

Power Series and Euler's Formula

At $x = 0$, the n th derivative of x^n is the number $n!$ Other derivatives are 0.
 Multiply the n th derivatives of $f(x)$ by $x^n/n!$ to match function with series

TAYLOR SERIES $f(x) = f(0) + f'(0)\frac{x}{1} + f''(0)\frac{x^2}{2} + \dots + f^{(n)}(0)\frac{x^n}{n!} + \dots$

EXAMPLE 1 $f(x) = e^x$ All derivatives = 1 at $x = 0$ Match with $x^n/n!$

Taylor Series
Exponential Series $= e^x = 1 + 1\frac{x}{1} + 1\frac{x^2}{2} + \dots + 1\frac{x^n}{n!} + \dots$

EXAMPLE 2 $f = \sin x$ $f' = \cos x$ $f'' = -\sin x$ $f''' = -\cos x$

At $x = 0$ this is 0 1 0 -1 0 1 0 -1 REPEAT

sin x $= 1 \cdot \frac{x}{1} - 1\frac{x^3}{3!} + 1\frac{x^5}{5!} - \dots$ ODD POWERS $\sin(-x) = -\sin x$

EXAMPLE 3 $f = \cos x$ produces 1 0 -1 0 1 0 -1 0 REPEAT

cos x $= 1 - 1\frac{x^2}{2!} + 1\frac{x^4}{4!} - \dots$ EVEN POWERS $\frac{d}{dx}(\cos x) = -\sin x$

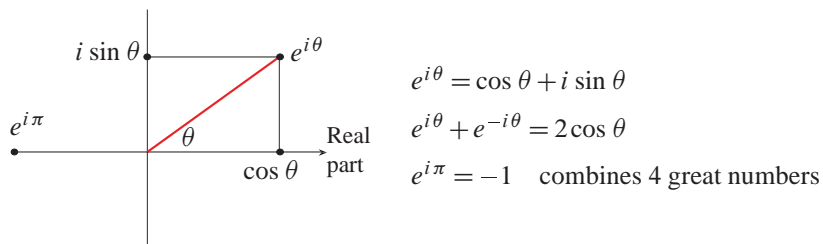
Imaginary $i^2 = -1$ and then $i^3 = -i$ Find the exponential e^{ix}

$$e^{ix} = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \dots$$

$$= \left(1 - \frac{x^2}{2!} + \dots\right) + i \left(x - \frac{x^3}{3!} + \dots\right)$$

Those are **cos x + i sin x**

EULER'S GREAT FORMULA $e^{ix} = \cos x + i \sin x$



Two more examples of Power Series (Taylor Series for $f(x)$)

$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$ "Geometric series"

$f(x) = -\ln(1-x) = \frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$ "Integral of geometric series"

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Resource: Highlights of Calculus
Gilbert Strang

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