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ELECTROMECHANICAL DYNAMICS

Part I: Discrete Systems

ELECTROMECHANICAL

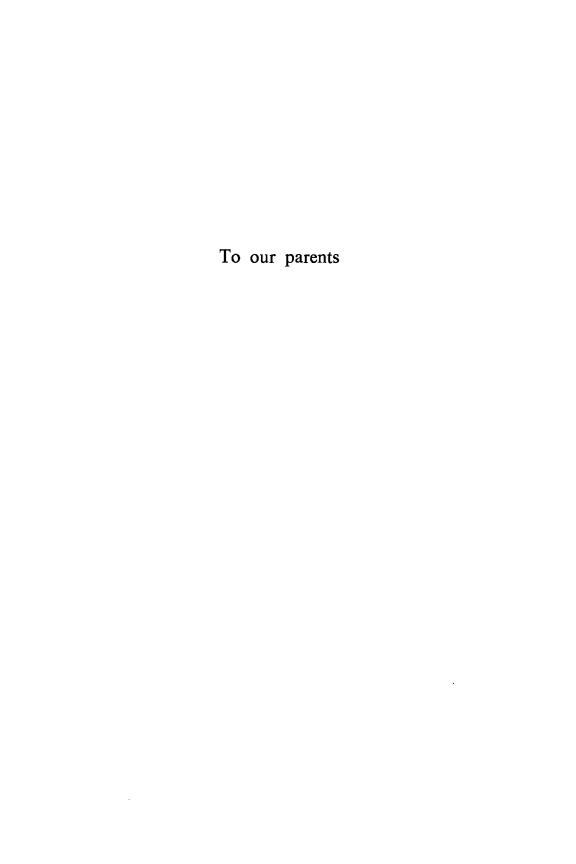# DYNAMICS

Part I: Discrete Systems

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To our parents

PREFACE

Part I: Discrete Systems

In the early 1950's the option structure was abandoned and a common core curriculum was instituted for all electrical engineering students at M.I.T. The objective of the core curriculum was then, and is now, to provide a foundation in mathematics and science on which a student can build in his professional growth, regardless of the many opportunities in electrical engineering from which he may choose. In meeting this objective, core curriculum subjects cannot serve the needs of any professional area with respect to nomenclature, techniques, and problems unique to that area. Specialization comes in elective subjects, graduate study, and professional activities.

To be effective a core curriculum subject must be broad enough to be germane to the many directions an electrical engineer may go professionally, yet it must have adequate depth to be of lasting value. At the same time, the subject must be related to the real world by examples of application. This is true because students learn by seeing material in a familiar context, and engineering students are motivated largely by the relevance of the material to the realities of the world around them.

In the organization of the core curriculum in electrical engineering at M.I.T. electromechanics is one major component. As our core curriculum has evolved, there have been changes in emphasis and a broadening of the topic. The basic text in electromechanics until 1954, when a new departure was made, was *Electric Machinery* by Fitzgerald and Kingsley. This change produced *Electromechanical Energy Conversion* by White and Woodson, which was used until 1961. At that time we started the revision that resulted in the present book. During this period we went through many versions of notes while teaching the material three semesters a year.

Our objective has always been to teach a subject that combines classical mechanics with the fundamentals of electricity and magnetism. Thus the subject offers the opportunity to teach both mechanics and electromagnetic theory in a context vital to much of the electrical engineering community.

Our choice of material was to some extent determined by a desire to give the student a breadth of background sufficient for further study of almost any type of electromechanical interaction, whether in rotating machinery,

plasma dynamics, the electromechanics of biological systems, or magneto-elasticity. It was also chosen to achieve adequate depth while maintaining suitable unity, but, most important, examples were chosen that could be enlivened for the engineering student interested in the interplay of physical reality and the analytical model. There were many examples from which to choose, but only a few satisfied the requirement of being both mathematically lucid *and* physically demonstrable, so that the student could "push it or see it" and directly associate his observations with symbolic models. Among the areas of electrical engineering, electromechanics excels in offering the opportunity to establish that all-important "feel" for a physical phenomenon. Properly selected electromechanical examples can be the basis for discerning phenomena that are remote from human abilities to observe.

Before discussing how the material can be used to achieve these ends, a review of the contents is in order. The student who uses this book is assumed to have a background in electrostatics and magnetostatics. Consequently, Chapter 1 and Appendix B are essentially a review to define our starting point.

Chapter 2 is a generalization of the concepts of inductance and capacitance that are necessary to the treatment of electromechanical systems; it also provides a brief introduction to rigid-body mechanics. This treatment is included because many curricula no longer cover mechanics, other than particle mechanics in freshman physics. The basic ideas of Chapter 2 are repeated in Chapter 3 to establish some properties of electromechanical coupling in lumped-parameter systems and to obtain differential equations that describe the dynamics of lumped-parameter systems.

Next, the techniques of Chapters 2 and 3 are used to study rotating machines in Chapter 4. Physical models are defined, differential equations are written, machine types are classified, and steady-state characteristics are obtained and discussed. A separate chapter on rotating machines has been included not only because of the technological importance of machines but also because rotating machines are rich in examples of the kinds of phenomena that can be found in lumped-parameter electromechanical systems.

Chapter 5 is devoted to the study, with examples, of the dynamic behavior of lumped-parameter systems. Virtually all electromechanical systems are mathematically nonlinear; nonetheless, linear incremental models are useful for studying the stability of equilibria and the nature of the dynamical behavior in the vicinity of an equilibrium. The second half of this chapter develops the classic potential-well motions and loss-dominated dynamics in the context of electromechanics. These studies of nonlinear dynamics afford an opportunity to place linear models in perspective while forming further insights on the physical significance of, for example, flux conservation and state functions.

Chapter 6 represents our first departure from lumped-parameter systems into continuum systems with a discussion of how observers in relative motion will define and measure field quantities and the related effects of material motion on electromagnetic fields. It is our belief that dc rotating machines are most easily understood in this context. Certainly they are a good demonstration of field transformations at work.

As part of any continuum electromechanics problem, one must know how the electric and magnetic fields are influenced by excitations and motion. In quasi-static systems the distribution of charge and current are controlled by magnetic diffusion and charge relaxation, the subjects of Chapter 7. In Chapter 7 simple examples isolate significant cases of magnetic diffusion or charge relaxation, so that the physical processes involved can be better understood.

Chapters 6 and 7 describe the electrical side of a continuum electromechanical system with the material motion predetermined. The mechanical side of the subject is undertaken in Chapter 8 in a study of force densities of electric and magnetic origin. Because it is a useful concept in the analysis of many systems, we introduce the Maxwell stress tensor. The study of useful properties of tensors sets the stage for later use of mechanical stress tensors in elastic and fluid media.

At this point the additional ingredient necessary to the study of continuum electromechanics is the mechanical medium. In Chapter 9 we introduce simple elastic continua—longitudinal motion of a thin rod and transverse motion of wires and membranes. These models are used to study simple continuum mechanical motions (nondispersive waves) as excited electromechanically at boundaries.

Next, in Chapter 10 a string or membrane is coupled on a continuum basis to electric and magnetic fields and the variety of resulting dynamic behavior is studied. The unifying thread of this treatment is the dispersion equation that relates complex frequency ω with complex wavenumber k. Without material convection there can be simple nondispersive waves, cut off or evanescent waves, absolute instabilities, and diffusion waves. The effect of material convection on evanescent waves and oscillations and on wave amplification are topics that make a strong connection with electron beam and plasma dynamics. The method of characteristics is introduced as a convenient tool in the study of wave propagation.

In Chapter 11 the concepts and techniques of Chapters 9 and 10 are extended to three-dimensional systems. Strain displacement and stress-strain relations are introduced, with tensor concepts, and simple electromechanical examples of three-dimensional elasticity are given.

In Chapter 12 we turn to a different mechanical medium, a fluid. We first study electromechanical interactions with inviscid, incompressible

fluids to establish essential phenomena in the simplest context. It is here that we introduce the basic notions of MHD energy conversion that can result when a conducting fluid flows through a transverse magnetic field. We also bring in electric-field interactions with fluids, in which ion drag phenomena are used as an example. In addition to these basically conducting processes, we treat the electromechanical consequences of polarization and magnetization in fluids. We demonstrate how highly conducting fluids immersed in magnetic fields can propagate Alfvén waves.

In Chapter 13 we introduce compressibility to the fluid model. This can have a marked effect on electromechanical behavior, as demonstrated with the MHD conduction machine. With compressibility, a fluid will propagate longitudinal disturbances (acoustic waves). A transverse magnetic field and high electrical conductivity modify these disturbances to magnetoacoustic waves.

Finally, in Chapter 14 we add viscosity to the fluid model and study the consequences in electromechanical interactions with steady flow. Hartmann flow demonstrates the effect of viscosity on the dc magnetohydrodynamic machine.

To be successful a text must have a theme; the material must be inter-related. Our philosophy has been to get into the subject where the student is most comfortable, with lumped-parameter (circuit) concepts. Thus many of the subtle approximations associated with quasi-statics are made naturally, and the student is faced with the implications of what he has assumed only after having become thoroughly familiar with the physical significance and usefulness of his approximations. By the time he reaches Chapter 4 he will have drawn a circle around at least a class of problems in which electro-magnetic fields interact usefully with media in motion.

In dealing with physical and mathematical subjects, as we are here, in which the job is incomplete unless the student sees the physical laws put to work in some kind of physical embodiment, it is necessary for the thread of continuity to be woven into the material in diverse and subtle ways. A number of attempts have been made, to which we can add our early versions of notes, to write texts with one obvious, pedagogically logical basis for evolving the material; for example, it can be recognized that classes of physical phenomena could be grouped according to the differential equation that describes the pertinent dynamics. Thus we could treat magnetic diffusion, diffusion waves on elastic continua, and viscous diffusion waves in one chapter, even though the physical embodiments are entirely different. Alternatively, we could devise a subject limited to certain technological applications or cover superficially a wide range of basically unrelated topics such as "energy conversion" under one heading. This was the preva-lent approach in engineering education a decade or so ago, even at the

undergraduate level. It seems clear to us that organizing material in a teachable and meaningful fashion is far more demanding than this. To confess our own mistakes, our material went originally from the general to the specific; it began with the relativistic form of Maxwell's equations, including the effects of motion, and ended with lumped-parameter devices as special cases. Even if this were a pedagogically tenable approach, which we found it was not, what a bad example to set for students who should be learning to distinguish between the essential and the superfluous! Ideas connected with the propagation of electromagnetic waves (relativistic ideas) must be included in the curriculum, but their connection with media in motion should be made after the student is aware of the first-order issues.

A meaningful presentation to *engineers* must interweave and interrelate mathematical concepts, physical characteristics, the modeling process, and the establishment of a physical "feel" for the world of reality. Our approach is to come to grips with each of these goals as quickly as possible (let the student "get wet" within the first two weeks) and then, while reinforcing what he has learned, continually add something new. Thus, if one looks, he will see the same ideas coming into the flow of material over and over again.

For the organization of this book one should look for many threads of different types. We can list here only a few, in the hope that the subtle reinforcing interplay of mathematical and physical threads will be made evident. Probably the essential theme is Maxwell's equations and the ideas of quasi-statics. The material introduced in Chapter 1 is completely abstract, but it is reinforced in the first few chapters with material that is close to home for the student. By the time he reaches Chapter 10 he will have learned that waves exist which intimately involve electric and magnetic fields that are altogether quasistatic. (This is something that comes as a surprise to many late in life.) Lumped-parameter ideas are based on the integral forms of Maxwell's equations, so that the dynamical effects found with lumped-parameter time constants L/R and RC in Chapter 5 are easily associated with the subjects of magnetic diffusion and charge relaxation. A close tie is made between the "speed voltage" of Chapter 5 and the effects of motion on magnetic fields, as described by field transformations in Chapters 6 to 14. Constant flux dynamics of a lumped coil in Chapter 5 are strongly associated with the dynamics of perfectly conducting continuous media; for example, Alfvén waves in Chapter 12.

Consider another thread of continuity. The book begins with the mathematics of circuit theory. The machines of Chapter 4 are essentially circuits in the sinusoidal steady state. In Chapter 5 we linearize to pursue lumped-parameter ideas of stability and other transient responses and then proceed to nonlinear dynamics, potential-well theory, and other approaches that should form a part of any engineer's mathematical background. By the time

the end of Chapter 10 is reached these ideas will have been carried into the continuum with the addition of tensor concepts, simple cases of the method of characteristics, and eigenvalue theory. The ω-k plot and its implication for all sorts of subjects in modern electrical engineering can be considered as a mathematical or a physical objective. The ideas of stability introduced with ordinary differential equations (exp st) in Chapter 5 evolve into the continuum stability studies of Chapter 10 [exp $j(\omega t - kx)$] and can be regarded as a mathematical or a physical thread in our treatment. We could list many other threads: witness the evolution of energy and thermodynamic notions from Chapters 3 to 5, 5 to 8, and 8 to 13.

We hope that this book is not just one more in the mathematics of electrical engineering or the technical aspects of rotating machines, transducers, delay lines, MHD converters, and so on, but rather that it is the mathematics, the physics, and, most of all, the engineering combined into one.

The material brought together here can be used in a variety of ways. It has been used by Professors C. N. Weygandt and F. D. Ketterer at the University of Pennsylvania for two subjects. The first restricts attention to Chapters 1 to 6 and Appendix B for a course in lumped-parameter electromechanics that both supplants the traditional one on rotating machines in the electrical engineering curriculum and gives the background required for further study in a second term (elective) covering Chapter 7 and beyond. Professors C. D. Hendricks and J. M. Crowley at the University of Illinois have used the material to follow a format that covers up through Chapter 10 in one term but omits much of the material in Chapter 7. Professor W. D. Getty at the University of Michigan has used the material to follow a one-term subject in lumped-parameter electromechanics taught from a different set of notes. Thus he has been able to use the early chapters as a review and to get well into the later chapters in a one-term subject.

At M.I.T. our curriculum seems always to be in a state of change. It is clear that much of the material, Chapters 1 to 10, will be part of our required (core) curriculum for the forseeable future, but the manner in which it is packaged is continually changing. During the fall term, 1967, we covered Chapters 1 to 10 in a one-semester subject taught to juniors and seniors. The material from Chapters 4 and 6 on rotating machines was used selectively, so that students had "a foot solidly in the door" on this important subject but also that the coverage could retain an orientation toward the needs of all the diverse areas found in electrical engineering today. We have found the material useful as the basis for early graduate work and as a starting point in several courses related to electromechanics.

Finally, to those who open this book and then close it with the benediction, "good material but unteachable," we apologize because to them we have not made our point. Perhaps not as presented here, but certainly as it is

represented here, this material is rich in teaching possibilities. The demands on the teacher to see the subject in its total context, especially the related problems that lie between the lines, are significant. We have taught this subject many times to undergraduates, yet each term has been more enjoyable than the last. There are so many ways in which drama can be added to the material, and we do not need to ask the students (bless them) when we have been successful in doing so.

In developing this material we have found lecture demonstrations and demonstration films to be most helpful, both for motivation and for developing understanding. We have learned that when we want a student to see a particular phenomenon it is far better for us to do the experiment and let the student focus his attention on what he should see rather than on the wrong connections and blown fuses that result when he tries to do the experiment himself. The most successful experiments are often the simplest—those that give the student an opportunity to handle the apparatus himself. Every student should "chop up some magnetic field lines" with a copper "axe" or he will never really appreciate the subject. We have also found that some of the more complex demonstrations that are difficult and expensive to store and resurrect each semester come through very well in films. In addition to our own short films, three films have been produced professionally in connection with this material for the National Committee on Electrical Engineering Films, under a grant from the National Science Foundation, by the Education Development Center, Newton, Mass.

Synchronous Machines: Electromechanical Dynamics by H. H. Woodson
Complex Waves I: Propagation, Evanescence and Instability by J. R. Melcher
Complex Waves II: Instability, Convection and Amplification by J. R. Melcher

An additional film is in the early stages of production. Other films that are useful have been produced by the Education Development Center for the National Committee on Fluid Mechanics Films and for the College Physics Film Program. Of particular interest, from the former series, is *Magnetohydrodynamics* by Arthur Shercliff.

A book like this can be produced only with plenty of assistance. We gratefully acknowledge the help we received from many directions and hope we have forgotten no one after seven years of work. First of all we want to acknowledge our students with whom we worked as the material developed. They are the one most essential ingredient in an effort of this sort. Next we want to thank Dr. S. I. Freedman, Professor H. H. Richardson, and Dr. C. V. Smith, Jr., for their assistance in framing worthwhile approaches to several of our key topics. In seven years we have had the help of many able

teachers in presenting this material to students. Their discussions and advice have been most useful. In this category we want particularly to mention Professors H. A. Haus, P. L. Penfield, D. C. White, G. L. Wilson, R. Gallager, and E. Pierson and Doctors J. Reynolds, W. H. Heiser, and A. Kusko. Professor Ketterer, who has taught this material at M.I.T. and the University of Pennsylvania, Professors C. D. Hendricks and J. M. Crowley, who have taught it at M.I.T. and the University of Illinois, and Professor W. D. Getty, who has taught it at M.I.T. and the University of Michigan, have been most generous with their comments. Messrs. Edmund Devitt, John Dressler, and Dr. Kent Edwards have checked the correctness of many of the mathematical treatments. Such a task as typing a manuscript repeatedly is enough to try the patience of anyone. Our young ladies of the keyboard, Miss M. A. Daly, Mrs. D. S. Figgins, Mrs. B. S. Morton, Mrs. E. M. Holmes, and Mrs. M. Mazroff, have been gentle and kind with us.

A lengthy undertaking of this sort can be successful only when it has the backing of a sympathetic administration. This work was started with the helpful support of Professor P. Elias, who was then head of the Department of Electrical Engineering at M.I.T. It was finished with the active encouragement of Professor L. D. Smullin, who is presently head of the Department.

Finally, and most sincerely, we want to acknowledge the perseverance of our families during this effort. Our wives, Blanche S. Woodson and Janet D. Melcher, have been particularly tolerant of the demands of this work.

This book appears in three separately bound, consecutively paged parts that can be used individually or in any combination. Flexibility is ensured by including with each part a complete Table of Contents and Index. In addition, for convenient reference, Parts II and III are supplemented by brief appendices which summarize the relevant material from the preceding chapters. Part I includes Chapters 1 to 6, hence emphasizes lumped-parameter models while developing background in field concepts for further studies.

H. H. Woodson
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Cambridge, Massachusetts
January 1968