Combinatorics: The Fine Art of Counting

Week Nine Menu
Graphic Grub

This week's colorful menu includes a number of graphic staples. Feel free to taste a small portion of each item or pick your favorite dish and chow down.

1. Prove that at a party where everyone has at least two friends there must be a circle of friends. Prove that this is still true even if one person has only one friend and everyone else has at least two friends.

2. Find all the non-isomorphic spanning trees in the graphs of the cube and the octahedron.

3. K_5 can be decomposed into cycles in two non-isomorphic ways, as two 5-cycles or as a 4-cycle and two 3-cycles. Find all non-isomorphic decompositions into cycles of the following graphs:

- Octahedron
- $K_{4,4}$
- K_7

4. Find a proper vertex-coloring of each of the five regular polyhedral graphs using the minimum number of colors possible.

5. Find a proper edge-coloring of each of the five regular polyhedral graphs using the minimum number of colors possible.

6. Construct a round-robin tournament schedule for 9 players {1,2,3,4,5,6,7,8,9} using 9 rounds. For each round list the matches played and any players who have a "bye" (i.e. no opponent in that round).

7. Prove or disprove: every 2-coloring of the edges of K_6 must contain a mono-chromatic 4-cycle.

8. Find the largest graph with 6 vertices which does not contain a sub-graph isomorphic to K_4.

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