15.997 Practice of Finance: Advanced Corporate Risk Management
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Valuation

MIT Sloan School of Management
15. 997 Advanced Corporate Risk Management
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Review

- The risk-neutral valuation framework starts by going one step deeper than the traditional risk-adjusted discount rate.
- The risk-adjusted discount rate (1) assumes a linear exposure to the underlying factor risk, and (2) applies a single discount factor to the expected cash flow.
- This is <u>equivalent</u> to applying <u>different discount factors</u> to the "up" and "down" states. These are the forward state prices.
- The risk-neutral valuation framework recovers these forward state prices.
- With these in hand, we are no longer restricted to valuing only linear exposures. We can value <u>any</u> non-linear pattern of exposure.
- The risk-adjusted discount rate method cannot do this because it stopped before recovering the two different discount factors, and only applied the "average" discount factor to the "average" or expected cash flow.

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Turbo-Charged Valuation

- Once we have recovered the risk-neutral probabilities (i.e. the forward state prices) we can value any non-linear pattern of exposure.
- Whole body of "option pricing" or "derivative pricing" theory is just working out a compendium of formulas for different problems. Behind them is the general principle.
- Illustrate this with a case study…

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Valuing Option-Like Investments

- Use risk-neutral method…
- Model the stochastic process for the underlying risk variable.
- Specify the risk-neutral process, i.e., calculate the risk neutral probabilities.
- Value *any* derivative claim using the risk-neutral method, i.e., using the risk-neutral probabilities to determine the certainty equivalent, and discounting at the risk-free rate.

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An Aluminum Supply Contract with a Structured Price Clause

- aluminum producer and major consuming firm
- 20-year term
- Fixed annual quantity
- Price clause
  - Floor price equal to $1,644/Ton
  - Between $1,644 and $1,808/Ton, the world market price
  - From $1,808 to $2,308/Ton, price rises by the world market
  - price,
  - From $2,308 to $3,008/Ton, price rises by 25% of the world
  - market price,
  - From $3,008/Ton, price rises by 50% of the world market
  - Price
  - All trigger values grown at the price of inflation.

Contract Payoff Diagram
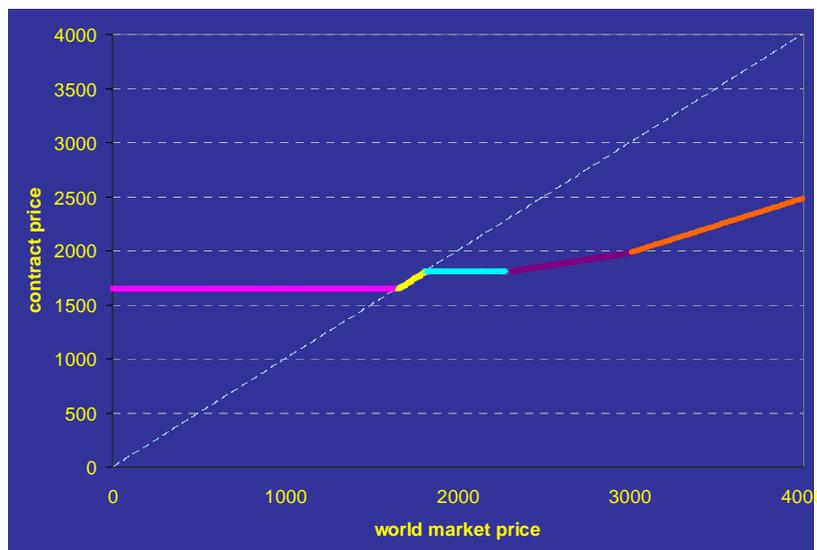
Option Analogy

- Payoff diagram can be converted into a batch of options on the aluminum price
  - Find the options!
  - The 20 year contract means a series of options, one set for each year
- Options can be priced using a model for the price evolution and a model for pricing risk
- The value of the contract is the value of the bundle of options reshaping the payoff, summed across the full series of maturities.

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Floor Price as a Put Option

- Receive the world market price,
- Plus a put option with the right to surrender the world market price in exchange for the floor price, i.e., a put option on aluminum with a strike price equal to the contract floor price.

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Effect of a Put on the Payoff

A put option reshapes the payoff at the low end…
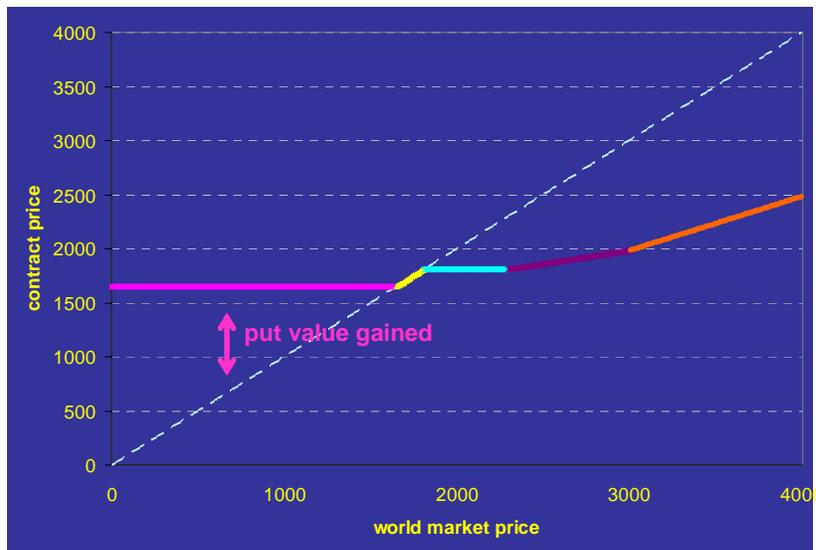
How to Capture the Remainder?

Full Contract Equals…

- Fixed price of $1,644/Ton
- Plus a Call with the exercise price of $1,644/Ton
- Minus a Call with the exercise price of $1,808/Ton
- Plus 25% of a Call with the exercise price of $2,308/Ton
- Plus 25% of a Call with the exercise price of $3,008/Ton.

Valuing the Options

- Suppose the world market price of aluminum follows a random walk…i.e., GBM
- Choose a convenience yield, …i.e., a dividend rate and expected rate of growth of aluminum prices
- Estimate the volatility
- Apply Merton's variation on Black-Scholes, the value of an option on dividend paying stock…

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Spot Price Model Matters

Valuation with the Estimated Process:
ATM Calls of Varying Maturity
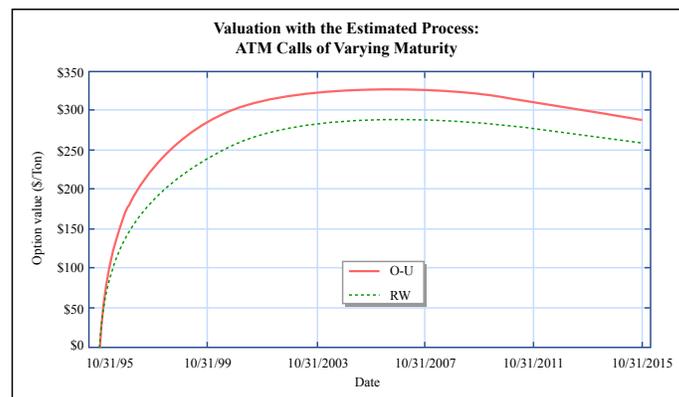
Figure by MIT OpenCourseWare.

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Spot Price Model Matters
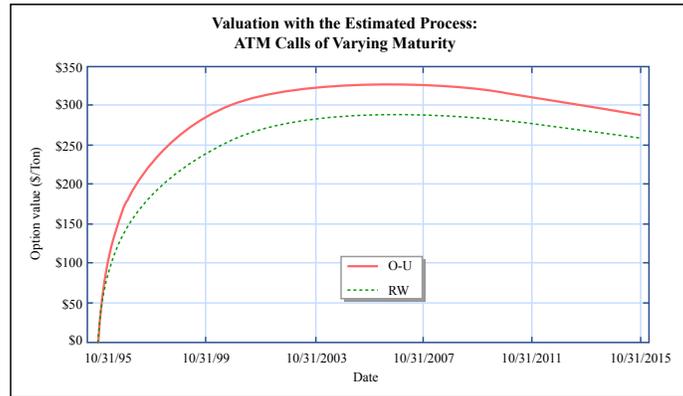
Valuation with the Estimated Process:
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Figure by MIT OpenCourseWare.

Spot Price Model Matters

Valuation with the Estimated Process:
ATM Puts of Varying Maturity

Figure by MIT OpenCourseWare.
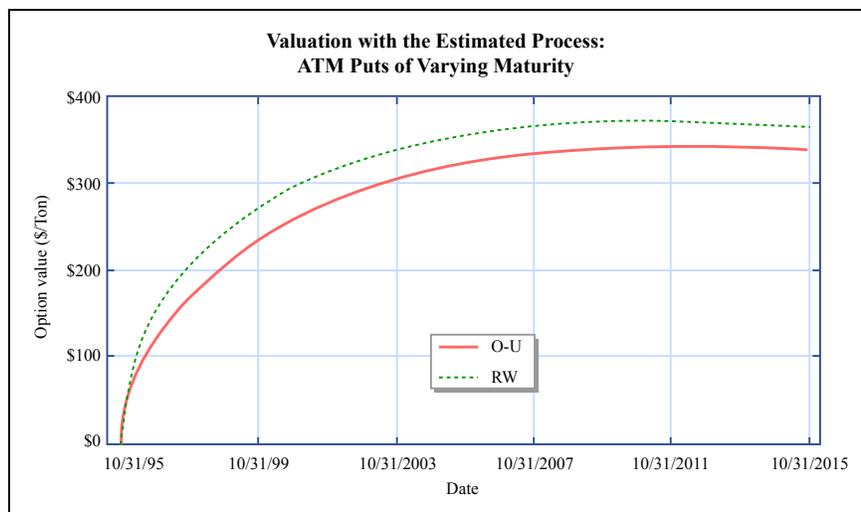
Spot Price Model Matters

Valuing the Contract for Alternative Processes
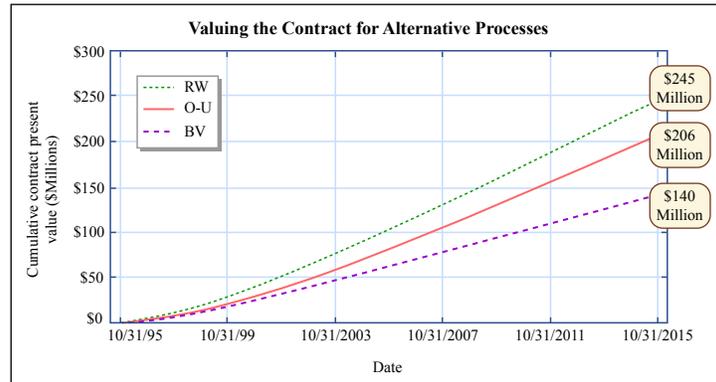
Figure by MIT OpenCourseWare.

Risk & Valuation: When Does It Matter

- Underlying state variable has asymmetric risk – extreme downside.
- Underlying state variable risk does not grow linearly with time – don't compound discount rate.
- Project payoff is non-linear – e.g., a call option on the underlying state variable.

Main Myth of Risk-Neutral Valuation

- Do you need traded securities?
- No.
  - RN valuation is just about consistent valuation of all non-linear payoffs.
  - Any equilibrium model like the CAPM implies a risk-neutral valuation framework.
- Traded securities make the starting point of the valuation more reliable. But this has nothing to do with the RN framework.

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Second Myth of Risk-Neutral Valuation

- Real-option pricing is a competitor to the DCF framework.
- RN is just an extension of the DCF framework.

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