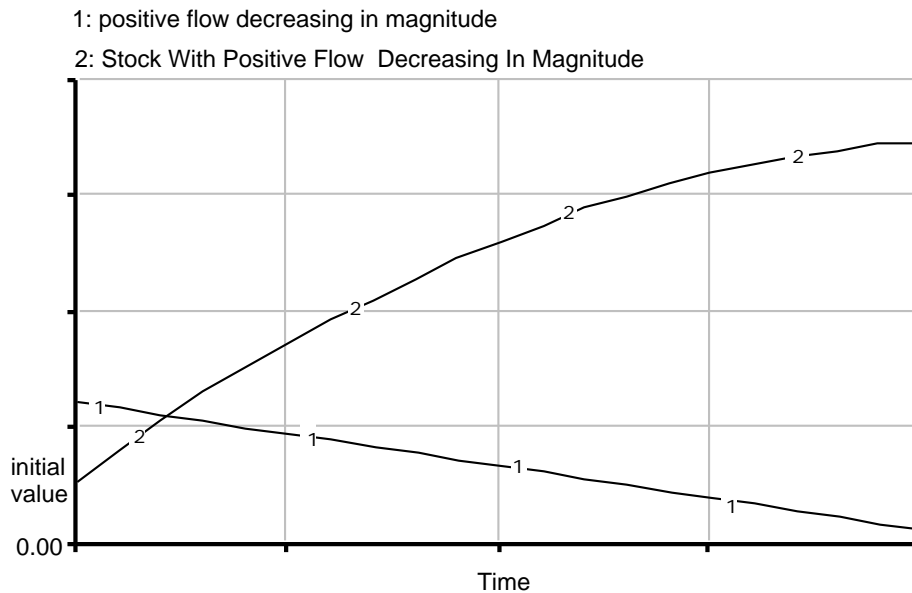
**Figure 3: Positive flow increasing in magnitude and the stock affected by the flow**

As seen in Figure 3, the slope of the stock graph increases as time passes. The slope of the stock graph at any time equals the value of the flow at that time. Because the magnitude of the flow is increasing with time, the stock grows more and more rapidly as time passes. The initial value of the stock has no effect on the behavior of the stock. A stock with any initial value will grow at a faster and faster rate with time if the net flow into the stock is positive and increasing in magnitude.

### 3.3.3 Positive Flows Decreasing in Magnitude

If the magnitude of a positive flow is decreasing over time, the stock will increase by a smaller and smaller amount as time passes. As the value of the flow approaches zero, the stock approaches a certain value. When the flow reaches and stays zero, the stock stops changing and attains an equilibrium value.

Figure 4 shows a linearly decreasing positive flow and the stock affected by the flow.



**Figure 4: Positive flow decreasing in magnitude and the stock affected by the flow**

Figure 4 shows that even if the positive flow is decreasing in magnitude, the value of the stock increases because as long as the flow is positive, the contents of the flow are being added to the stock. As time passes, the stock increases by a smaller and smaller amount, approaching an equilibrium value as the flow approaches zero.

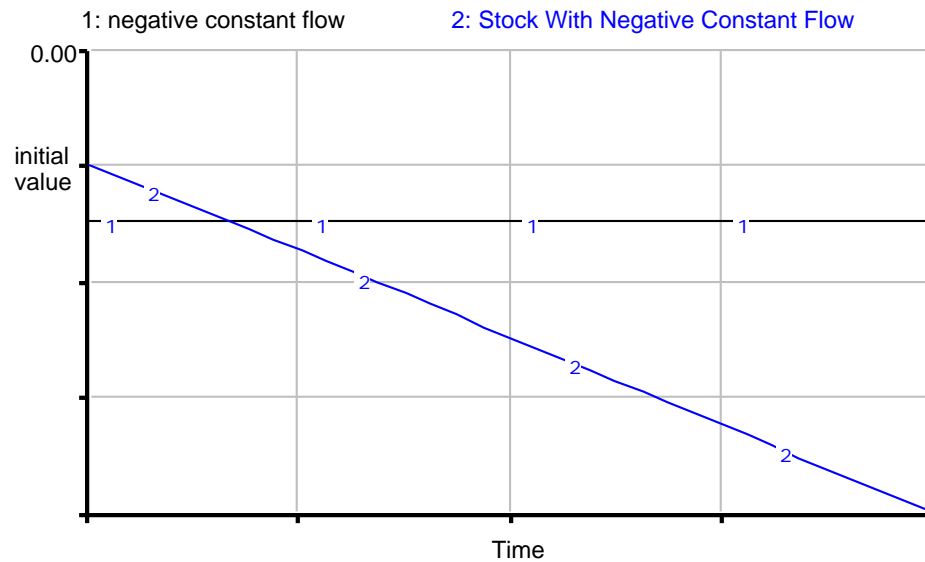
Thus, a positive flow always increases the value of the stock irrespective of the initial value of the stock. The stock may have an initial value greater than, equal to, or less than zero, but the value of a stock with a positive flow increases with time because the contents of the flow are continuously being added to the stock. The rate at which the value of the stock increases depends on the magnitude of the positive flow. The behavior of the stock over time depends on whether the positive flow is constant, increasing, or decreasing over time.

### 3.4 Negative Flows

Negative flows correspond to outflows from the stock. A negative flow decreases the value of the stock. A negative flow may be constant, increasing, or decreasing in magnitude.

#### 3.4.1 Constant Negative Flows

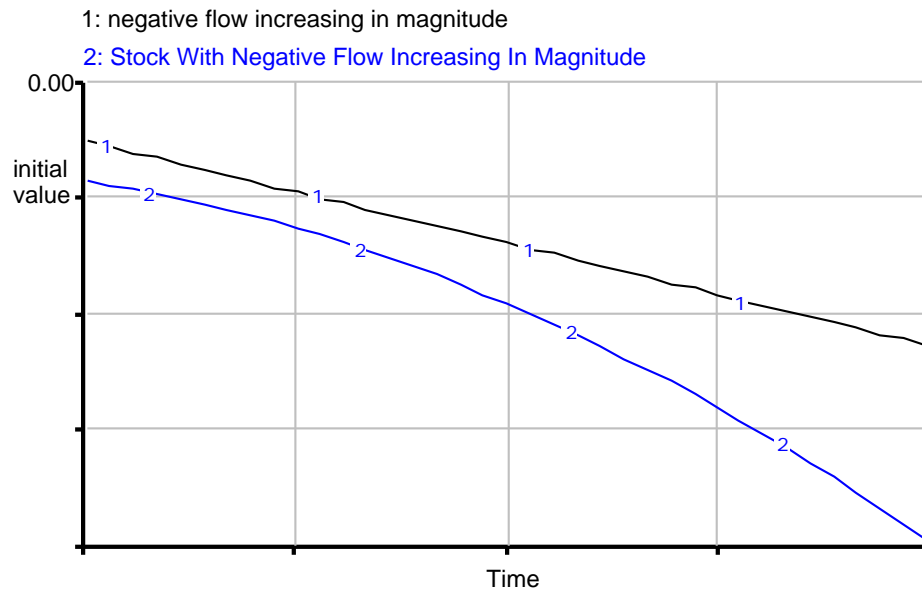
The graph in Figure 5 shows a stock with a constant negative flow. The value of the stock decreases from the initial value, irrespective of whether the initial value of the stock is positive, zero, or negative. Figure 5 shows the stock with an initial value equal to “initial value.” The amount by which the stock decreases in each time period is constant because the outflow (negative flow) from the stock is constant.



**Figure 5: Negative constant flow and the stock affected by the flow**

### 3.4.2 Negative Flows Increasing in Magnitude

A negative flow increasing in magnitude will remove the contents of the stock from the stock at an increasing rate as time passes because the magnitude of the flow increases with time. Figure 6 shows a negative flow increasing in magnitude and the stock behavior produced by the flow.

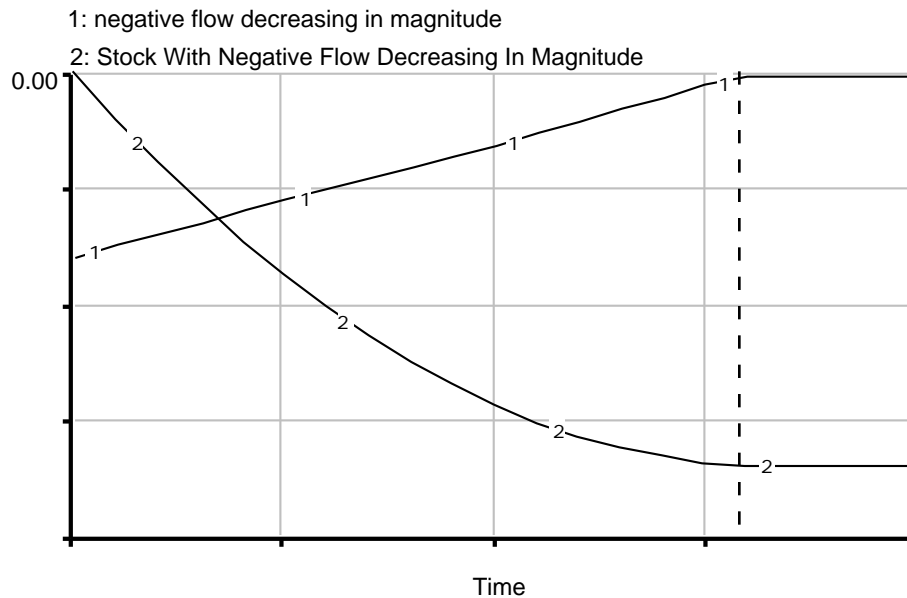


**Figure 6: Negative flow increasing in magnitude and the stock affected by the flow**

In Figure 6, the value of the stock decreases more and more rapidly over time because the flow becomes more and more negative. The magnitude of the flow, however, increases over time; therefore, the flow is a negative flow increasing in magnitude. Because the flow is negative, the value of the stock decreases over time.

### 3.4.3 Negative Flows Decreasing in Magnitude

A negative flow linearly decreasing in magnitude reduces the value of the stock by a smaller and smaller amount in each successive time period. Figure 7 shows a negative flow decreasing in magnitude and the stock affected by the flow. The stock in Figure 7 has an initial value of zero, but the behavior of the stock would be the same as in Figure 7 even if the initial value was not equal to zero.



**Figure 7: Negative flow decreasing in magnitude and the stock affected by the flow**

Figure 4 and Figure 7 both show flows that are decreasing in magnitude. In Figure 4, the flow is positive, and the stock approaches an equilibrium value. The flow in Figure 4 does not reach zero, and so the stock does not reach an equilibrium value, at least in the time period shown in the graph. In Figure 7, however, the negative flow decreasing in magnitude reaches zero after a certain time period and then remains zero. The stock reaches an equilibrium value when the flow reaches zero, and stays at the equilibrium value because the flow remains zero (the value of a stock cannot change if the net flow into the stock is zero).

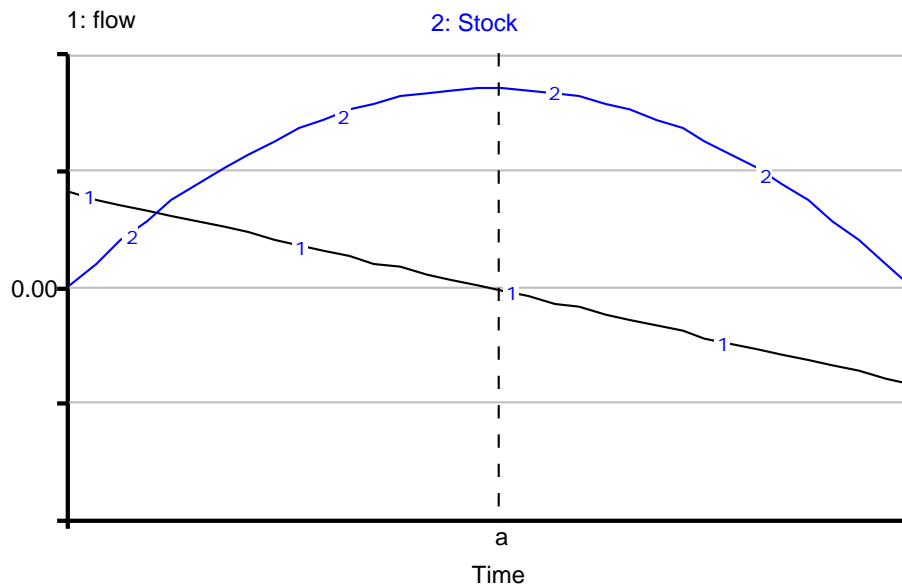
A negative flow, being an outflow, always decreases the value of a stock over time. The initial value of the stock does not affect the behavior of the stock. Therefore, a stock may have a positive, zero, or negative initial value, but if the net flow out of the stock is negative, the value of the stock will decrease over time.

## 4. EXAMPLES

### 4.1 Example 1: Positive and Negative Flows: Crossing the Time Axis<sup>5</sup>

Often, a flow is initially positive but decreases linearly and becomes negative, or an initially negative flow increases linearly and becomes positive. Such a flow is continuously decreasing or increasing in value, but not in magnitude.

An initially positive flow that decreases linearly and becomes negative has a negative slope and a constantly decreasing value. The magnitude of the flow, however, first decreases as the flow approaches zero. When the flow becomes negative, the magnitude of the flow starts increasing. Thus, the flow is a combination of a positive flow with decreasing magnitude and a negative flow with increasing magnitude. Therefore, the behavior of the stock changes when the flow crosses the time axis. Figure 8 shows an initially positive flow with decreasing magnitude that crosses the time axis and becomes negative with increasing magnitude, and the stock behavior produced by the flow.



**Figure 8: System with a linear flow that crosses the time axis**

In Figure 8, the flow has a positive value at time zero. From time 0 to 'a,' the flow is positive and decreasing in magnitude. The stock increases in value because the flow is

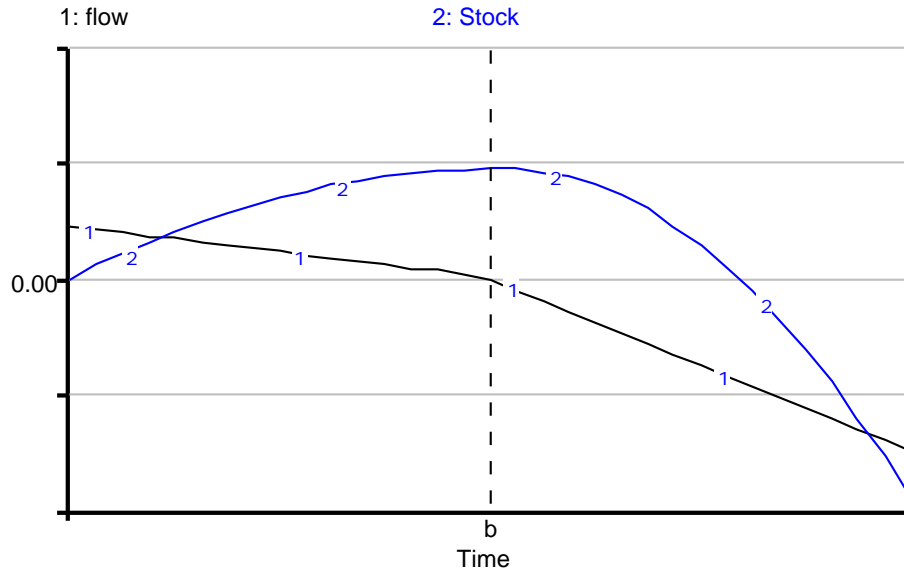
<sup>5</sup> In Figure 8, the flow graph does not appear to cross the time axis. Consider the time axis to be the line where 'y = 0.' If the line 'y = 0' is the time axis, then clearly the flow graph crosses the time axis at time 'a.'

positive, but grows at a slower and slower rate because the magnitude of the flow is decreasing. The flow decreases linearly and equals zero at time 'a.' After time 'a,' however, the flow is negative and increasing in magnitude, so the stock decreases faster and faster.

Two observations can thus be made from the graph in Figure 8:

1. As long as the flow is positive, the value of the stock increases, irrespective of the magnitude of the flow. A positive flow is an inflow; therefore, a positive flow always increases the value of the stock. Similarly, a negative flow always decreases the value of the stock, irrespective of the magnitude of the flow or whether the flow is constant, increasing, or decreasing in magnitude.
2. The shape of the stock graph depends on the rate of change of the flow magnitude. If the flow is decreasing in magnitude, the stock grows or declines more and more slowly as time passes, as in the time between zero and 'a.' If the flow is increasing in magnitude, the stock grows or declines more and more rapidly as time passes. Note that the rate at which the stock grows until time 'a' and the rate at which the stock declines after time 'a' are equal in magnitude because the slope of the flow is constant. The stock behaviors before and after time 'a' are mirror images of each other. If the slope of the flow changes, the shape of the stock also changes.

Figure 9 shows a system where the slope of the flow increases in magnitude after time 'b.'<sup>6</sup> As a result, the stock decays faster after time 'b' than the stock grows before time 'b.'



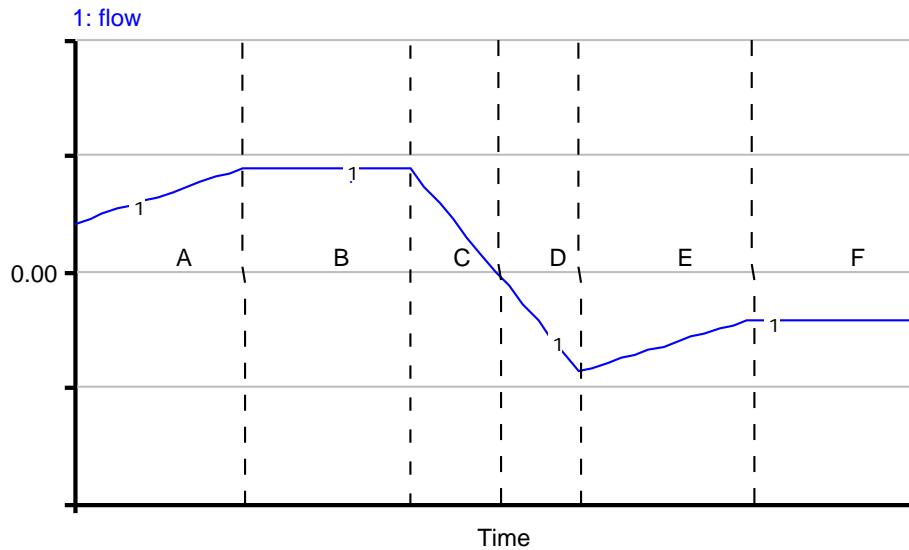
**Figure 9: Flow with changing slope that crosses the time axis**

<sup>6</sup> The flow in Figure 9 has a negative slope. When the slope becomes more negative after time 'b,' the value of the slope decreases because the value of the slope becomes more negative, but the magnitude of the slope increases.



## 4.2 Example 2: Constant and Linearly Changing Flow

Figure 10 shows a flow that changes characteristics several times.

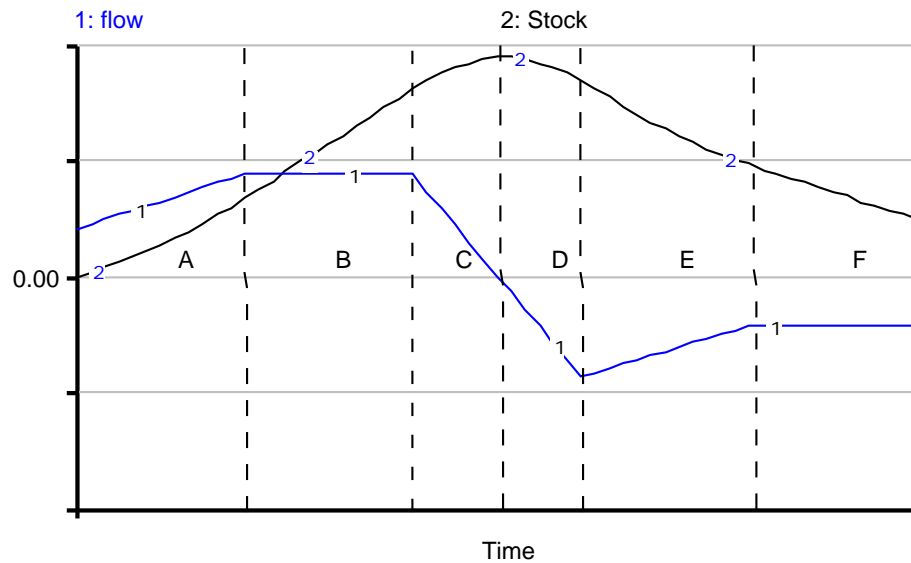


**Figure 10: A changing linear flow**

The flow can be divided into six segments:

- During segment A, the flow is positive and increasing in magnitude.
- During segment B, the flow is positive and constant.
- During segment C, the flow is positive and decreasing in magnitude.
- During segment D, the flow is negative and increasing in magnitude.
- During segment E, the flow is negative and decreasing in magnitude.
- During segment F, the flow is constant and negative.

Finally, note that the slope magnitudes in segments C and D are greater than the slopes of segments A and E. Therefore, the stock changes more rapidly during segments C and D than in segments A and E. The reader should be able to intuitively estimate the stock behavior, shown in Figure 11. Assume that the initial value of the stock is zero.

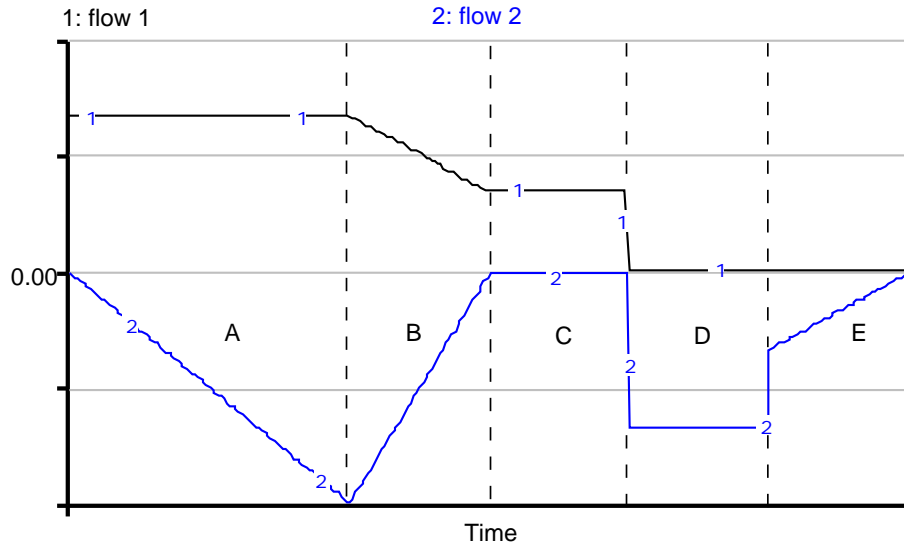


**Figure 11: Stock corresponding to the changing linear flow**

Estimating the stock behavior:

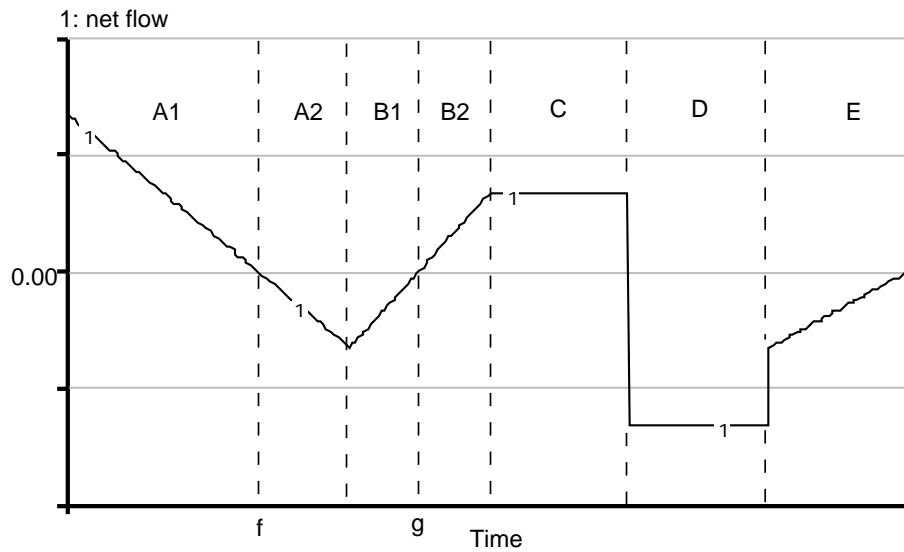
- During segment A, the flow is positive and increasing: the stock grows faster and faster with time.
- During segment B, the flow is constant: the stock graph is a straight line with a positive slope.
- During segment C, the flow is positive and decreasing: the stock grows at a decreasing rate. At the end of segment C, the stock stops growing because the value of the flow reaches zero.
- The point of time between segment C and segment D is important because the stock behavior changes from positive and decreasing in magnitude to negative and increasing in magnitude.
- During segment D, the flow is negative and increasing in magnitude: the stock decreases faster and faster.
- During segment E, the flow is negative and decreasing in magnitude: the stock continues to decrease but at a slower and slower rate.
- During segment F, the flow is negative and constant: the stock graph is a straight line with a negative slope.

### 4.3 Example 3: Graphical Integration of Combined Flows



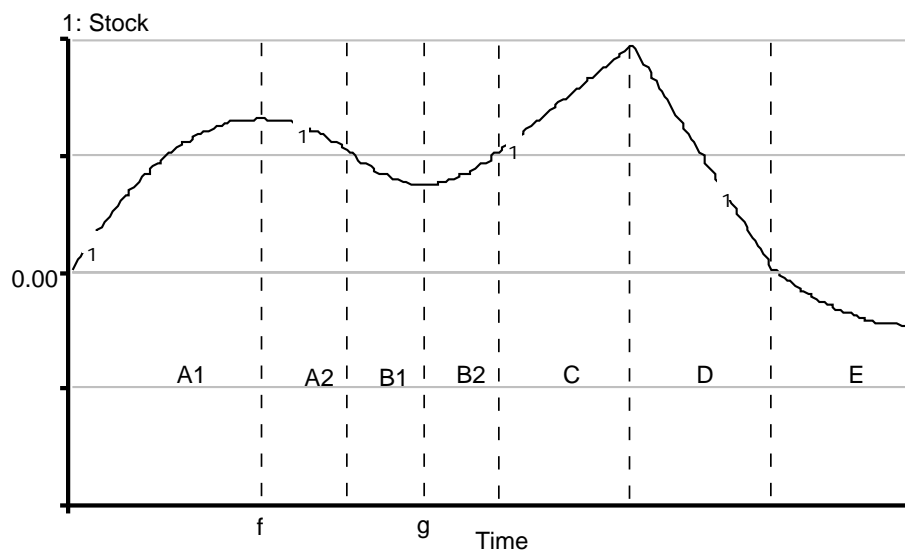
**Figure 12: Two flows into the same stock**

Figure 12 shows two flows going into the same stock. To estimate the behavior of the stock, the flows in each segment have to be added. The net flow can be calculated by arithmetically adding the values of the flows at every point of time. Figure 13 shows the net flow, the sum of flow 1 and flow 2 in Figure 12.



**Figure 13: The net flow**

The next step is to graphically integrate the net flow. Note that the net flow is divided into seven rather than five segments. At time 'f,' the net flow changes from positive and decreasing in magnitude to negative and increasing in magnitude. Segment A is divided into A1 and A2 because while the slope of the flow is constant in segment A, the characteristics<sup>7</sup> of the flow change at time 'f.' At time 'g,' the flow changes from negative and decreasing in magnitude to positive and increasing in magnitude. Segment B is divided into B1 and B2 because the characteristics of the flow change at time 'g.' The graphical integration of the net flow in Figure 13 is the first flow in this paper that requires the graphical integration of a step function.<sup>8</sup> A segment-by-segment graphical integration yields the stock behavior shown in Figure 14. Assume the initial value of the stock is zero.



**Figure 14: Stock corresponding to the net flow in Figure 13**

Estimating the stock behavior:

- During segment A1, the net flow is positive and decreasing in magnitude: the stock grows but at a decreasing rate.
- During segment A2, the net flow is negative and increasing in magnitude: the stock declines at an increasing rate.
- During segment B1, the net flow is negative and decreasing in magnitude: the stock declines at a decreasing rate.

<sup>7</sup> The sign of the flow changes from positive to negative, and the flow changes from being decreasing in magnitude to increasing in magnitude. Note, however, that even though the sign of the flow changes, the value of the flow is constantly decreasing.

<sup>8</sup> The "STEP" function is a built-in function in system dynamics simulation softwares. The STEP function is used to model an instantaneous change in the value of a variable.

- During segment B2, the net flow is positive and increasing in magnitude: the stock grows at an increasing rate.
- During segment C, the net flow is positive and constant: the stock grows linearly.
- At the time point between segment C and segment D, the net flow steps down to a negative value.
- During segment D, the net flow is negative and constant: the value of the stock declines linearly.
- At the time point between segment D and segment E, the flow steps up. The flow is still negative, but has a smaller magnitude.
- During segment E, the net flow is negative and decreasing in magnitude: the stock declines in value but at a decreasing rate.

At the time point between segment C and segment D, the net flow steps down and changes instantaneously from a positive to a negative value. In Figure 14, the stock responds by decreasing linearly for the time period that the flow is negative and constant, segment D. Thus, even though the flow changes instantaneously, the stock changes gradually. When the type of linear flow changes, the stock graph changes gradually depending on the characteristics of the flow. The change in the slope of the stock graph is also continuous and gradual. A step function, however, creates a certain point at which a sudden change occurs in the slope of the stock. The slope of the stock graph equals the value of the flow. Thus, when the value of the flow suddenly changes, the slope of the stock graph changes suddenly too. It is important to keep in mind that the value of a stock always changes gradually. There are no sudden jumps or discontinuities in the value of a stock; a stock changes continuously over time, even though flows may change values instantaneously.

## 5. GENERAL STEPS FOR PERFORMING GRAPHICAL INTEGRATION

The behavior of a stock affected by linear flows can be determined by applying the following general steps:

1) Divide the flow into segments. A segment is a section of the flow diagram during which the characteristics of the flow do not change. Often a flow will be a combination of more than one of the types of flows mentioned in section 3 of this paper: the flow changes from one type to another after a certain time interval. Step 2 below lists the various types of linear flows studied in this paper. Each time the type of flow changes, it marks the beginning of a new segment. Usually, a new segment starts when the slope of the linear flow changes. Each time the flow changes from positive to negative or negative to positive, the slope of the flow may not change but the type of flow does change.

2) Determine the type of linear flow in each segment. Flows can be:

1. zero
2. constant and positive
3. positive and increasing in magnitude
4. positive and decreasing in magnitude
5. constant and negative
6. negative and increasing in magnitude
7. negative and decreasing in magnitude

3) After identifying the different segments of the flow, use the following table to determine the behavior of the stock in each segment:

| # | Sign of Flow | Type of Flow            | Stock Behavior                |
|---|--------------|-------------------------|-------------------------------|
| 1 | zero         | constant                | no change in the stock        |
| 2 | positive     | constant                | increasing in a straight line |
| 3 | positive     | increasing in magnitude | increasing faster and faster  |
| 4 | positive     | decreasing in magnitude | increasing slower and slower  |
| 5 | negative     | constant                | decreasing in a straight line |
| 6 | negative     | increasing in magnitude | decreasing faster and faster  |
| 7 | negative     | decreasing in magnitude | decreasing slower and slower  |

**Table 1: Stock behavior corresponding to different types of linear flows**

4) After determining the stock behavior in each segment, combine the behavior of the stock in different segments to estimate the stock behavior over the entire time range being studied.

## 6. CONCLUSION

Section 3 studied various types of linear flows. Two characteristics of linear flows determine the behavior of the stock:

- the sign of the flow (zero, positive, or negative)
- the magnitude of the flow (constant, increasing, or decreasing)

The exact value or equation of the flow at any time is not crucial in determining the dynamics of the stock. The value of the flow is required, however, to calculate the exact value of a stock at any time. Estimating the behavior of a stock is often more useful than the ability to calculate the exact value of the stock.

When the type of linear flow changes over time, one must divide the flow into distinct segments for each type of behavior and integrate each section separately. The behavior of a stock with more than one flow can be estimated by graphically integrating the sum of all the flows, the net flow.

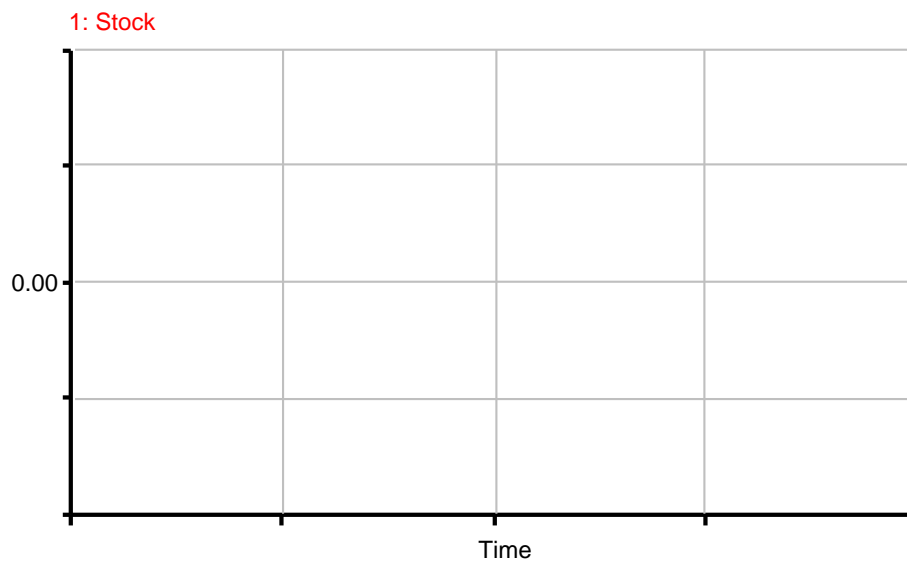
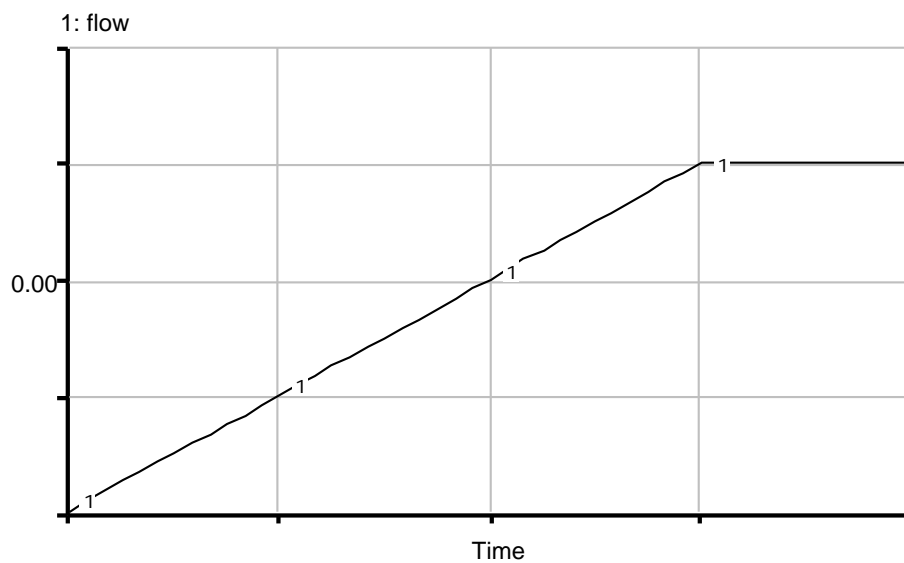
## 7. EXERCISES

Complete the following exercises, then refer to the solutions in section 8.

### 7.1 Exercise 1

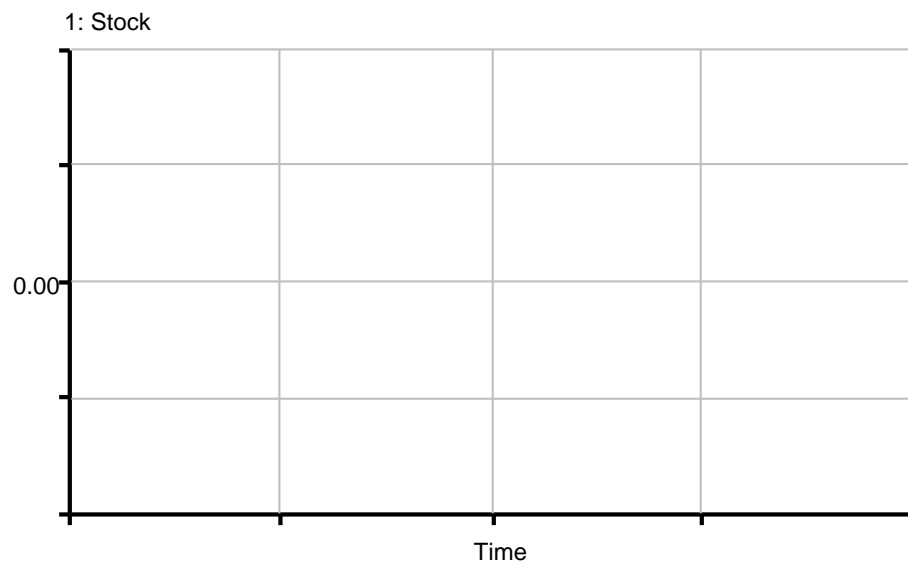
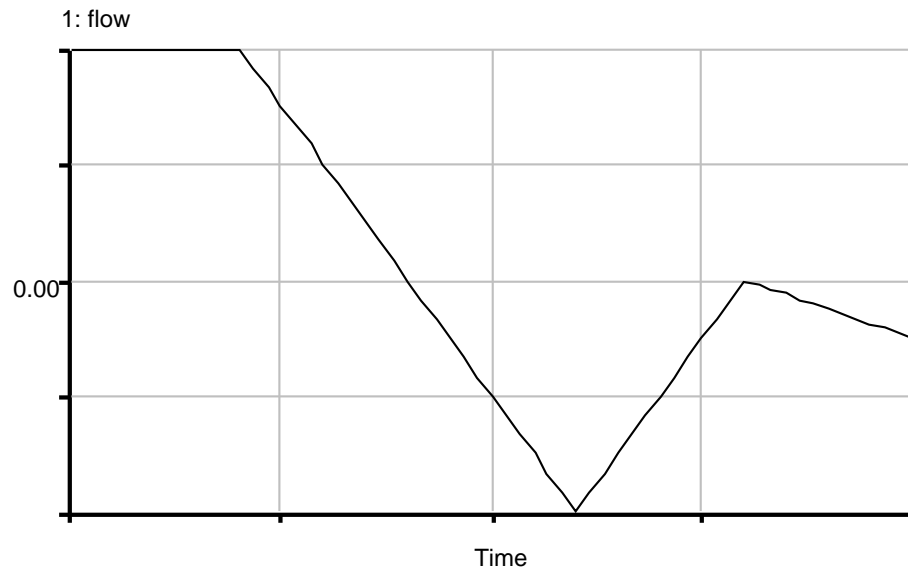
Divide each of the flows in the exercise into appropriate time segments for integration. Draw the corresponding stock behavior on the blank graphs. Assume that the initial values of the stocks are zero (the stock behavior will be the same irrespective of the initial value chosen).

A.



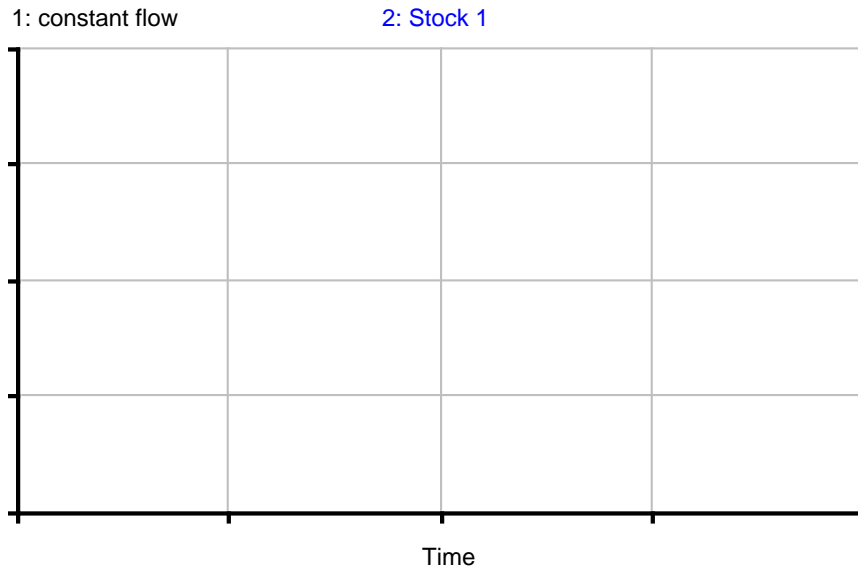
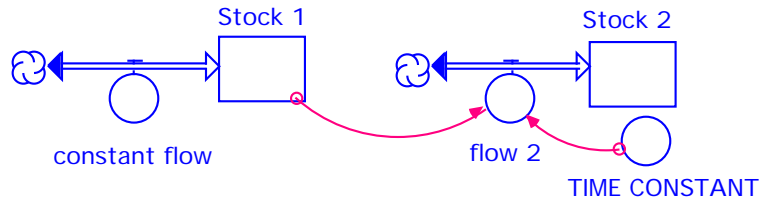


B.



### 7.2 Exercise 2

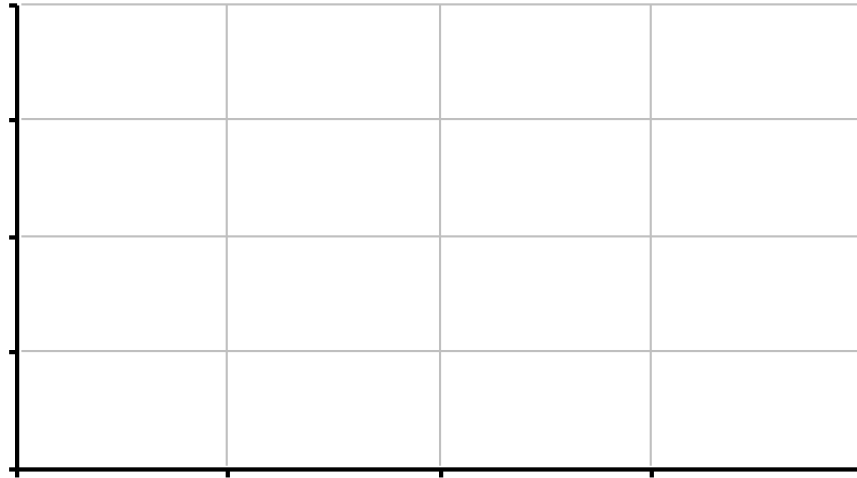
The flow into “Stock 1,” “constant flow,” has a constant positive value. If “flow 2” is equal to “Stock 1” for all time, estimate the behavior of “Stock 1” and “Stock 2” over time.<sup>9</sup> Assume that the initial value of both stocks is zero.



<sup>9</sup> The variable “TIME CONSTANT” has been introduced for dimensional consistency of the model. The graph of “flow 2” will have the same shape as the graph of “Stock 1.” The slope of the graph of “flow 2” will equal the slope of the graph of “Stock 1” if the “TIME CONSTANT” is equal to one. Assume that the value of the “TIME CONSTANT” is equal to one.

1: flow 2

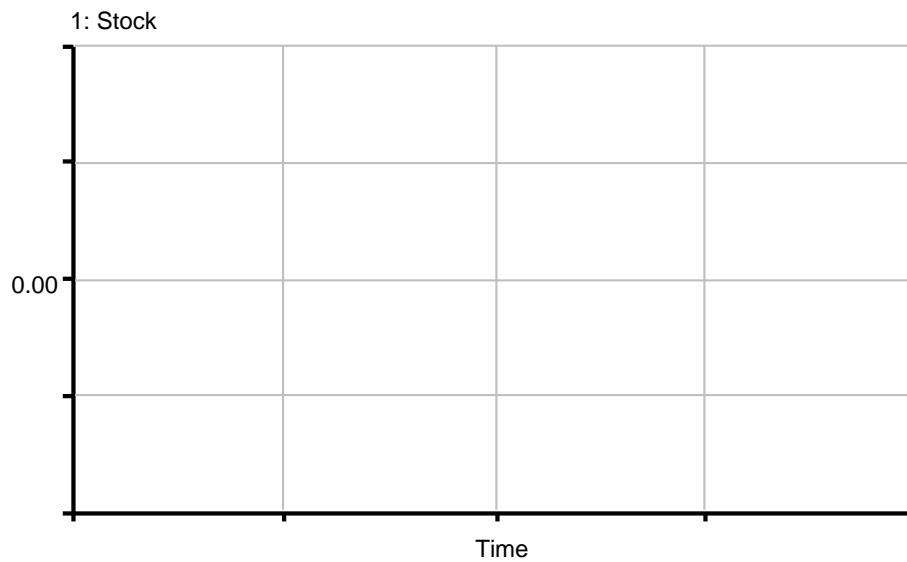
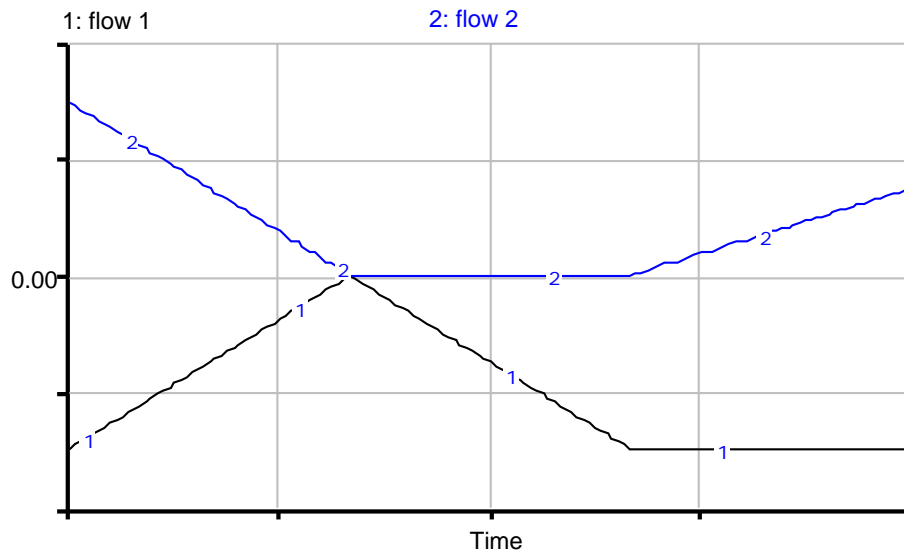
2: Stock 2



Time

### 7.3 Exercise 3

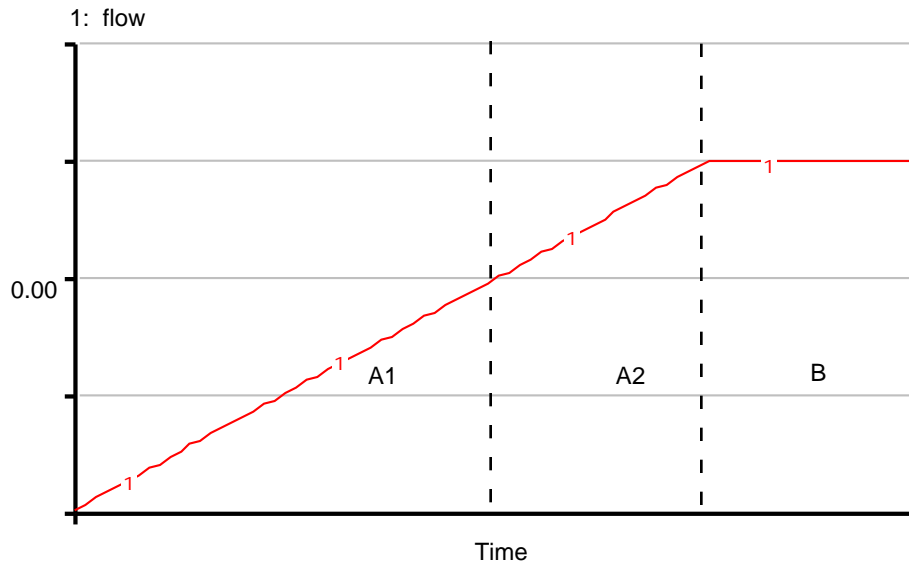
Estimate the behavior of a stock with two flows as shown below. Assume that the initial value of the stock is zero. For convenience, plot the net flow on the graph with the flows.



## 8. SOLUTIONS

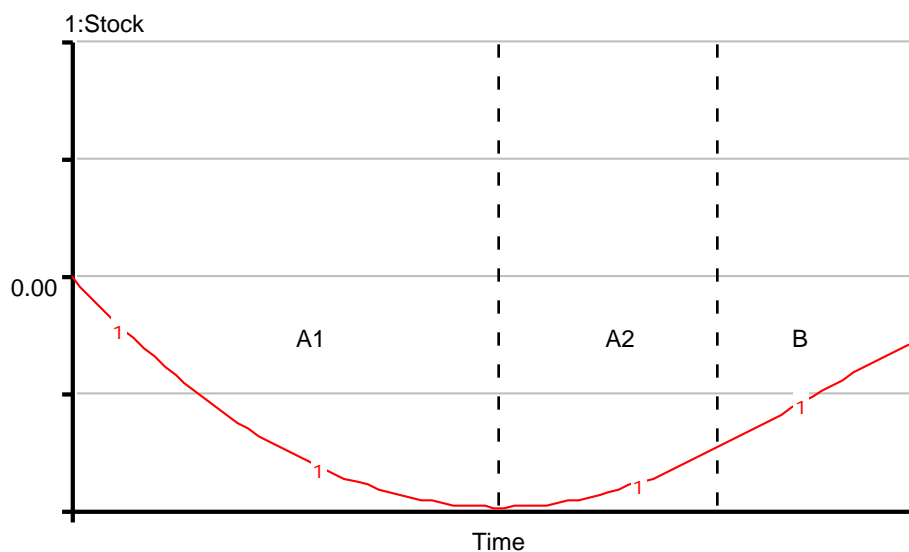
### 8.1 Solutions to Exercise 1

A. The correct segmentation is shown in the graph below.

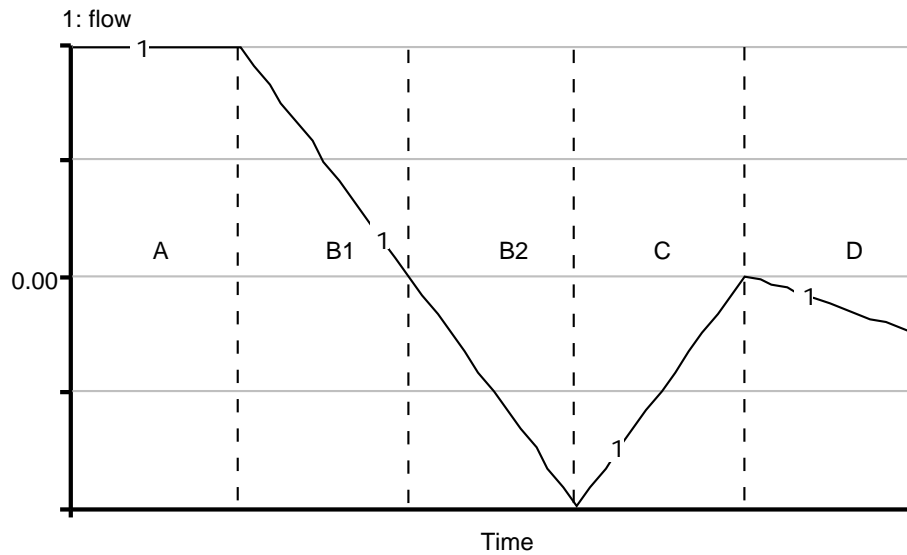


During segments A1 and A2, the slope of the flow does not change, but the sign of the flow does change from negative to positive.

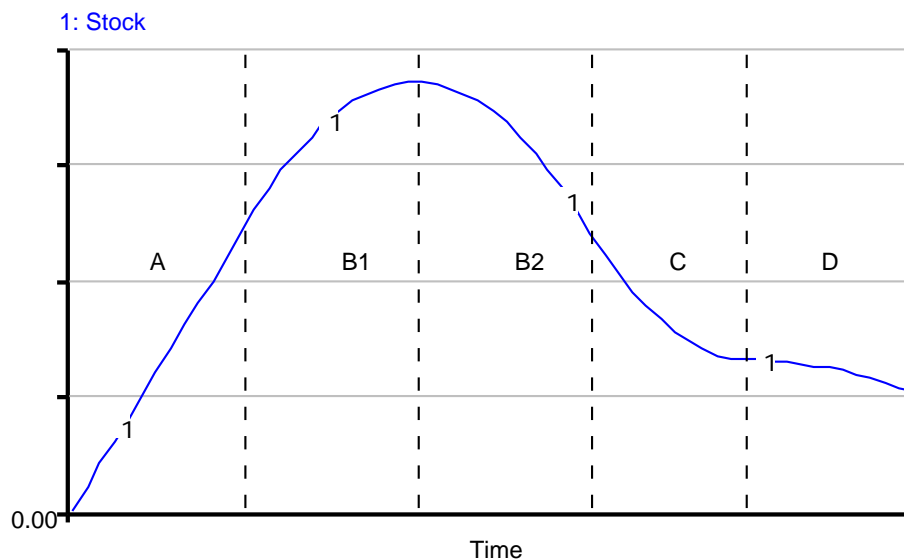
During time A1, the flow is negative and decreasing in magnitude; thus, the value of the stock falls below the initial value of zero, but decreases slower and slower as time passes. As the flow becomes positive and the magnitude of the flow increases in period A2, the stock starts growing at an increasing rate. During time B, the flow is positive and constant, causing linear stock growth. The overall stock behavior is shown in the following graph.



B. The flow in this exercise can be segmented at the obvious turning points. Notice, however, that the flow also crosses the time axis and should be segmented at the point of crossing the time axis also. The correct segmentation is shown below.

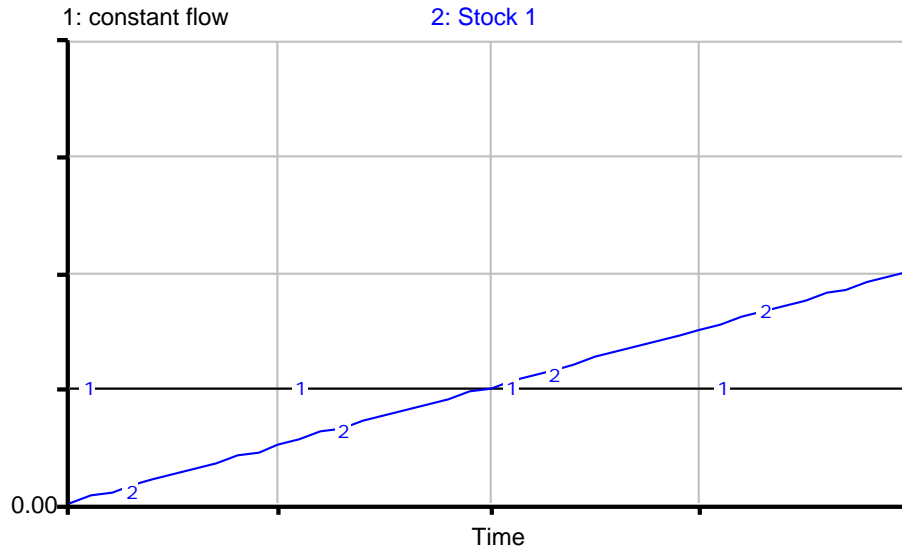


During segment A, the flow is positive and constant, so the stock grows linearly. During segment B1, the flow is positive but decreasing in magnitude, driving the stock to increase in value but at a slower and slower rate. After crossing the time axis, during segment B2, the flow is negative but now increasing in magnitude. The stock starts decreasing faster and faster during segment B2. During segment C, the flow is still negative but decreasing in magnitude, so the stock is still decreasing but slower and slower. During segment D, the flow is similar to the flow during segment B2, but the slope is smaller in magnitude. Thus, the stock decreases at an increasing rate until the end of the period shown on the graph, but does not decrease as rapidly as in section B2.

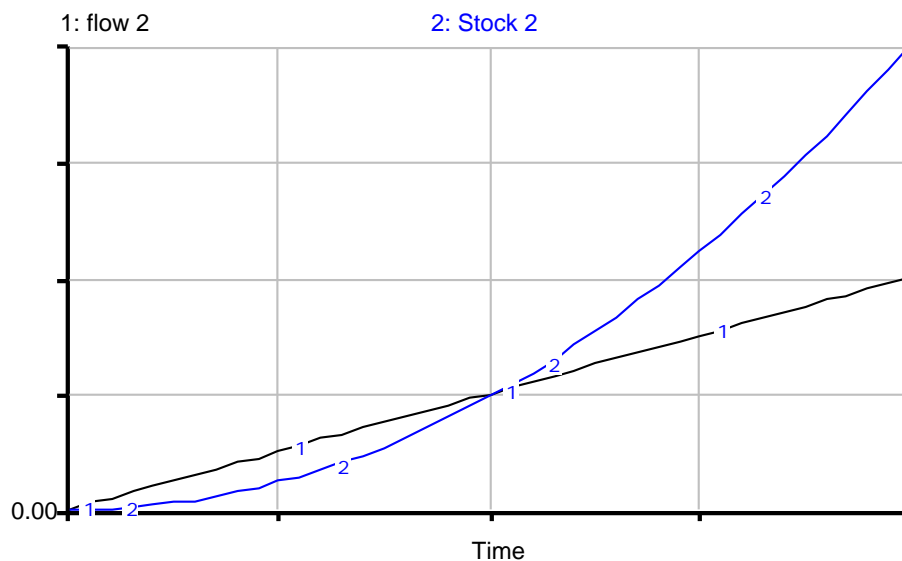


## 8.2 Solutions to Exercise 2

Solve the problem one step at a time. First, graphically integrate “constant flow.” Because “constant flow” is positive and constant, “Stock 1” will grow at a constant rate from the initial value of zero. The rate at which “Stock 1” grows (the slope of the “Stock 1” curve) depends on the magnitude of “constant flow.” The following graph shows the behavior of “constant flow” and “Stock 1.”

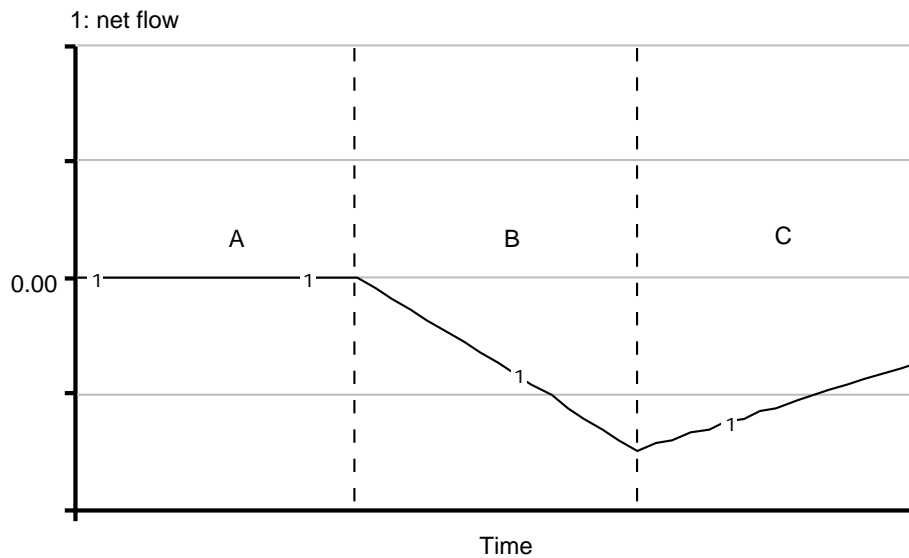


Because “flow 2” is equal to “Stock 1,” “flow 2” will behave exactly the same way as “Stock 1.” Thus, “flow 2” will be a positive flow increasing in magnitude. A positive flow increasing in magnitude causes the stock to grow faster and faster as time passes. Thus, “Stock 2” grows at an increasing rate, as the next graph shows.



### 8.3 Solutions to Exercise 3

First, add flow 1 and flow 2. The following graph shows the net flow.



The next step is to segment and then graphically integrate the graph of the net flow. Because the graph is always negative or zero, the flow has to be segmented at only the points where the slope of the flow graph changes. With a constant net flow of zero, as in segment A, the stock remains at the initial value. When the flow becomes negative and increasing in magnitude during segment B, the stock decreases faster and faster. As the direction of the flow graph changes and the flow becomes negative and decreasing in magnitude in segment C, the stock is still decreasing, but slower and slower. The stock behavior is shown in the next graph.

