

| <i>Cartesian</i> |   | <i>Cylindrical</i>                                   |   | <i>Spherical</i>   |
|------------------|---|--|---|--|
| $x$              | = | $r \cos \phi$  | = | $r \sin \theta \cos \phi$  |
| $y$              | = | $r \sin \phi$  | = | $r \sin \theta \sin \phi$  |
| $z$              | = | $z$  | = | $r \cos \theta$  |
| $\mathbf{i}_x$   | = | $\cos \phi \mathbf{i}_r - \sin \phi \mathbf{i}_\phi$ | = | $\sin \theta \cos \phi \mathbf{i}_r + \cos \theta \cos \phi \mathbf{i}_\theta - \sin \phi \mathbf{i}_\phi$ |
| $\mathbf{i}_y$   | = | $\sin \phi \mathbf{i}_r + \cos \phi \mathbf{i}_\phi$ | = | $\sin \theta \sin \phi \mathbf{i}_r + \cos \theta \sin \phi \mathbf{i}_\theta + \cos \phi \mathbf{i}_\phi$ |
| $\mathbf{i}_z$   | = | $\mathbf{i}_z$                                       | = | $\cos \theta \mathbf{i}_r - \sin \theta \mathbf{i}_\theta$   |

| <i>Cylindrical</i> |   | <i>Cartesian</i>                                   |   | <i>Spherical</i>   |
|--------------------|---|--|---|--|
| $r$                | = | $\sqrt{x^2 + y^2}$                                 | = | $r \sin \theta$  |
| $\phi$             | = | $\tan^{-1} y/x$                                    | = | $\phi$   |
| $z$                | = | $z$  | = | $r \cos \theta$  |
| $\mathbf{i}_r$     | = | $\cos \phi \mathbf{i}_x + \sin \phi \mathbf{i}_y$  | = | $\sin \theta \mathbf{i}_r + \cos \theta \mathbf{i}_\theta$ |
| $\mathbf{i}_\phi$  | = | $-\sin \phi \mathbf{i}_x + \cos \phi \mathbf{i}_y$ | = | $\mathbf{i}_\phi$  |
| $\mathbf{i}_z$     | = | $\mathbf{i}_z$                                     | = | $\cos \theta \mathbf{i}_r - \sin \theta \mathbf{i}_\theta$ |

| <i>Spherical</i>    |   | <i>Cartesian</i>   |   | <i>Cylindrical</i>                                    |
|---------------------|---|--|---|---|
| $r$                 | = | $\sqrt{x^2 + y^2 + z^2}$   | = | $\sqrt{r^2 + z^2}$                                    |
| $\theta$            | = | $\cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}$   | = | $\cos^{-1} \frac{z}{\sqrt{r^2 + z^2}}$                |
| $\phi$              | = | $\cot^{-1} x/y$  | = | $\phi$  |
| $\mathbf{i}_r$      | = | $\sin \theta \cos \phi \mathbf{i}_x + \sin \theta \sin \phi \mathbf{i}_y + \cos \theta \mathbf{i}_z$ | = | $\sin \theta \mathbf{i}_r + \cos \theta \mathbf{i}_z$ |
| $\mathbf{i}_\theta$ | = | $\cos \theta \cos \phi \mathbf{i}_x + \cos \theta \sin \phi \mathbf{i}_y - \sin \theta \mathbf{i}_z$ | = | $\cos \theta \mathbf{i}_r - \sin \theta \mathbf{i}_z$ |
| $\mathbf{i}_\phi$   | = | $-\sin \phi \mathbf{i}_x + \cos \phi \mathbf{i}_y$   | = | $\mathbf{i}_\phi$                                     |

Geometric relations between coordinates and unit vectors for Cartesian, cylindrical, and spherical coordinate systems.



*Cartesian Coordinates* ( $x, y, z$ )

$$\nabla f = \frac{\partial f}{\partial x} \mathbf{i}_x + \frac{\partial f}{\partial y} \mathbf{i}_y + \frac{\partial f}{\partial z} \mathbf{i}_z$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \mathbf{i}_x \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \mathbf{i}_y \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \mathbf{i}_z \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

*Cylindrical Coordinates* ( $r, \phi, z$ )

$$\nabla f = \frac{\partial f}{\partial r} \mathbf{i}_r + \frac{1}{r} \frac{\partial f}{\partial \phi} \mathbf{i}_\phi + \frac{\partial f}{\partial z} \mathbf{i}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \mathbf{i}_r \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \mathbf{i}_\phi \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \mathbf{i}_z \frac{1}{r} \left[ \frac{\partial (r A_\phi)}{\partial r} - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

*Spherical Coordinates* ( $r, \theta, \phi$ )

$$\nabla f = \frac{\partial f}{\partial r} \mathbf{i}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \mathbf{i}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \mathbf{i}_\phi$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta A_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\begin{aligned} \nabla \times \mathbf{A} = & \mathbf{i}_r \frac{1}{r \sin \theta} \left[ \frac{\partial (\sin \theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] \\ & + \mathbf{i}_\theta \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial r} \right] + \mathbf{i}_\phi \frac{1}{r} \left[ \frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \end{aligned}$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$



## MAXWELL'S EQUATIONS

| <i>Integral</i>  | <i>Differential</i>  | <i>Boundary Conditions</i>  |
|--|--|---|
| <b>Faraday's Law</b>   |  |   |
| $\oint_L \mathbf{E}' \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$  | $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$               | $\mathbf{n} \times (\mathbf{E}'_2 - \mathbf{E}'_1) = 0.$                                    |
| <b>Ampere's Law with Maxwell's Displacement Current Correction</b>   |  |   |
| $\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J}_f \cdot d\mathbf{S} + \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{S}$                | $\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$ | $\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{K}_f$                            |
| <b>Gauss's Law</b>   |  |   |
| $\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho_f dV$  | $\nabla \cdot \mathbf{D} = \rho_f$   | $\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \sigma_f$                                 |
| $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$   | $\nabla \cdot \mathbf{B} = 0$  | $\mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0$  |
| <b>Conservation of Charge</b>  |  |   |
| $\oint_S \mathbf{J}_f \cdot d\mathbf{S} + \frac{d}{dt} \int_V \rho_f dV = 0$   | $\nabla \cdot \mathbf{J}_f + \frac{\partial \rho_f}{\partial t} = 0$               | $\mathbf{n} \cdot (\mathbf{J}_2 - \mathbf{J}_1) + \frac{\partial \sigma_f}{\partial t} = 0$ |
| <b>Usual Linear Constitutive Laws</b>  |  |   |
| $\mathbf{D} = \epsilon \mathbf{E}$   |  |   |
| $\mathbf{B} = \mu \mathbf{H}$  |  |   |
| $\mathbf{J}_f = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \sigma \mathbf{E}'$ [Ohm's law for moving media with velocity $\mathbf{v}$ ] |  |   |

## PHYSICAL CONSTANTS

| Constant                      | Symbol                                     | Value   | units                                |
|-------------------------------|--|---|--------------------------------------|
| Speed of light in vacuum      | $c$  | $2.9979 \times 10^8 \approx 3 \times 10^8$            | m/sec                                |
| Elementary electron charge    | $e$  | $1.602 \times 10^{-19}$                               | coul                                 |
| Electron rest mass            | $m_e$                                      | $9.11 \times 10^{-31}$                                | kg                                   |
| Electron charge to mass ratio | $\frac{e}{m_e}$                            | $1.76 \times 10^{11}$                                 | coul/kg                              |
| Proton rest mass              | $m_p$                                      | $1.67 \times 10^{-27}$                                | kg                                   |
| Boltzmann constant            | $k$  | $1.38 \times 10^{-23}$                                | joule/°K                             |
| Gravitation constant          | $G$  | $6.67 \times 10^{-11}$                                | nt-m <sup>2</sup> /(kg) <sup>2</sup> |
| Acceleration of gravity       | $g$  | 9.807   | m/(sec) <sup>2</sup>                 |
| Permittivity of free space    | $\epsilon_0$                               | $8.854 \times 10^{-12} \approx \frac{10^{-9}}{36\pi}$ | farad/m                              |
| Permeability of free space    | $\mu_0$                                    | $4\pi \times 10^{-7}$                                 | henry/m                              |
| Planck's constant             | $h$  | $6.6256 \times 10^{-34}$                              | joule-sec                            |
| Impedance of free space       | $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ | $376.73 \approx 120\pi$                               | ohms                                 |
| Avogadro's number             | $N$  | $6.023 \times 10^{23}$                                | atoms/mole                           |

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