PROBLEMS

12.1. The mechanism shown in Fig. 12P.1 is to be used as an electrically driven rocket. An insulating fluid of constant density ρ is compressed by a piston. The fluid is then ejected through a slit (nozzle) with a velocity V_p; because $dD \ll LD$, V_p is approximately a constant, and the system is approximately in a steady-state condition ($\partial/\partial t = 0$):

(a) What is the pressure p at the inside surface of the piston? (Assume that $p = 0$ outside the rocket.)
(b) Under the assumption that $d \ll L$, what is V_p?
(c) What is the total thrust of the rocket in terms of the applied voltage V_o and other constants of the system?

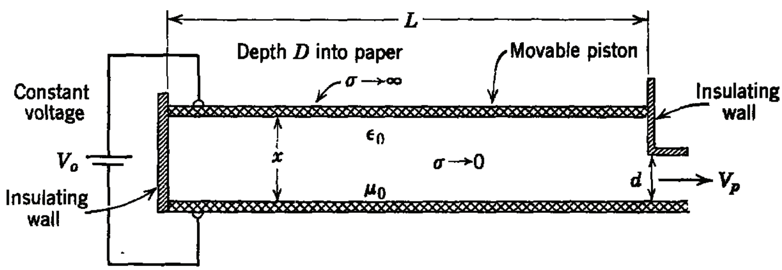
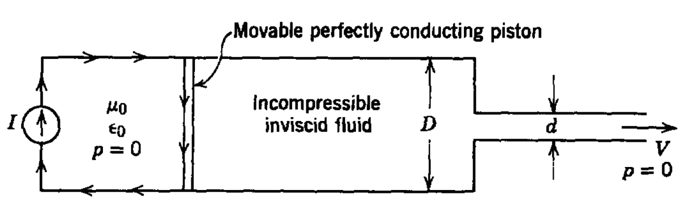


Fig. 12P.1

12.2. A magnetic rocket is shown in Fig. 12P.2. A current source (distributed over the width W) drives a circuit composed in part of a movable piston. This piston drives an incompressible fluid through an orifice of height d and width W. Because $D \gg d$, the flow is essentially steady.

(a) Find the exit velocity V as a function of I.
(b) What is the thrust developed by the rocket? (You may assume that it is under static test.)
(c) Given that $I = 10^3$ A, $d = 0.1$ m, $W = 1$ m, and the fluid is water, what are the numerical values for V and the thrust? Would you prefer to use water or air in your rocket?



The rocket has a depth W into the paper
Fig. 12P.2

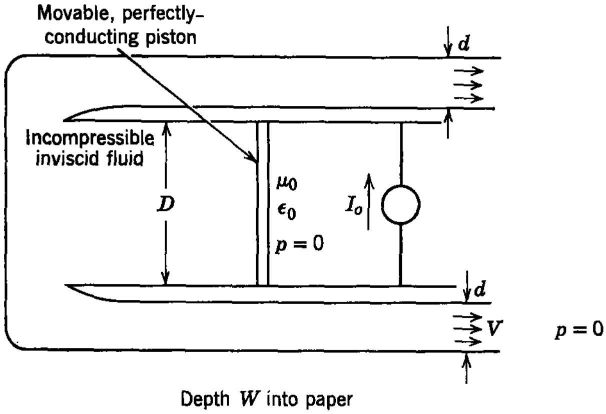


Fig. 12P.3

12.3. A magnetic rocket is shown in Fig. 12P.3. A current source I_o (distributed over the depth W) drives a circuit composed in part of a movable piston. This piston drives an incompressible, inviscid, nonconducting fluid through two orifices, each of height d and depth W. Because $D \gg d$, the flow is essentially steady.

(a) Find the exit velocity V as a function of I_o.

(b) What is the thrust developed by the rocket? (You may assume that it is under static test.)

12.4. An incompressible, inviscid fluid of density ρ flows between two parallel walls as shown in Fig. 12P.4. The bottom wall has a small step of height d in it at $x_1 = 0$. At $x_1 = -L$, the velocity of the fluid is $\mathbf{v} = V_o\mathbf{i}_1$ and the pressure is p_o. Also, at $x_1 = +L$ the velocity of the fluid is uniform with respect to x_2 and is in the x_1-direction, since $d \ll L$. Assuming that the flow is steady ($\partial/\partial t = 0$) and irrotational, find the x_1-component of the force per unit depth on the bottom wall. The system is uniform in the x_3-direction and has the x_2 dimensions shown in Fig. 12P.4.

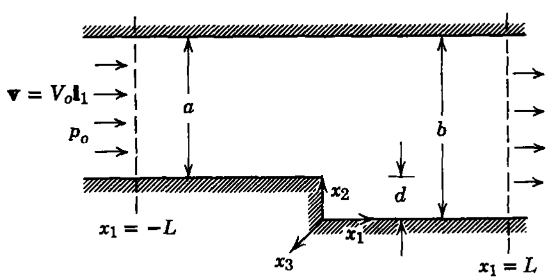


Fig. 12P.4

12.5. Far away from the rigid cylinder shown in Fig. 12P.5 the velocity of a fluid with density ρ is a constant $\mathbf{V} = V_o\mathbf{i}_1$ and its pressure is p_o. Assume that the fluid is incompressible and that the flow is steady and irrotational:

(a) Find the velocity of the fluid everywhere.
(b) Sketch the velocity field.
(c) Find the pressure everywhere.
(d) Use the stress tensor to find the total pressure force (give magnitude and direction) exerted by the fluid on the rigid cylinder. Assume that $\partial/\partial x_3 = 0$.

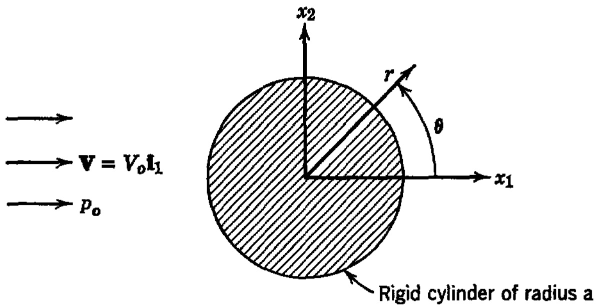


Fig. 12P.5

12.6. An inviscid incompressible fluid flows around a rigid sphere of radius a, as shown in Fig. 12P.6. At $x_1 = \pm\infty$ the fluid velocity becomes $\mathbf{v} = V_o\mathbf{i}_1$.

(a) Compute the velocity distribution $\mathbf{v}(x_1, x_2, x_3)$.
(b) Find the pressure $p(x_1, x_2, x_3)$. [Assume that the pressure is zero at $(x_1, x_2, x_3) = (-a, 0, 0)$.]
(c) Use the results of (b) to compute the force exerted on the sphere in the x_1-direction by the fluid.

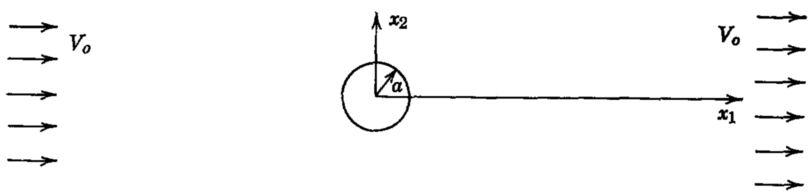


Fig. 12P.6

12.7. The velocity distribution of an inviscid fluid is given as $\mathbf{v} = -\nabla\phi$, where $\phi = (V_o/a)x_1x_2$ and V_o and a are constants.

(a) Show by means of a sketch the direction and magnitude of the velocity in the x_1-x_2 plane.
(b) Compute the fluid acceleration. Sketch the direction and magnitude of the acceleration in the x_1-x_2 plane.
(c) In what physical situation would you expect the flow to have this distribution?

12.8. In the configuration of Fig. 12P.8 an incompressible, inviscid fluid of mass density ρ flows without rotation ($\nabla \times \mathbf{v} = 0$), between two rigid surfaces shown, with velocity

$$\mathbf{v} = \mathbf{i}_1 v_o \frac{x_2}{a} + \mathbf{i}_2 v_o \frac{x_1}{a},$$

where v_o and a are positive constants. Neglect gravity.

(a) Find the fluid acceleration at all points in the flow.

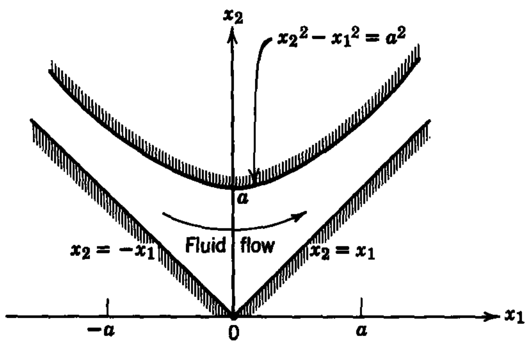


Fig. 12P.8

(b) The pressure is constrained at the origin ($x_1 = x_2 = 0$) to be p_o. Find the pressure at all other points in the fluid.

12.9. Consider the situation of Prob. 12.8, but now with gravity acting in the $-x_2$ direction.

(a) Find the velocity between the rigid walls.
(b) Show that the boundary conditions are satisfied at the walls.

12.10. Figure 12P.10 shows an irrotational flow in a corner formed by a rigid wall.

(a) Let $\mathbf{v} = -\nabla\phi$. What are the boundary conditions on ϕ? Sketch the contour of constant ϕ in the x-y plane.
(b) What function $\phi(x, y)$ satisfies both $\nabla \cdot \mathbf{v} = 0$ and $\nabla \times \mathbf{v} = 0$ and the boundary conditions of part (a)?
(c) Assuming that the pressure $p = p_o$ at $(x, y) = 0$ and that $p = p_o$ to the left and below the wall, what is the force exerted by the fluid on the section of the wall between $x = c$ and $x = d$?
(d) Compute the fluid acceleration. Make a sketch to show the magnitude and direction of the acceleration in the x-y plane.

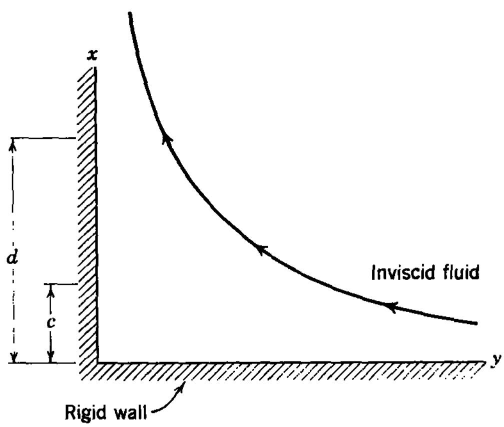


Fig. 12P.10

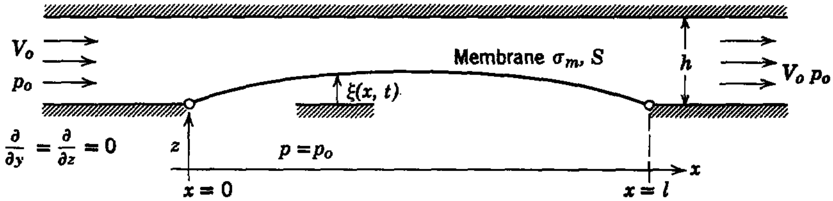


Fig. 12P.11

12.11. An inviscid incompressible fluid enters the channel shown in Fig. 12P.11, with the velocity V_0 and pressure p_0. One wall of the channel includes a section of length l composed of a taut membrane with the deflection ξ.

(a) Assume that spatial variations in the membrane deflection occur slowly so that the velocity v_x is independent of z. Relate $v_x(x)$ to V_0, ξ, and h.

(b) Determine the pressure on the upper surface of the membrane using the fact that $p = p_0$ at the inlet where $v = V_0$.

(c) Find an expression of the form $T_z = C\xi$ for the force per unit area T_z on the membrane as a function of ξ and a constant C. To do this assume that perturbations in ξ are small and use the fact that the pressure below the membrane is p_0.

(d) Now assume that the dynamics occur slowly enough that the result of part (c) will remain true even if $v = v(x, t)$ and $\xi = \xi(x, t)$. The membrane has a tension S and mass per unit area σ_m. For what values of V_0 will the static equilibrium of the membrane at $\xi = 0$ be stable?

(e) Explain physically why the instability of part (d) occurs.

12.12. A perfectly conducting membrane with tension S and mass per unit area σ_m is fixed at $x = 0$ and $x = L$. An inviscid, incompressible fluid with mass density ρ_0 flows underneath the membrane (Fig. 12P.12). An electric field exists in the region above the membrane. The upper region is filled with a light gas that is at atmospheric pressure p_0 everywhere.

(a) Find the value of the pressure p_1 in the fluid for $-d \le y \le 0$ in terms of given parameters if $\xi(x, t) = 0$ is a state of equilibrium.

(b) Under what conditions can a small signal sinusoidal oscillation exist about the equilibrium position $\xi(x, t) = 0$?

Note. The gravitational field affects both the equilibrium and small signal solutions.

(c) Make a dimensioned ω-k plot for a real wavenumber.

(d) Justify the validity of imposing boundary conditions at $x = 0$ and $x = L$ such that these conditions affect the membrane for $0 \le x \le L$.

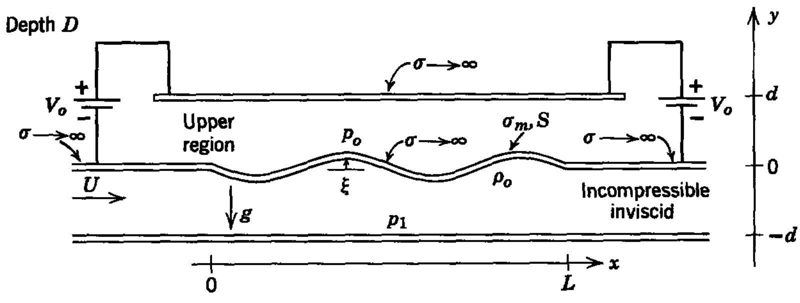


Fig. 12P.12

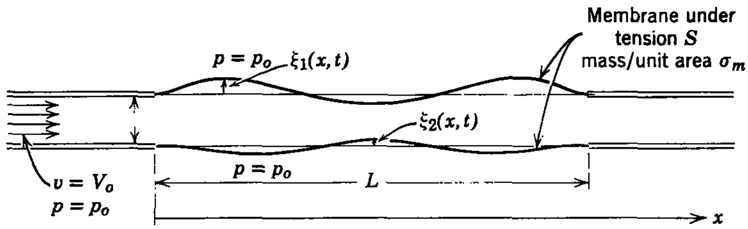


Fig. 12P.13

12.13. A duct is formed by stretching membranes between plane parallel rigid plates, as shown in Fig. 12P.13. An inviscid fluid flows through this duct, entering at the left with velocity V_0. The pressure outside the duct is p_o, so that the membrane can be in static equilibrium with $\xi_1 = \xi_2 = 0$.

(a) What is the largest velocity V_0 that can be used and have the membranes remain in a state of stable equilibrium?

(b) What would the appearance of the membranes be if V_0 were just large enough to make the equilibrium $\xi_1 = \xi_2 = 0$ unstable?

12.14. An inviscid, incompressible fluid rests on a rigid bottom, as shown in Fig. 12P.14. In the absence of any disturbances it is static and has a depth a. If disturbed, the surface of the fluid has the position $\xi(x, t)$. As is obvious to anyone who has observed ocean waves, disturbances of the interface propagate as waves. It is our object here to derive an equation for the propagation of these gravity waves, which have "wavelengths" that are long compared with the depth a of the fluid. To do this we make the following assumptions:

(a) The effects of inertia in the y-direction are negligible. Hence the force equation in the y-direction is

$$\frac{\partial p}{\partial y} = -\rho g.$$

Because $y = \xi(x, t)$ is a free surface, the pressure there is constant (say zero). What is p in terms of y and ξ?

(b) Because the fluid is very shallow, we can assume that $v_x = v_x(x, t)$; that is, the horizontal fluid velocity is independent of y. On the basis of this assumption, use the conservation of mass equation for the incompressible fluid ($\nabla \cdot \mathbf{v} = 0$) to find $v_y(x, y, t)$ in terms of v_x.

(c) Use the result of (a) to write the horizontal component of the force equation as one equation in $v_x(x, t)$ and $\xi(x, t)$.

(d) Use the result of (b) to write an additional equation in ξ and v_x (assume $\xi \ll a$ so that only linear terms need be retained).

(e) Combine equations from parts (c) and (d) to obtain the wave equation for gravity waves. What is the phase velocity of these waves?

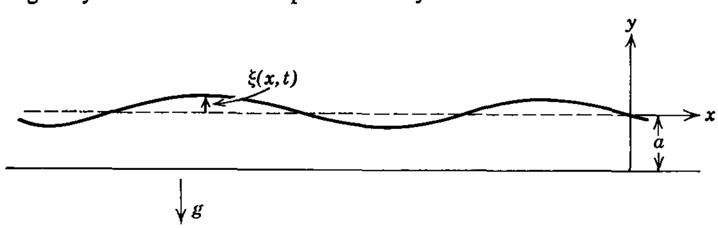


Fig. 12P.14

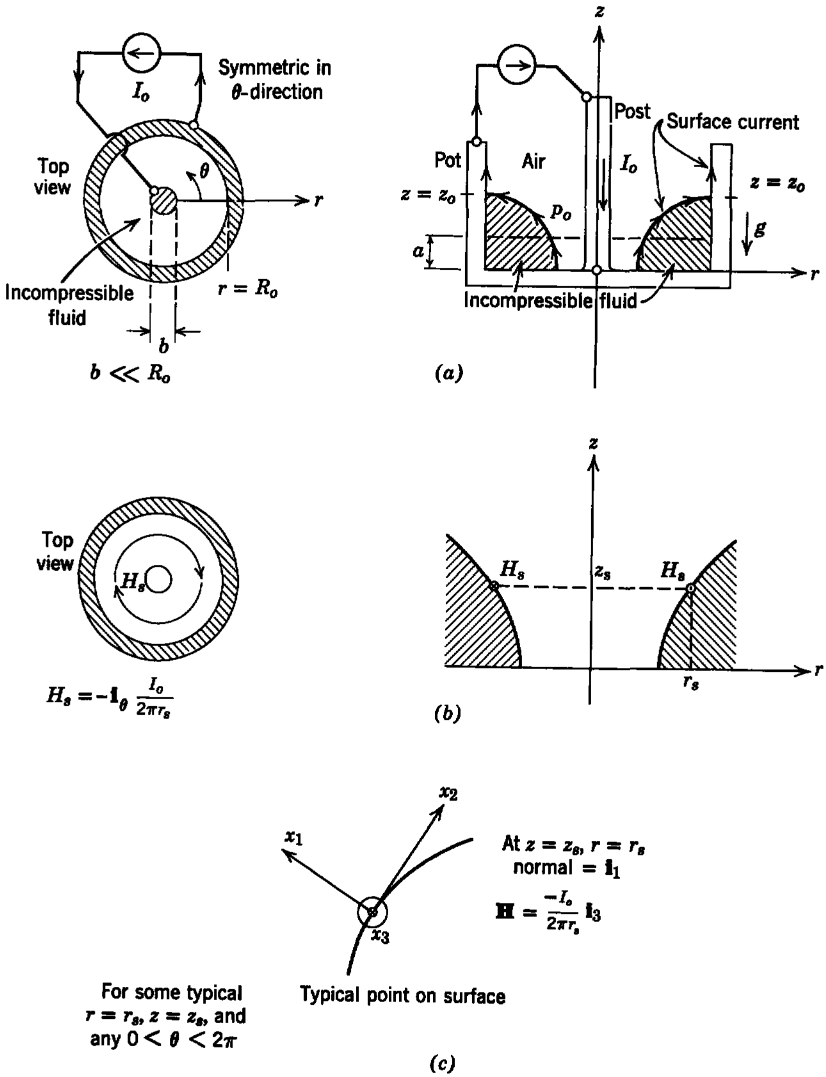


Fig. 12P.15

12.15*. A perfectly conducting, cylindrical pot contains a perfectly conducting fluid. A center coaxial post is placed inside the cylinder. A current source is attached between the center post and the outer wall of the pot to cause a current to flow on the perfectly conducting surfaces, as shown in Fig. 12P.15*a*. When the current source is turned off, the fluid comes to rest with its surface at $x = a$. When the current source is turned on, the magnetic field pressure (normal surface traction) causes the surface to deform (e.g. as shown).

* Colgate, Furth, and Halliday, *Rev. Mod. Phys.*, **32**, No. 4, 744 (1960).

(a) At any typical point on the surface (θ-direction symmetry exists; see Fig. 12P.15*b*), the normal traction on the surface can be found by using the Maxwell Stress Tensor and a coordinate system arranged for the sample point, as shown in Fig. 12P.15*c*. Since the MST result is good for any surface point (r, z), the normal traction is known everywhere as a function of r, the radial position of the surface point. Find the normal traction due to the magnetic field as a function of r.

(b) The hydrostatic pressure of the dense fluid varies appreciably with z, due to gravity, whereas the pressure of the light gas (air) may be assumed to be p_0 (atmospheric) everywhere. Using the fact that the forces acting normal to the fluid-air interface must balance, find an equation for the surface. Neglect surface tension and call the top point on the surface $z = z_0$, $r = R_0$. *Hint.* Remember that at $z = z_0$, $r = R_0$, *the magnetic traction* $+ p_0$ exists on the air side of the interface and is counterbalanced by the hydrostatic pressure on the fluid side of the interface. No magnetic field exists in the fluid; hydrostatic pressure exerts a normal force on the interface.

(c) In part (b) the value of z_0 remains unknown. Because the total mass of the fluid (or volume for an incompressible fluid) must be conserved, *set up* an expression that will determine z_0. Integration need not be carried out.

12.16. The MHD machine for which dimensions and parameters are defined in Fig. 12P.16 can be assumed to operate with incompressible, uniform flow velocity v in the z-direction. The fluid has constant, scalar conductivity. There is a uniform applied flux density B in the

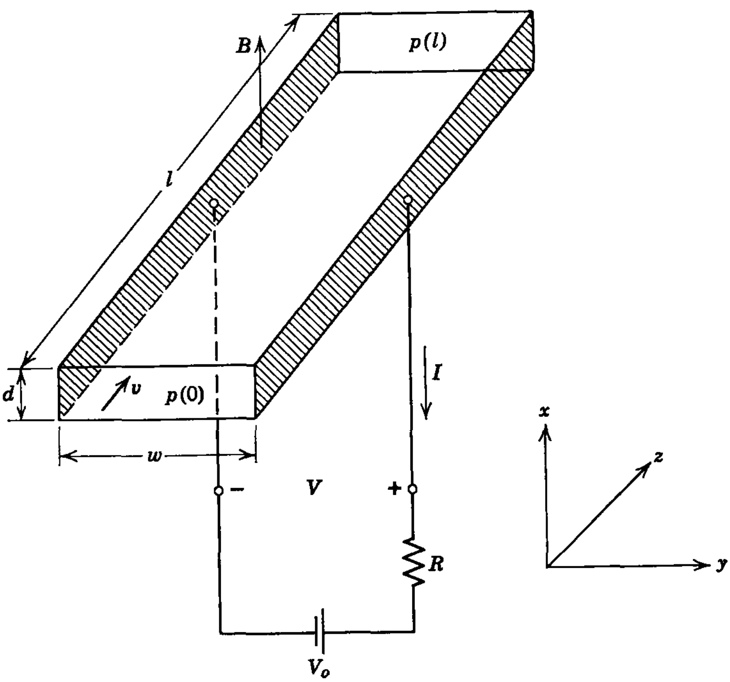


Fig. 12P.16

x-direction and the magnetic field due to current through the fluid may be neglected. The electrical terminals are connected to a battery of constant voltage V_0 and a constant resistance R in series. For steady-state operation calculate and sketch the electric power out of the generator $P_e = VI$ and the mechanical power into the generator $P_m = [p(0) - p(l)]wdv$ as functions of the fluid velocity v. Specify the range of velocity over which the system operates as a generator, pump, and brake.

12.17. From a conformal-mapping analysis of end effects in an MHD generator,* for a generator having

channel width	w,	channel depth	d,
electrode length	l,	uniform velocity	v_0,
fluid conductivity	σ,	flux density	B_0 over length of electrodes,

the electrical output power is

$$P_{\text{out}} = \frac{1}{R_i}(V_{\text{oc}} - V)V - \frac{1}{R_i}aV^2,$$

where

$$R_i = \frac{w}{\sigma l d},$$

$$V_{\text{oc}} = v_0 B_0 w,$$

$$a = \frac{2}{\pi}\left(\frac{w}{l}\right)\ln 2$$

$$V = \text{terminal voltage.}$$

(a) Show that the mechanical power input is given by

$$P_m = \int_0^l \int_0^w dJ_y B_0 \, dy \, dz = \frac{1}{R_i}(V_{\text{oc}} - V)V_{\text{oc}}$$

by direct integration. *Note.* All that is needed for this integration is the facts that $\mathbf{E} = -\nabla\phi$ and the difference in potential between the electrodes is V.

(b) Defining efficiency as $\eta = P_{\text{out}}/P_m$, find the efficiency at maximum power output and the maximum efficiency and plot them as functions of $1/a$ for $0 < 1/a < 10$.

12.18. For the MHD machine with solid electrodes, for which parameters, dimensions, and variables are defined in Fig. 12P.18*a*, assume that the fluid is incompressible, inviscid, and has a constant, scalar conductivity σ. Neglect the magnetic field due to current in the fluid. The source that supplies the pressure $\Delta p = p_i - p_o$ has the linear characteristic $\Delta p = \Delta p_0(1 - v/v_0)$, where Δp_0 and v_0 are positive constants. The machine with this mechanical source can be represented electrically by the equivalent circuit of Fig. 12P.18*b*. Find the open-circuit voltage V_{oc} and the internal resistance R_i' in terms of the given data.

* G. W. Sutton and A. Sherman, *Engineering Magnetohydrodynamics*, McGraw-Hill, New York, 1965, Section 14.6.1.

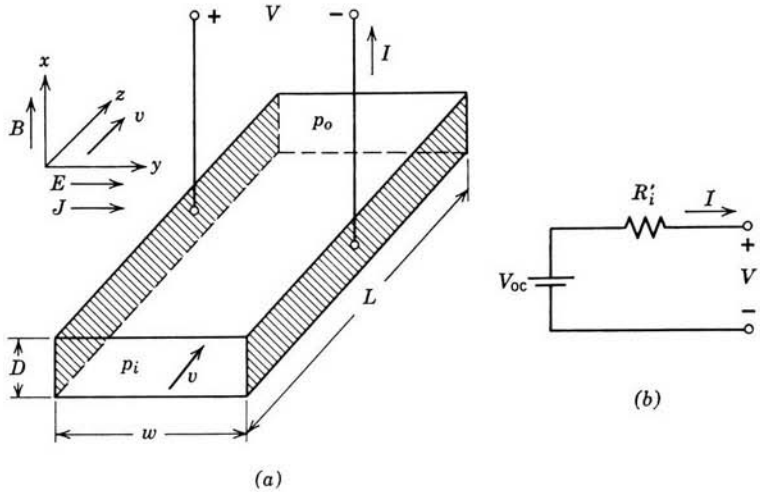


Fig. 12P.18

12.19. An MHD conduction generator has the configuration and dimensions defined in Fig. 12P.19. The fluid is inviscid, has conductivity σ, and is flowing with a uniform, constant velocity v in the x-direction. The field intensity H_o, in the y-direction, is produced by the system shown, which consists of a magnetic yoke with two windings; one (N_o) carries a constant current I_o and the other (N_L) carries the load current I_L. The resistance of winding N_L, fringing effects at the ends and sides of the channel, and the magnetic field due to current in the fluid may be neglected (the magnetic Reynolds number based on length l is small). For steady-state operating conditions find the number of turns N_L necessary to make the terminal voltage V_L independent of load current I_L.

System has length l perpendicular to the paper

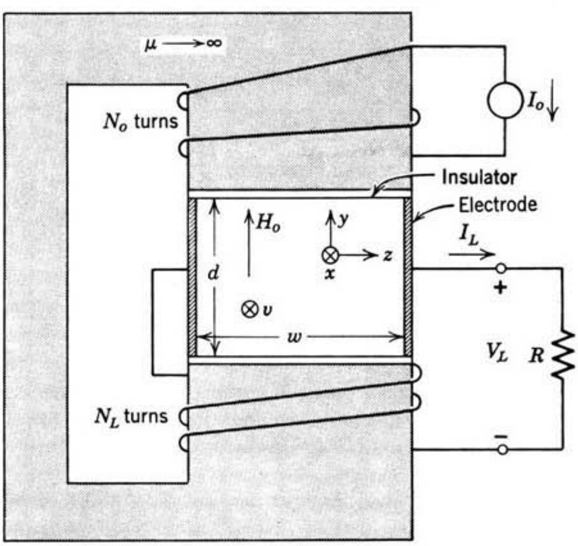


Fig. 12P.19

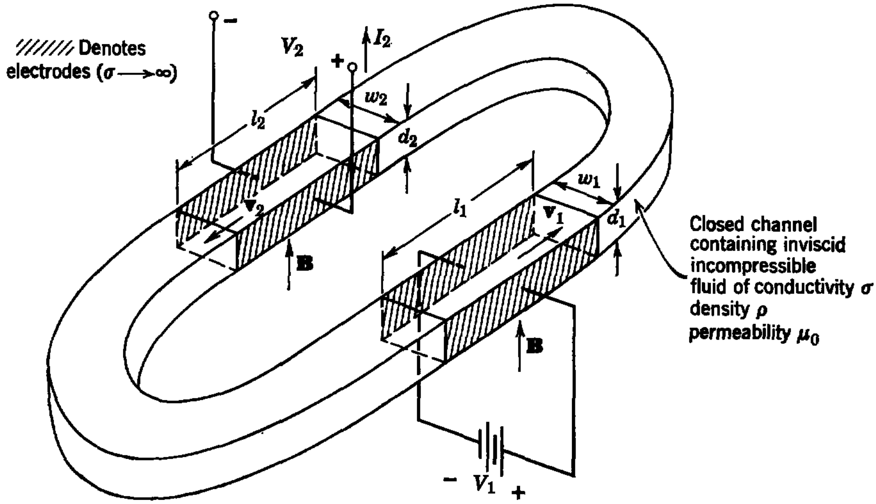


Fig. 12P.20

12.20. A dc transformer is to be made by using a closed channel of incompressible, inviscid fluid of conductivity σ, permeability μ_0 and density ρ, and two MHD conduction machines, as illustrated in Fig. 12P.20. Both machines have the same applied uniform flux density **B**, but their dimensions are different, as indicated. Make the usual assumptions of uniform velocity in the machines, neglect the magnetic fields induced by current in the fluid, and neglect end and edge effects and fringing. For steady-state conditions find a relation between V_2 and I_2 in terms of input voltage V_1, conductivity σ, and the dimensions. Draw the Thevenin equivalent circuit this relation implies.

12.21. A conducting liquid flows with a constant velocity v in the closed channel shown in Fig. 12P.21. The motion is produced by an MHD pump which provides a pressure rise

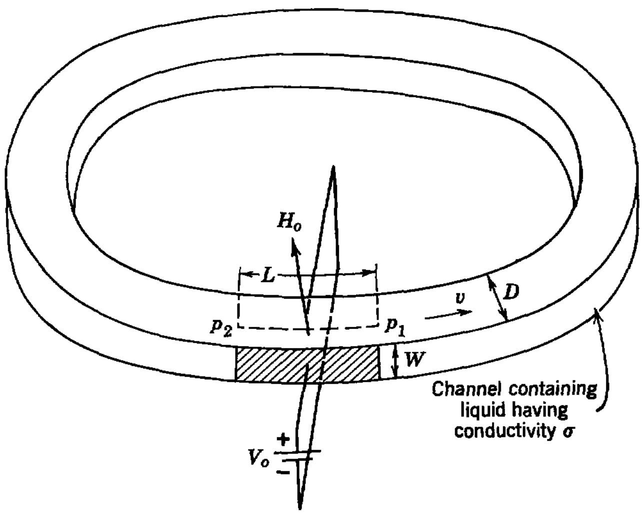


Fig. 12P.21

$p_1 - p_2 > 0$ where p_1 and p_2 are the outlet and inlet pressures. The fluid, as it flows through the remainder of the channel, undergoes the pressure drop $p_1 - p_2 = kv$, where k is a known constant. Determine the velocity v in terms of the imposed magnetic field H_0 and the other constants of the system.

12.22. The rectangular channel with the dimensions shown in Fig. 12P.22 is to be used in an MHD pump for a highly conducting liquid of conductivity σ. The channel has two sides which are perfectly conducting electrodes, and an electrical circuit is connected to them. An

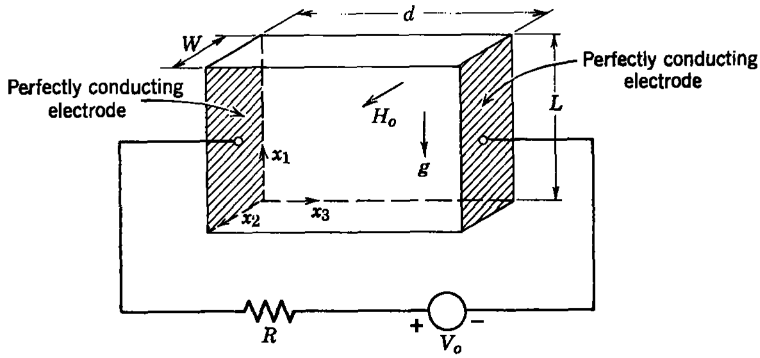


Fig. 12P.22

external magnet produces a constant magnetic field H_0 in the x_2-direction. The device is to pump against a gravitational field (which is in the $(-x_1)$-direction):

(a) What range of values for the voltage source V_0 will make the liquid flow upward in the $+x_1$-direction? Assume that the pressure at $x_1 = 0$ is the same as the pressure at $x_1 = L$.

(b) Under the conditions of part (a) show clearly that the voltage source is supplying power to the liquid.

12.23. Two large reservoirs of water are connected by a large duct, as shown in Fig. 12P.23*a*. Over a length l of this duct the walls are highly conducting electrodes short-circuited together by an external circuit, as shown in Fig. 12P.23*b*. A uniform, constant magnetic field B_0 is imposed perpendicular to the direction of flow. Because the water has a conductivity σ, there is a current through the water between the electrodes. Assume that the reservoirs are so large that h_1 and h_2 remain constant and that the fluid is incompressible and inviscid. What is the velocity v of the fluid between the electrodes?

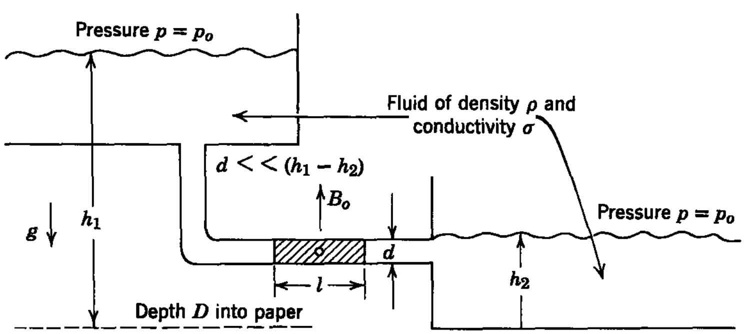


Fig. 12P.23 (*a*)

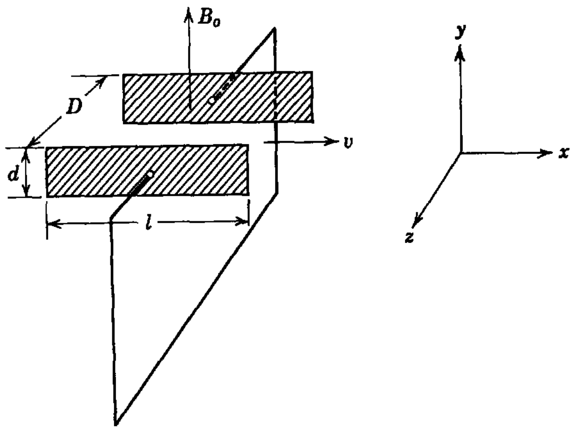


Fig. 12P.23 (*b*)

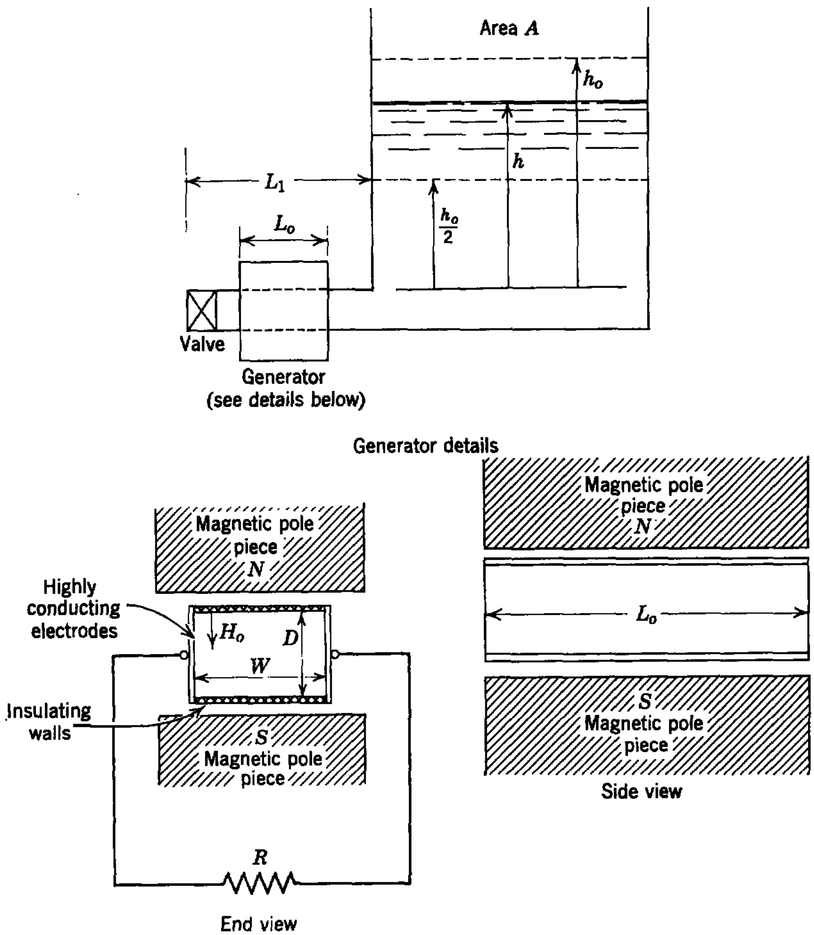


Fig. 12P.24

12.24. The system shown in Fig. 12P.24 consists of a storage tank of horizontal cross-sectional area A, open to the atmosphere at the top, and an MHD generator through which fluid stored in the tank can flow to atmospheric pressure. The generator is loaded by resistance R as shown. The tank is initially filled to a height h_o with mercury. At $t = 0$ the valve is opened to allow mercury to discharge through the generator. When the height has decreased to $h_o/2$ the valve is closed again. Do the following calculations, using the numerical data given below. Make any approximations that are justified by the numerical data. Assume all flow to be incompressible ($\nabla \cdot \mathbf{v} = 0$) and irrotational ($\nabla \times \mathbf{v} = 0$).

(a) Calculate the height h of mercury in the tank as a function of time.
(b) Calculate the current in the load resistance R as a function of time.

Numerical Data

	Mercury	Generator
Density:	1.35×10^4 kg/m^3	$H_o = 1.6 \times 10^5$ A/m
		$D = 0.1$ m
Conductivity:	10^6 mhos/m	$W = 0.2$ m
		$L_o = 1$ m
	Tank Dimensions	$L_1 = 2$ m
	$A = 100$ m^2	$R = 2 \times 10^{-5}\ \Omega$
	$h_o = 10$ m	Acceleration of Gravity
		$g = 9.8$ m/sec^2

12.25. A simple, bulk-coupled MHD system is used to pump mercury from one storage tank to another, as shown in Fig. 12P.25*a*. Figure 12P.25*b* shows the details of the MHD system. The MHD system is driven with a voltage source V_o in series with a resistance R.

Each storage tank has area A and is open to atmospheric pressure at the top. Consider all flow to be incompressible and irrotational ($\nabla \cdot \mathbf{v} = 0$ and $\nabla \times \mathbf{v} = 0$). Use the following numerical data for your solutions.

	Mercury	MHD System
Density:	1.35×10^4 kg/m^3	$H_o = 5 \times 10^5$ A/m
Conductivity:	10^6 mhos/m	$D = 0.01$ m
		$W = 0.02$ m
Tank Area:	$A = 0.1$ m^2	$L_1 = 0.1$ m
Acceleration of Gravity:	$g = 9.8$ m/sec^2	$L_2 = 1.9$ m
		$R = 10^{-5}\ \Omega$

(a) What voltage V_o is required to maintain the levels in the tanks $h_1 = 0.4$ m and $h_2 = 0.5$ m. How much current and power does the voltage source supply in this case?
(b) With the equilibrium conditions of (a) established, the voltage V_o is doubled at $t = 0$. Find $h_2(t)$ and the source current $i(t)$ for $t > 0$. Sketch and label curves of these time functions. To solve this problem exactly it is necessary to know the three-dimensional flow pattern in the tanks. For this problem, however, it is sufficient to make the following approximations. In writing dynamical equations neglect the acceleration of the fluid in the tanks compared with the acceleration of the fluid in the pipe. Be sure to estimate the error caused by making this assumption *after* the solution has been completed. Also neglect the magnetic field due to current in the mercury.

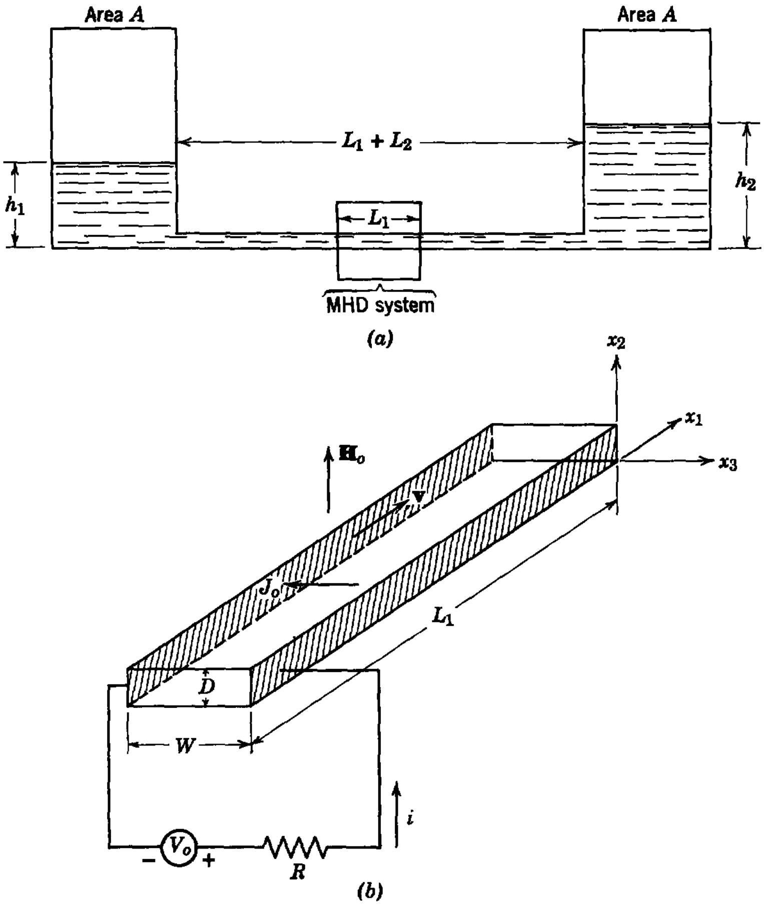


Fig. 12P.25

12.26. Figure 12P.26 shows schematically an ac, series-excited, liquid metal conduction pump. The liquid metal has mass density ρ and electrical conductivity σ. The excitation winding has N turns ($N \gg 1$) and the magnetic path is closed externally by infinitely permeable, nonconducting magnetic material. The dimensions are given in the figure. The electrical terminals are driven by an alternating current source: $i(t) = I \sin \omega t$, where I and ω are positive constants. The pump works against a velocity dependent pressure rise $p(l) - p(0) = \Delta p_0(v/v_0)$, where Δp_0 and v_0 are positive constants. For steady-state operation complete the following:

(a) Find the velocity v as a function of time.
(b) Evaluate the ratio of the amplitudes of the ac and dc components of velocity.

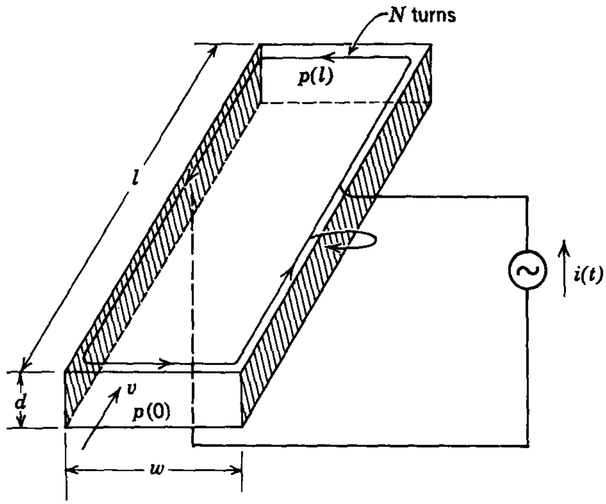


Fig. 12P.26

12.27. A pair of magnetohydrodynamic conduction generators is shown in Fig. 12P.27. In each generator a conducting fluid flows through a channel of width w and height a with approximately uniform constant velocity V. A magnetic field is applied to each generator by means of a magnetic circuit that produces (approximately) a uniform magnetic field in the vertical direction. The magnetic circuits can be considered as having infinite permeability with all the drop in mmf across the channels. Currents are passed through the channels by means of highly conducting electrodes of height a and length b, as shown in Fig. 12P.27. The current i_1 (amperes) through the left generator is used to produce a magnetic field in the left generator and in the right generator, as shown, and to deliver power to the load R_L. The current i_2 through the right generator follows a similar path, except that the turns N and N_m have different directions. We wish to establish the dynamics that would be expected for two generators interconnected in this way, with the objective of producing ac rather than dc power delivered to the loads R_L.

(a) Find the pair of ordinary differential equations in $i_1(t)$ and $i_2(t)$ that defines the system dynamics under the given conditions.

(b) Determine the condition, in terms of the given system parameters, that the generators be stable.

(c) Under what condition will the system operate in the sinusoidal steady state? Given that $R_L = 0$, $\sigma = 50$ mhos/m, $V = 4000$ m/sec, and $N = 1$ turn, what is the length b required to meet this condition?

(d) Compute the frequency at which the system will operate in the sinusoidal steady state under the above conditions, given that $N_m = 1$ turn.

12.28. The system shown in Fig. 12P.28 has been proposed for an ac, self-excited, MHD power generator. It consists of a *single* channel with length l, width D, and height W, through which an incompressible, inviscid, highly conducting liquid with conductivity σ and permeability μ_0 flows. The velocity of the liquid *is constrained* externally to be a *constant* and always in the x_1-direction; that is, $\mathbf{v} = V\mathbf{i}_1$ everywhere in the channel. (The scalar V is a known fixed constant.) The sides of the channel at $x_2 = 0$ and $x_2 = W$ are insulating, but the other two sides, namely those at $x_3 = 0$ and $x_3 = D$, are perfectly conducting electrodes.

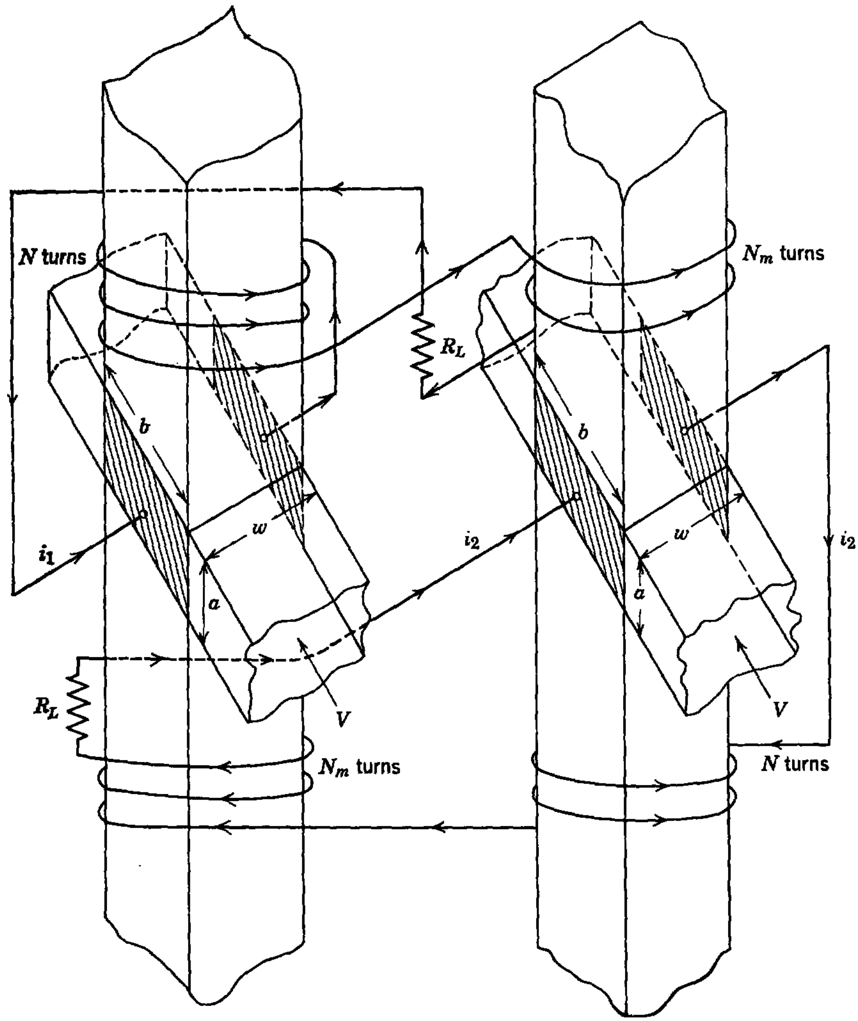


Fig. 12P.27

An external circuit connected to the electrodes consists of three elements in series. The first two elements are a load resistance R_L and a capacitance C. The last element is a *lossless* coil of N turns which is wound on the top ($x_2 = W$) surface of the channel to produce a *uniform* x_2-directed magnetic field everywhere in the channel. (Reasonable assumptions may be made about this field; namely, that it is equivalent to the field produced as if the N turns were distributed uniformly through the depth W of the channel but on the outer perimeter of the channel.)

(a) Find the value of the load resistance R_L that will make the device act as an ac generator (so that the current i is a pure sinusoid). Note that ac power will then be continuously dissipated in the load resistance R_L.

(b) What is the frequency of the resulting sinusoid of part (a)?

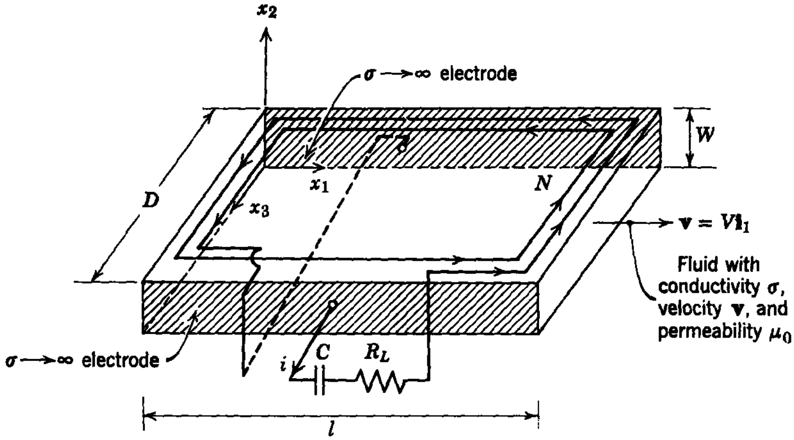


Fig. 12P.28

12.29. The MHD generator of Fig. 12P.29 has a channel of length l, width w, and depth d. An external system not shown establishes the magnetic flux density $\mathbf{B}_0 = \mathbf{i}_x B_0$. Two equal resistances of R Ω each are connected in series between the electrodes, with a switch S in parallel with one resistance. The channel contains an inviscid, incompressible fluid of mass density ρ flowing under the influence of a pressure difference $\Delta p = p_i - p_o$, which is positive and maintained constant by external means. The fluid has an electrical conductivity σ and the magnetic field due to current flow in the fluid can be neglected.

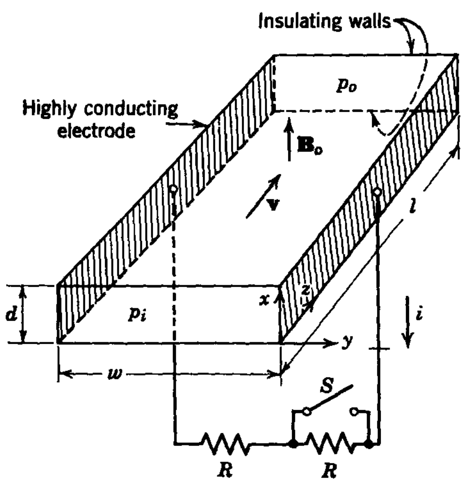


Fig. 12P.29

(a) For steady-state conditions with switch S open, find the velocity v and load current i in terms of given data.

(b) With the system operating in the steady state as defined in part (a), the switch S is closed at $t = 0$. Find the velocity v and load current i as functions of the given data and time for $t > 0$.

12.30. This problem concerns the self-excitation of a dc generator. The variables and dimensions are given in Fig. 12P.30. The channel has a constant cross-sectional area, the fluid is incompressible and inviscid, and the conductivity is constant and scalar. Assume

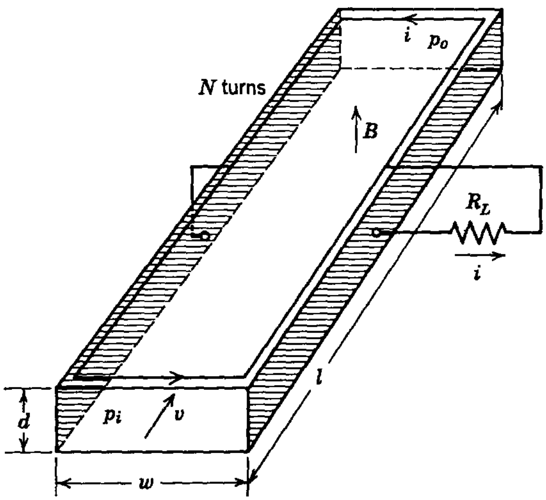


Fig. 12P.30

that the magnetic circuit is closed outside the channel with infinitely permeable iron. The mechanical source providing power has the pressure-velocity characteristic $\Delta p = p_i - p_o = \Delta p_o(1 - v/v_o)$, where Δp_o and v_o are positive constants. The volume, geometry, and space factor of the field winding are constants so that the field coil resistance varies as the square of the number of turns $R_c = N^2 R_{co}$. The numerical constants of the system are

$\Delta p_o = 2 \times 10^5\ n/\text{m}^2$	$v_o = 10^3$ m/sec	$d = 0.2$ m
$l = 2$ m	$w = 0.4$ m	$R_L = 2.5 \times 10^{-2}\ \Omega$
$\sigma = 40$ mhos/m	$R_{co} = 10^{-6}\ \Omega$	

(a) Find the number of turns necessary to produce a load power in R_L of 1.5×10^6 W. If there is more than one solution, pick the most efficient.

(b) For the number of turns in part (a), find the start up transient in current and plot it as a function of time. Assume an initial current of 10 A, provided by external means.

(c) For the number of turns found in part (a) find the steady-state load power as a function of R_L. Plot the curve.

12.31. The system of Fig. 12P.31 represents an MHD transverse-current generator with continuous electrodes. We make the usual assumptions about incompressible, inviscid, uniform flow. The fluid has mass density ρ and conductivity σ. The pressure drop along the

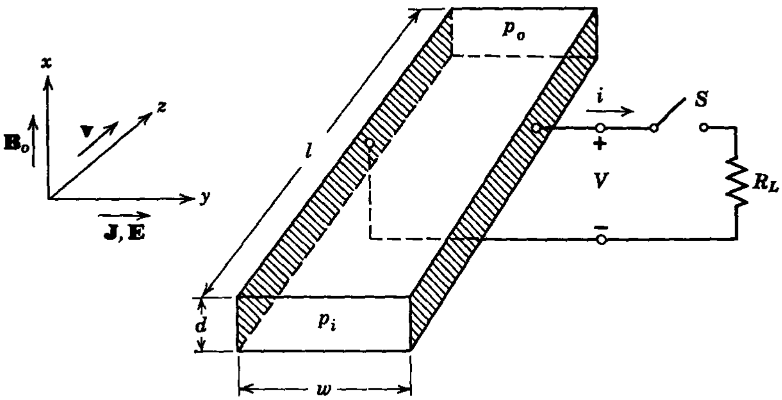


Fig. 12P.31

length of the channel is constrained by a mechanical source to be

$$p_i - p_o = \Delta p = \Delta p_o\left(1 - \frac{v}{v_o}\right),$$

where Δp_o and v_o are positive constants. The flux density B_o is uniform and constant and is supplied by a system not shown. Neglect the magnetic field due to current in the fluid. The switch S is open initially and the system is in the steady state:

(a) Find the terminal voltage V.

(b) At $t = 0$ switch S is closed. Find and sketch the ensuing transients in fluid velocity and load current.

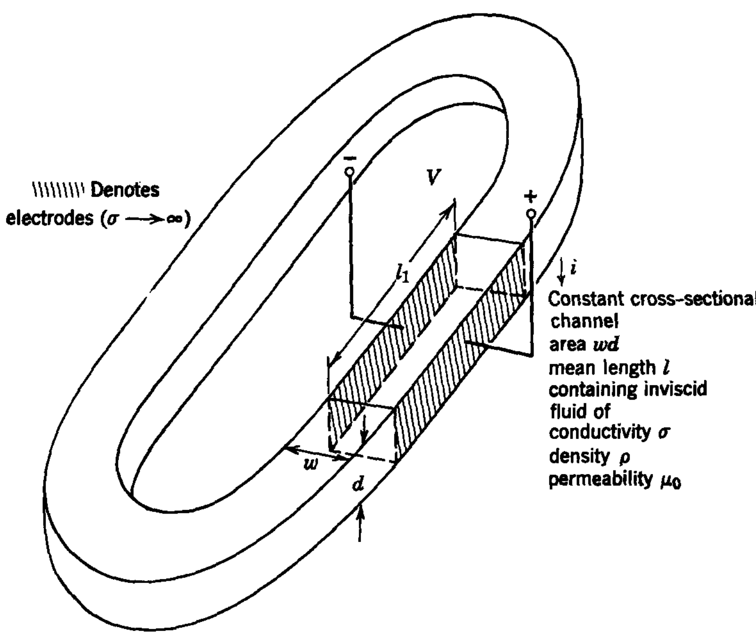


Fig. 12P.32

12.32. An energy storage element is to be made by using a closed channel of incompressible inviscid fluid of conductivity σ, permeability μ_0, and mass density ρ, with an MHD conduction machine for coupling (Fig. 12P.32). The channel has constant cross-sectional area wd and mean length l and the radius of the bends is large compared with the channel width w. The flux density **B** is supplied by a system not shown. Assume that the velocity is uniform across the channel and neglect end and edge effects and the magnetic field induced by current in the fluid. Find an equivalent electric circuit as seen from the electrical terminals and evaluate the circuit parameters.

12.33. In the system of Fig. 12P.33 an MHD generator is to be used to charge a capacitor. The MHD generator has a channel of constant cross-sectional area with the dimensions and arrangements shown in Fig. 12P.33. The working fluid has electrical conductivity σ and is incompressible and inviscid; it is constrained by external means to flow through the channel with a constant velocity v_o that is uniform across the cross section. The constant uniform flux density B_o is established by an external magnet not shown. Neglect the magnetic field due to current in the fluid and neglect fringing effects at the ends.

(a) Find the capacitor voltage V_c as a function of time and evaluate the final energy stored in capacitance C.
(b) Find the pressure difference supplied by the fluid source as a function of time and evaluate the total energy supplied by the fluid source.
(c) Account for the difference between your answers for energy in parts (a) and (b).

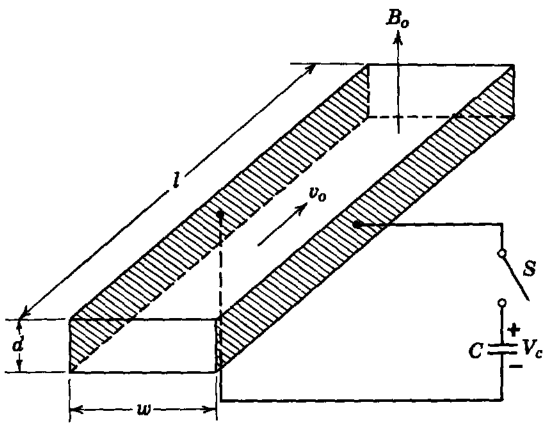
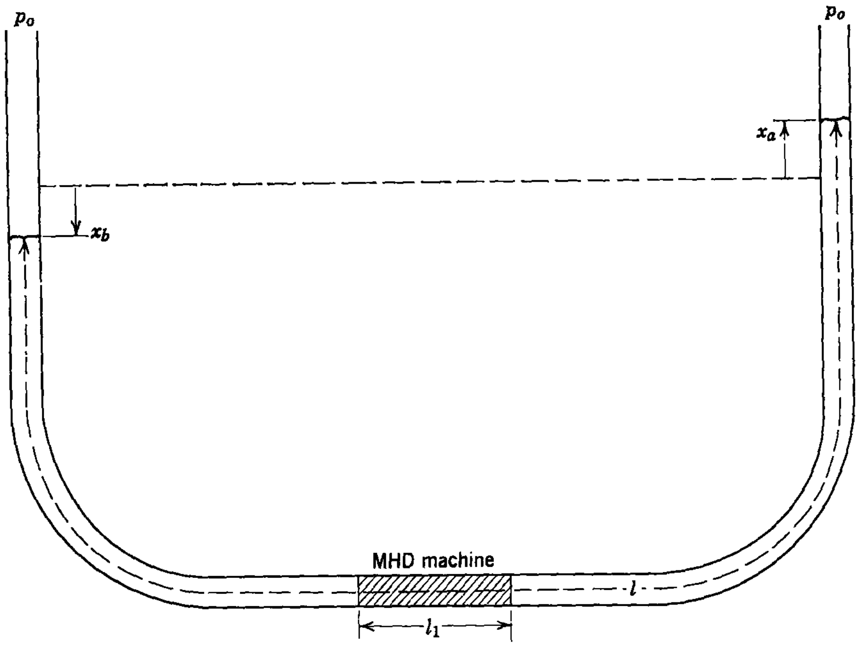


Fig. 12P.33

12.34. This problem is similar to the example worked in Section 12.2.1b. A U-tube of constant, rectangular cross section contains an inviscid, incompressible conducting fluid of mass density ρ and conductivity σ. The fluid has a total length l between the two surfaces which are open to atmospheric pressure as illustrated in Fig. 12P.34*a*. A conduction type MHD machine of length l_1 is inserted at the bottom of the U-tube. The details of the MHD channel are illustrated in Fig. 12P.34*b*. Neglect end effects and the magnetic field due to current flow in the fluid. The system parameters are

$$l = 1 \text{ m}, \quad l_1 = 0.1 \text{ m}, \quad w = 0.01 \text{ m}, \quad d = 0.01 \text{ m},$$
$$B_o = 2 \text{ Wb/m}^2, \quad V = 0.001 \text{ V}, \quad g = 9.8 \text{ m/sec}^2.$$

The fluid is mercury with constants $\sigma = 10^6$ mhos/m, $\rho = 1.36 \times 10^4$ kg/m^3. With the system in equilibrium, with switch S open, $x_a = x_b = 0$; switch S is closed at $t = 0$. Calculate the ensuing transients in fluid position x_a and electrode current i. Sketch and label curves of these transients.



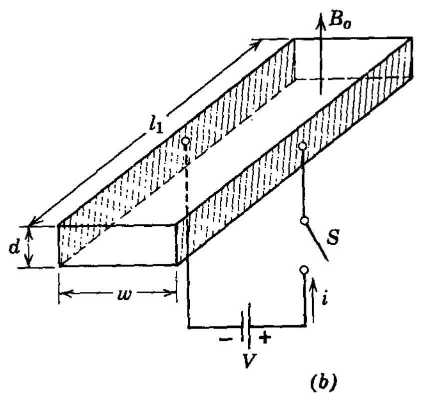


Fig. 12P.34

12.35. An incompressible inviscid conducting liquid fills the conduit shown in Fig. 12P.35. A current density J_0 (known constant) flows through the fluid over a length L of the channel. This section of the fluid is also subjected to a magnetic field produced by a magnetic circuit. (The gap D is the only portion of the circuit where $\mu \neq \infty$.) This magnetic field is produced by two N turn coils wound as shown. The liquid has two free surfaces denoted by x and y. When the fluid is stationary, $x = L/2$ and $y = L/2$. In the regions of the free surfaces, electrodes carrying the currents I_1 and I_2 are arranged as shown. It is seen that the resistance

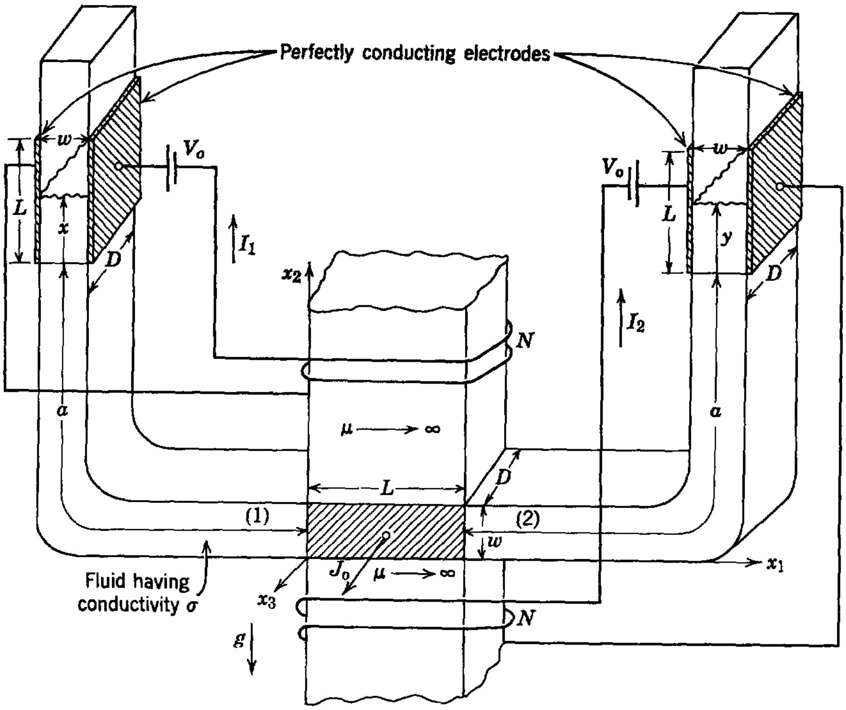


Fig. 12P.35

of the circuits is related to the heights of the interfaces. Find an equation for $x(t)$ that describes the system. Specifically, when $t = 0$, $x = L$, and $dx/dt = 0$, what is $x(t)$?

Assumptions

(a) v is directed along the channel and is independent of cross-sectional position.
(b) Ignore $L(dI_1/dt)$ and $L(dI_2/dt)$ (the rates of change with time are slow because of the fluid inertia).
(c) Ignore the magnetic field produced by J_o.

12.36. Rework the Alfvén-wave problem in cylindrical geometry in Section 12.2.3 with the following change in the boundary condition at $z = 0$. The end of the tube at $z = 0$ is fixed, rigid, and perfectly conducting. Your answer should consist of solutions for v_θ, B_θ, J_z, and J_r which are similar to (12.2.121)–(12.2.124). The functional dependences on z will be different in your answer because of the different boundary conditions.

12.37. A perfectly conducting inviscid fluid is bounded on the right by a perfectly conducting rigid wall, as shown in Fig. 12P.37. On the left a perfectly conducting plate also makes electrical contact with the liquid while being forced to execute an oscillatory motion in the x-direction. The system is immersed in a magnetic field so that when there are no motions $\mathbf{B} = B_0\mathbf{i}_z$, where B_0 is a constant.

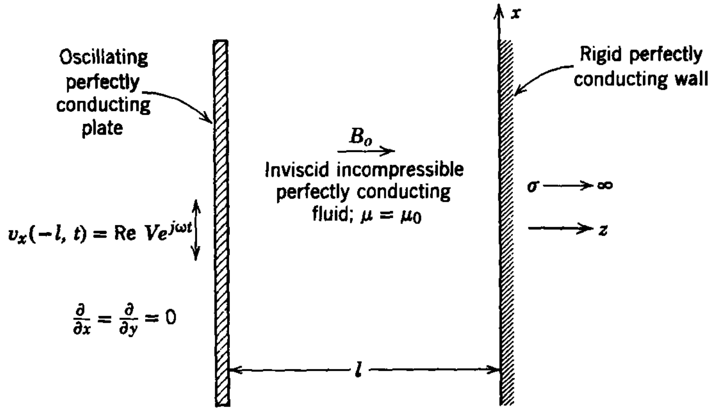


Fig. 12P.37

(a) Find the equations of motion, which together predict the transverse fluid velocity $v_x(z, t)$ and field intensity $H_x(z, t)$.

(b) Use appropriate boundary conditions to find $H_x(z, t)$ in the sinusoidal steady state.

(c) Compute the current density implied by (b). If you were to do this experiment, how would you construct the walls of the container that are parallel to the x-z plane? Explain in words why the fluid can transmit shearing motions even though it lacks viscosity.