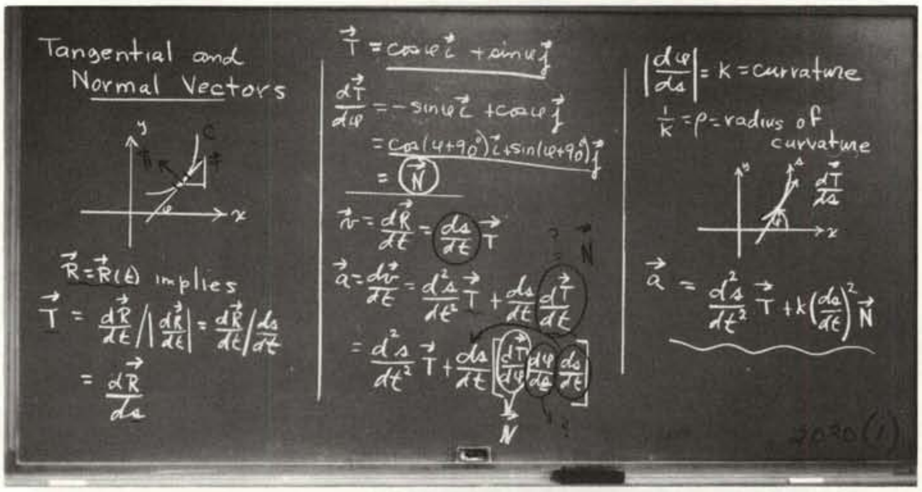
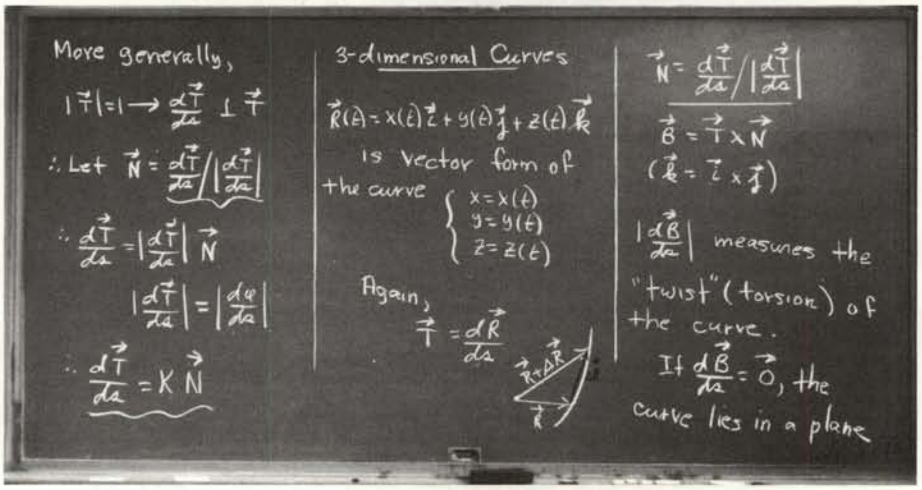
Study Guide
Block 2: Vector Calculus

Unit 2: Tangential and Normal Vectors

1. Lecture 2.020



a.



b.

2. Read Thomas, Sections 14.3, 14.4, 14.5 (Note: if you are not too familiar with these vectors, you may find it helpful to view the lecture a second time, after you have read the appropriate sections of Thomas.)

3. Exercises:

2.2.1(L)

A particle moves in the plane according to the equation of motion:

$$\vec{R} = t\,\vec{i} + \tfrac{1}{3}(t^2 + 2)^{3/2}\vec{j}.$$

a. Find the tangent vector to the curve at any point (x(t),y(t)).

b. How far does the particle travel between t = 0 and t = 6?

2.2.2

A particle moves according to the equation of motion:

$$\vec{R} = \frac{t^2}{2}\vec{i} + \tfrac{1}{3}(2t + 1)^{3/2}\vec{j}.$$

a. Find the position, velocity, speed, and acceleration of the particle at t = 4.

b. How far does the particle travel between t = 0 and t = 4?

c. Find a unit tangent vector to the curve t = 4.

2.2.3(L)

a. Show that if $\vec{v} = \frac{ds}{dt}\vec{T}$, then the acceleration, $\vec{a}$, is given by

$$\vec{a} = \frac{d^2s}{dt^2}\vec{T} + \kappa\left(\frac{ds}{dt}\right)^2\vec{N} \text{ where } \vec{T}\cdot\vec{N} = 0 \text{ and } \kappa = \left|\frac{d\vec{T}}{ds}\right| .$$

b. Use the expressions for $\vec{a}$ and $\vec{v}$ in part a. to find an expression for $\vec{v} \times \vec{a}$. From this deduce an expression for κ in terms of $\vec{v}$ and $\vec{a}$.

c. Use b. to find the curvature of the path followed by the particle which moves according to the equation in Exercise 2.2.2 at t = 4.

2.2.4

Use part a. to Exercise 2.2.3 to show that the acceleration of a particle is always normal to its path of motion <u>if and only if</u> its speed is constant.

2.2.5(L)

Show that if the equation of a curve is given in the Cartesian form $y = f(x)$, the curvature, κ, as defined in Exercise 2.2.3 a. can be computed from the "recipe"

$$\kappa = \left| \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} \right| .$$

2.2.6

a. Use the result of Exercise 2.2.5 to find the curvature of $y = e^{2x}$ at the point (0,1).

b. If a particle moves according to the equation $\vec{R} = t\,\vec{i} + e^{2t}\,\vec{j}$, its path of motion in Cartesian coordinates is $y = e^{2x}$ (this should be easily checked by the reader). Use this fact together with the result of Exercise 2.2.3 b. to solve part a. by a different method.

2.2.7

a. Find the curvature of $y = ax + b$, where a and b are given constants.

b. Find the curvature of $y = \sqrt{a^2 - x^2}$ where a is a positive constant.

2.2.8(L)

A particle moves according to the equation

$$\vec{R} = t\,\vec{i} + (t^2 + 1)\,\vec{j}.$$

a. Find its normal and tangential components of acceleration at any time t.

b. Find the curvature of the path of motion both from the results of part a. and from Exercise 2.2.3 b.

2.2.9

Compute the tangential and normal components of acceleration of a particle which moves according to the equation, $\vec{R} = t^3 \vec{i} + \sin t \vec{j}$.