For Section 3.3:

Prob. 3.3.1 In writing Eq. 3.2.3, the inertia of the charge carriers is ignored. Add inertial terms to the equations, assume that the magnetic field is zero and consider an imposed electric field $\vec{E} = \mathrm{Re}\,\hat{E}\exp(j\omega t)$. Show that the effects of inertia are negligible if $\omega << \nu_{\pm}$. For copper, the electron mobility is about 3×10^{-3} m^2/volt sec, while $q_-/m_- = 1.76 \times 10^{11}$ m^2/sec^2 volt. What must the frequency be to make the electron inertia significant?

For Section 3.5:

Prob. 3.5.1 For the system of Probs. 2.11.1 and 2.13.1,

(a) Show that the reciprocity condition requires that $C_{21} = C_{12}$.

(b) Find the electrical forces (f_1,f_2) in terms of (v_1,v_2,ξ_1,ξ_2) that tend to displace the movable plate in the directions $(\xi_1,\xi_2,)$ respectively.

Prob. 3.5.2 In Fig. 3.6.1, a dielectric slab is pictured as being pulled upward between plane parallel electrodes from a dielectric fluid having the same permittivity as the slab.

(a) What is the total coenergy, $w'(v,\xi)$? (Ignore fringing fields.)

(b) Use the force-energy relation, Eq. 3.5.9, to find the polarization force tending to make the slab rise.

Prob. 3.5.3 Determine the electrical force tending to increase the displacement ξ of the saturable dielectric material of Prob. 2.13.2.

Prob. 3.5.4 For the MQS configuration described in Probs. 2.12.1 and 2.14.1,

(a) Find the radial surface force density T_r by using the coenergy function to obtain $T_r(i_1,i_2,\xi)$.

(b) Compare the operations necessary to obtain $T_r(\lambda_1,\lambda_2,\xi)$ using the energy function w to those using w'. Even though the coenergy formulation is more convenient for this problem, the energy function is more convenient if one or more flux linkages are constrained.

(c) If the inner coil is shorted at a time when its flux linkage is $\lambda_2 = 0$, what is $T_r(\lambda_1,\xi)$?

For Section 3.6:

Prob. 3.6.1 In a fluid at rest, external force densities are held in equilibrium by the gradient of the fluid pressure p. Hence, force equilibrium for each incremental volume of the fluid subject to a force density $\vec{F}$ is represented by

$$\nabla p = \vec{F}$$

Suppose that the bottom of the dielectric slab pictured in Fig. 3.6.1 is well above the lower edges of the electrodes, so that the fringing field, and hence the ∇E^2, is confined to the liquid dielectric. Then there is no Kelvin force density acting on the slab, and the force density of Eq. 3.6.7 prevails in the liquid. Use Eq. 3.6.7 in Eq. 3.6.1 and integrate from the exterior free surface to the bottom of the slab to find the fluid pressure acting on the bottom of the slab. Show that this pressure, acting over the bottom of the slab, gives a net upward force that is consistent with the result of Prob. 3.5.2.

Prob. 3.6.2 Use arguments similar to those leading to Eq. 3.6.4 to show that the torque on an electric dipole is

$$\vec{\tau} = \vec{P} \times \vec{E}$$

Based on arguments similar to those used in deducing Eq. 3.6.12 from Eq. 3.6.5, argue that the torque on a magnetic dipole is

$$\vec{\tau} = \mu_o \vec{m} \times \vec{H}$$

For Section 3.7:

Prob. 3.7.1 Show that the last paragraph in Sec. 3.7 is correct.

For Section 3.9:

Prob. 3.9.1 One way to show that Eq. 3.9.17 can be used to compute $\vec{\tau}$ is to write Eq. 3.9.16 in Cartesian coordinates and use the symmetry of the stress tensor to bring the components of $\vec{r}$ inside the spatial derivatives. Carry out these steps and then use the tensor form of Gauss' theorem to obtain Eq. 3.9.17.

For Section 3.10:

Prob. 3.10.1 For certain purposes, the electric force density in an incompressible liquid with no free charge density might be represented as

$$\vec{F} = \frac{1}{2}\varepsilon\nabla(\vec{E}\cdot\vec{E})$$

where ε is a function of the spatial coordinates. Show that this differs from Eq. 3.7.22 by the gradient of a pressure and that the accompanying stress components are

$$T_{ij} = \varepsilon E_i E_j$$

Prob. 3.10.2 A fluid has the electrical constitutive law

$$\vec{D} = \alpha_1\vec{E} + \alpha_2(\vec{E}\cdot\vec{E})\vec{E}$$

It is inhomogeneous, so that α_1 and α_2 are functions of the spatial coordinates. There is no free charge density and the fluid can be assumed incompressible. Integrate the conservation of coenergy equations to show that the coenergy density is

$$W' = \frac{1}{2}\alpha_1\vec{E}\cdot\vec{E} + \frac{\alpha_2}{4}(\vec{E}\cdot\vec{E})^2$$

Find the force density $\vec{F}$ in terms of $\vec{E}$, α_1 and α_2. Find the stress tensor T_{ij} associated with this force density. Prove that $\vec{F}$ can be written in the form $\vec{F} = \vec{P}\cdot\nabla\vec{E} + \nabla\pi$, where $\vec{P}$ is the polarization density.

Prob. 3.10.1 For certain purposes, the electrical force density in an incompressible liquid with no free charge density might be represented as

$$\vec{F} = \frac{1}{2}\varepsilon\nabla(\vec{E}\cdot\vec{E})$$

where ε is a function of the spatial coordinates. Show that this differs from Eq. 3.7.22 by the gradient of a pressure, and that the accompanying stress components are

$$T_{ij} = \varepsilon E_i E_j$$

Prob. 3.10.2 A fluid has the electrical constitutive law

$$\vec{D} = (\varepsilon_o + \alpha_1)\vec{E} + \alpha_2(\vec{E}\cdot\vec{E})\vec{E}$$

It is inhomogeneous, so that α_1 and α_2 are functions of the spatial coordinates. There is no free charge density and the fluid can be assumed incompressible. Integrate the conservation of coenergy equations to show that the coenergy density is

$$W' = \frac{1}{2}(\varepsilon_o + \alpha_1)\vec{E}\cdot\vec{E} + \frac{\alpha_2}{4}(\vec{E}\cdot\vec{E})^2$$

Find the force density $\vec{F}$ in terms of $\vec{E}$, α_1 and α_2. Find the stress tensor T_{ij} associated with this force density. Prove that $\vec{F}$ can be written in the form

$$\vec{F} = \vec{P}\cdot\nabla\vec{E} + \nabla\pi$$

where $\vec{P}$ is the polarization density.

Prob. 3.10.3 Fig. P3.10.3 shows a circular cylindrical tube of inner radius a into which a second tube of outer radius b projects half way. On top of this inner tube is a "blob" of liquid metal (shown inside the broken-line box) having an arbitrary shape, but having a base radius equal to that of the inner tube. The outer and inner tubes, as well as the blob, are all essentially perfectly conducting on the time scale of interest. When $t=0^-$, there are no magnetic fields. When $t=0^+$, the outer tube is used to produce a magnetic flux which has density $B_o\vec{i}_z$ a distance $\ell >> a$ above the end of the inner tube. What is the magnetic flux density over the cross section of the annulus between tubes a distance ℓ ($\ell >> a$) below the end of the inner tube? Sketch the distribution of surface current on the perfect conductors (outer and inner tubes and blob), indicating the relative densities. Use qualitative arguments to state whether the vertical magnetic force on the blob acts upward or downward. Use the stress tensor to find the magnetic force acting on the blob in the z direction. This expression should be exact if $\ell >> a$, and be written in terms of a, b, B_o and the permeability of free space μ_o.

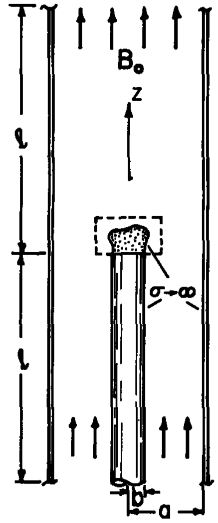

Fig. P3.10.3

Prob. 3.10.4 The mechanical configuration is as in Prob. 3.10.3. But, instead of the magnetic field, an electric field is produced by making the outer cylinder have the potential V_o relative to the inner one. Sketch the distribution of the electric field, and give qualitative arguments as to whether the electrical force on the blob is upward or downward. What is the electric field in the annulus at points well removed from the tip of the inner cylinder? Use the electric stress tensor to determine the z-directed electric force on the blob.

Prob. 3.10.5 In an EQS system with polarization, the force density is not $\vec{F} = \rho_p\vec{E} + \rho_f\vec{E}$, where ρ_p is the polarization charge. Nevertheless, this force density can be used to correctly determine the total force on an object isolated in free space. The proof follows from the argument given in the paragraph following Eq. 3.10.4. Show that the stress tensor associated with this force density is

$$T_{ij} = \varepsilon_o E_i E_j - \frac{1}{2}\delta_{ij}\varepsilon_o E_k E_k$$

Show that the predicted total force will agree with that found by any of the force densities in Table 3.10.1.

Prob. 3.10.6 Given the force density of Eq. 3.8.13, show that the stress tensor given for this force density in Table 3.10.1 is correct. It proves helpful to first show that

$$[(\nabla\times\vec{H}) \times \vec{B}]_i = \left(\frac{\partial H_i}{\partial x_j} - \frac{\partial H_j}{\partial x_i}\right) B_j$$

Prob. 3.10.7 Given the Kelvin force density, Eq. 3.5.12, derive the consistent stress tensor of Table 3.10.1. Note the vector identity given in Prob. 3.10.6.

Prob. 3.10.8 Total forces on objects can sometimes be found by the energy method "ignoring" fringing fields and yet obtaining results that are "exact." This is because the change in total energy caused by a virtual displacement leaves the fringing field unaltered. There is a "theorem" than any configuration that can be described in this way by an energy method can also be described by integrating the stress tensor over an appropriately defined surface. Use Eqs. 3.7.22 of Table 3.10.1 to find the force derived in Prob. 2.13.2.
3.7.19

For Section 3.11:

Prob. 3.11.1 An alternative to the derivation represented by Eq. 3.11.7 comes from exploiting an integral theorem that is analogous to Stokes's theorem.[1]

1. C. E. Weatherburn, Advanced Vector Analysis, G. Bell and Sons, Ltd., London, 1966, p. 126.

Prob. 3.11.1 (continued)

$$\oint_C \vec{V} \times d\vec{\ell} = \int_S [\vec{n}\nabla\cdot\vec{V} - \vec{n}\cdot(\vec{\nabla}V)]da \tag{1}$$

Here $\vec{\nabla}V$ is a dyadic operator defined in Cartesian coordinates such that, "premultiplied" by $\vec{n}$, it has the components

$$[n_x \; n_y \; n_z] \begin{bmatrix} \frac{\partial V_x}{\partial x} & \frac{\partial V_x}{\partial y} & \frac{\partial V_x}{\partial z} \\ \frac{\partial V_y}{\partial x} & \frac{\partial V_y}{\partial y} & \frac{\partial V_y}{\partial z} \\ \frac{\partial V_z}{\partial x} & \frac{\partial V_z}{\partial y} & \frac{\partial V_z}{\partial z} \end{bmatrix} \tag{2}$$

Hence,

$$\begin{aligned} \vec{n}\cdot\vec{\nabla}V = \vec{i}_x &\left[n_x \frac{\partial V_x}{\partial x} + n_y \frac{\partial V_y}{\partial x} + n_x \frac{\partial V_z}{\partial x}\right] \\ \vec{i}_y &\left[n_x \frac{\partial V_x}{\partial y} + n_y \frac{\partial V_z}{\partial y} + n_z \frac{\partial V_z}{\partial y}\right] \\ \vec{i}_z &\left[n_x \frac{\partial V_x}{\partial z} + n_y \frac{\partial V_y}{\partial z} + n_z \frac{\partial V_z}{\partial z}\right] \end{aligned} \tag{3}$$

Show that if $\vec{V} = \gamma_E\vec{n}$, it follows that

$$-\oint_C \gamma_E\vec{n} \times d\vec{\ell} = \int_S [-\vec{n}\gamma_E(\nabla\cdot\vec{n}) - \vec{n}(\vec{n}\cdot\nabla\gamma_E) + \nabla\gamma_E]da \tag{4}$$

Thus if it is recognized that

$$\vec{n}\gamma_E \; \nabla\cdot\vec{n} = \vec{n}\gamma_E(\frac{1}{R_1} + \frac{1}{R_2})$$

(see Sec. 7.6) and that

$$\nabla_\Sigma\gamma_E \equiv \nabla\gamma_E - \vec{n}(\vec{n}\cdot\nabla\gamma_E)$$

then Eq. 3.11.7 follows.

Prob. 3.11.2 A force density is concentrated in interfacial regions where it can be represented by a surface force density $\vec{T}$. The total force on any material supporting this surface force density is then found by integrating the surface force density over the surface upon which it acts:

$$\vec{f} = \int_S \vec{T}\, da \tag{1}$$

Suppose that the surface S is closed and that the external stress contributions to the surface force density are negligible, so that it is given by the second and third terms in Eq. 3.11.8. Use the integral theorem given in Prob. 3.11.1 to show that the resulting net force is zero.