Hi!

My name is Vaibhav, and today we will learn about functions.

Functions are very important concept, both for your engineering entrance exams and also to understand various concepts in mathematics.

It's a very fundamental concept, and is used to describe a lot of phenomenon, including physical and chemical phenomena.

And, also a lot of chapters we will learn further on such as limits, derivatives, continuity or be it any other chapter in mathematics, knowing functions is very important.

It is a very fundamental chapter.

It has various definitions, and so a lot of use for the next topics that you will learn.

So, we will start with an intuitive definition of functions.

Something that we will try to understand using examples, and then we will try to go into a more mathematically precise definition.

Functions are used to describe, a special relation between two variables.

So, here the word "relation" is a mathematical concept that we will understand later; but this (functions) is a very intuitive concept.

We have two variables of interest, and we are trying to define what is the relationship between them.

And, that is something which is important in a lot of places, because you always have something such as an input to a system and an output to a system and you want to figure out what the system is doing.

Functions come in handy at that place a lot.

This is a very intutitive description of functions.

In order to define it for our use, we ... mathematicians ... use different types of descriptors for functions.

Typically, functions are described using a graph, a rule or an equation.

Now, we will see how this is done.

When you have two variables either you can drive a graph, a plot OR you can have a very intuitive rule which you can easily describe OR at times as we will see more and more we will be using equations to describe a function.

So, let's begin with a very simple example.

We will take life of a typical student who is studying for an exam and going to school.

We want to figure out who his or her day goes.

Let's call the student Sonu, and we want to find out how Sonu's day went.

And, the variable that we are interested in is "How far Sonu is from home?" Variable 1 is time.

Variable 2 will be Sonu's distance from home.

Now, let's go to the first definition that we wrote.

So, our first description for function is that function is a "special" relation between two variables.

And, here what we have are two varibles of interest - one is "time of the day" and (the other is) "how far Sonu is from the home." So, let's try to make plot for a typical day.

I will try to remember how my school days were, and try to make a plot based on that.

On x-axis I will draw time of day, and on y-axis I will draw distance from home.

We will try time count from midnight, and in 24-hour format.

A day has 24 hours.

And, then we want to figure out how the day went.

In morning, Sonu was home till 6 o'clock.

Then he when to school, so the distance was how much farther the school was.

He was at school till roughly 2pm, which is 14 hours.

Then let's say he goes to a friends place, and spends some time there.

Then he comes back home, and spends the rest of the time at home.

So, here we see very intuitively we can describe the relationship between two variables using a plot.

This plot is essentially describing a function of where Sonu is throughout the day - how far Sonu is from his home - throughout the day with respect to time of the day.

Similarlt, we can draw such graphs or such relations for different variables.

So, taking an example from physics.

Let's say we want to study a similar thing for an object which is moving with a constant velocity.

So, let's take another example.

And if we want to draw a plot of distance of this object from the starting point it will be something very similar to this.

The nature of the graph will be different, but essentially we are plotting similar things.

Variable one will be time.

And, variable two will be distance from start.

And (from physics) we know if an object is moving with a constant velocity ... it can be described by an equation.

Let's say we denote the variable two as 'd', and this is described by 't'.

And let's say that the speed is one which is given.

This is an example where we described the function as an equation.

So, we saw that a function can be described by a graph, a rule.

At first the rule was where you are from home.

Or it could be a rule like object moving with a constant speed.

At times we also use equations.

And, they are interchangeable.

We can write this rule as an equation and this equation as a graph.

This is how distance varies with time.

These are some examples of function, which occur in day to day activities.

And in these examples we saw that function's input was time, but thay doesn't have to be the case.

Functions can have different notions of variable one and variable two depending on the application.

Another interesting way to look at function is as an input output relation.

Here, in the examples that we studied we had two variables.

We can see them as a relation between input which goes through the function and out comes the output.

So, in example 1 input was time of day.

And, output was distance from home.

Similarly, in example two the input was time in hours and output was distance from starting location.

So, we have seen multiple ways of intuitively describing a function.

Either it could be a special relation between two variables, it could be described using a graph, rule or equation ...

It could be thought of as a system which converts an input to an output.

And whenever we have anything of this sort, we can use functions to describe it.

Once we have these functions, they are pretty useful and they can be manipulated to figure out to find out properties of a system.

So far, we have seen a few examples.

Now, I would request you to take some time to think about other functions that you encounter in daily life.

For those of who you like sports, you can think of something in cricket.

Your favorite hits a ball and you want to figure out how fast the ball is moving.

So, you can have time after which the ball was hit and the speed of the ball.

It does not necessarily have to be speed or time, it can be something all together different.

Let's say you are going from Bombay to Delhi, and the number of passangers change at each station.

Based on the distance from Mumbai, you can figure out the number of passangers in the train.

In the train example, distance is the input variable or the variable 1.

And output variable is number of passengers.

And similarly you can find other functions in your day to day activities.

They occur in various physical and chemical phenomena.

So, a lot of things that you are learning in physics and chemistry - functions are also useful there to describe those phenomena.

Again, take some time to think about examples of functions.

Also, one thing that is important to see here is what makes variable one go on the x axis and variable two on y.

Is it possible to flip the variables on the graph?

Why is variable one on the x axis and variable two on the y axis?

One reason that you could think of is that variable one is input and variable two is the output; but there are other reasons that we will see as we go ahead.

This was a very intuitive definition of function, now let's study it from a mathemaically precise perspective.

For that we will go back to our orginial statement - "functions are used to describe special relation between two variables.

" Here we have a few mathematical terms, we have relations and w have variables.

We will try to see how they relate to the mathematical concepts that we already know.

One thing that we have already learnt is sets.

Sets are a collection of items.

So, let's say we have sets A and B.

A and B could be any sets of objects of interest.

Let's denote elements of A with small a, and that of set B with small b.

Once we have these sets, we have variables from those sets.

Whenever there is any relationship between these variables, we can define them using certain mathematical constructs.

One construct is the Cartesian products of two sets.

So, a Cartesian products of two sets A, B is given by A cross B. And, it includes all the elements a comma b.

Let's say A has two variables a1 and a2.

And, set B has one variable b1.

Then, A cross B will have two variables (a1,b1) and (a2,b2) (cut near ) But, if set B had another variable b2.

Then the total variables will be four.

Cartesian products of two sets includes number of elements of set A multiplied by number of elements of set B. T This is a very big set, and a Relation is defined as a subset of the Cartesian product.

So, let's say for some reason you are only interested in the subset of the Cartesian product set.

We call it by R.

R let's say has (a1, b1) and (a2, b2).

Any subset of the cartesian product is a relation.

Now, that we have defined the important concepts that we need for the precise defintion of fucntions,... we will see in the next lecture how we combine sets, cartesian products and relations - specifically relations to figure out what is special about functions.