15.997 Practice of Finance: Advanced Corporate Risk Management
Spring 2009

Problem Set #1

Simulate the risk distribution of the copper price using the random walk.

References

This assignment requires that you implement a binomial model and a monte carlo simulation for the copper price. A relevant reference is Parsons and Mello, Lecture Notes on Advanced Corporate Financial Risk Management, Chapter 6: Measuring Risk–Dynamic Models, Part A–The Random Walk Model of Stock Prices.

Binomial Tree

(1) Construct a spreadsheet to simulate copper prices. Implement the binomial model with T = 2 years and N=2, i.e. using a two-step tree, one step for each year. Set the expected rate of appreciation in the price to 10%, the annual volatility to 28%, the risk-free rate to 5%, and the initial copper spot price to $2.65/pound.

a) Draw the tree showing the price and the appreciation to date at each node. Show the actual probability of reaching each node.
b) What is the expected price of copper in one year and in two years?
c) Graph the probability distribution for the price at t=2.
d) What are the expected cumulative growth rates at t=1,2?
e) Calculate the standard deviation of the cumulative growth rates at t=1,2.
f) Graph the probability distribution for the cumulative growth rate at t=2.
g) Move one period forward, to t=1, assuming that the price moved up. What is the expected price at t= 2?

(2) Build the binomial model for T = 10

a) Build the 10-step binomial tree for the copper price.
b) Draw the tree showing the price and the appreciation to date at each node. Show the actual probability of reaching each node.
c) Graph the probability distribution for the price at t=10.
d) What is the expected price at t= 1..10? Graph the expected price through time.
e) What are the expected cumulative growth rates at t=1,..10?
f) Calculate the standard deviation of the cumulative growth rates at t=1…10.
g) Graph the probability distribution for the cumulative growth rate at t=10.
h) Move one period forward, to t=1, assuming that the price moved down. What is the expected price at t= 2..10? Graph the expected price through time on top of your previous graph of the expected price.
i) What is the probability that the price is below $3 at t=10?

j) What is the probability that the price is between $3 and $7 at t=10?
k) Extra Credit: Think about how to answer the question “What is the probability that the average price during the ten years is less than $5?”

Monte Carlo Simulation

(3) Construct a Monte Carlo simulation of the copper price. Use the same assumptions as before… T=10 years, N=10, the expected rate of appreciation in the price is 10%, the annual volatility is 28%, the risk-free rate is 5%, and the initial copper spot price is $2.65/pound..

a) Produce at least 100 simulations of the price.
b) Make a histogram for the price at t=10.
c) Use the simulation to estimate the expected price at t= 1..10? Graph the estimated expected price through time.
d) Estimate the expected cumulative growth rate at t=1…10? Graph it through time.
e) Calculate the standard deviation of the cumulative growth rate at t=1…10 Graph it. How does it change with the horizon?
f) Graph the probability distribution for the cumulative growth rate at t=10.
g) Move one period forward, to t=1, assuming that the price moved down to $2. What is the expected price at t= 2..10? Graph the expected price through time on top of your previous graph of the expected price.
h) What is the probability that the price is below $3 at t=10?
i) What is the probability that the price is between $3 and $7 at t=10?
j) What is the probability that the average price during the ten years is less than $4? Why is this easier to solve here than in the binomial tree?

Additional References

John C. Hull, Options, Futures & Other Derivatives, is good cookbook for many things in derivatives; in particular, Chapter 9 in the 4th edition discusses constructing simulations.

Robert McDonald, Derivatives Markets, also provides a full introduction to binomial trees and simulations; in the 1st edition see Chapter 10 for binomial trees. As I did in my lecture notes, he starts with the “forward tree” method when it is more common to use the Cox-Ross-Rubenstein method which he describes in Chapter 11 section 3.

Richard A Brealey and Stewart C. Myers, Principles of Corporate Finance, various editions, discuss the binomial method for simulating stock prices in the material on valuing options; in the 7th edition see Chapter 21, section 21.2.