

NETWORK STRUCTURES FOR IIR SYSTEMS

1. Lesson 12 - 40 minutes

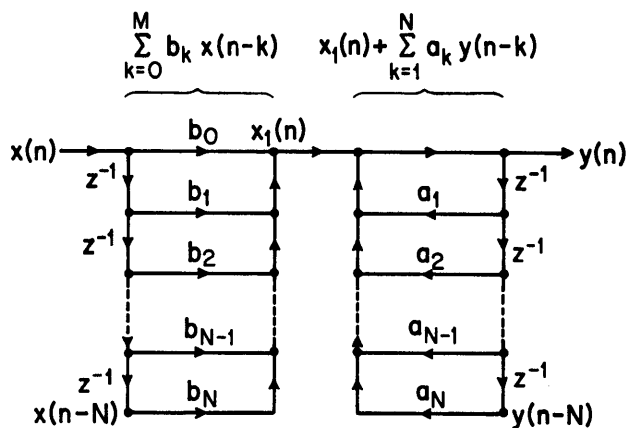
$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 - \sum_{k=1}^N a_k z^{-k}}$$

$$y(n) = \sum_{k=0}^M b_k x(n-k) + \sum_{k=1}^N a_k y(n-k)$$

$$H(z) = \left[\sum_{k=0}^M b_k z^{-k} \right] \left[\frac{1}{1 - \sum_{k=1}^N a_k z^{-k}} \right]$$

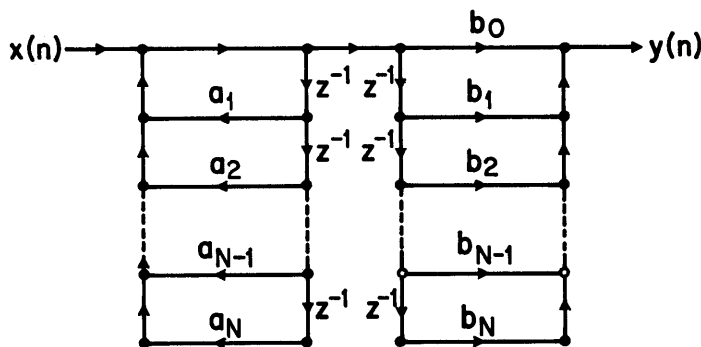
z-transform and difference equation for a general IIR system.

a.



Flow-graph representation of a general difference equation based on the factorization in b. (Direct form I realization.)

b.



Flow-graph representation of a general difference equation based on interchanging the order in which the poles and zeros are cascaded.

c.

$$y(n) = \sum_{k=0}^M b_k x(n-k) + \sum_{k=1}^N a_k y(n-k)$$

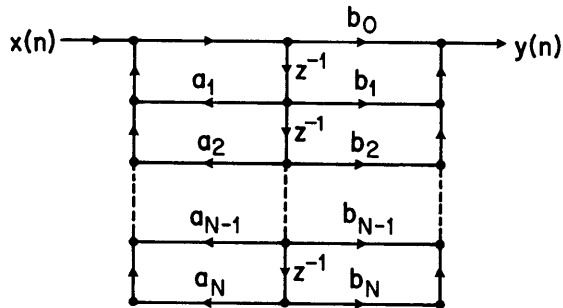
z-transform factorization and difference equation corresponding to the network in c.

$$H(z) = \left[\frac{1}{1 - \sum_{k=1}^N a_k z^{-k}} \right] \left[\sum_{k=0}^M b_k z^{-k} \right]$$

$$y_1(n) = x(n) + \sum_{k=1}^N a_k y_1(n-k)$$

$$y(n) = \sum_{k=0}^M b_k y_1(n-k)$$

d.



Flowgraph of c. collapsed to share delays (direct form II realization.)

e.

TRANSPOSITION THEOREM

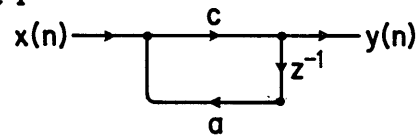
Transposition theorem for signal flow-graphs.

1. REVERSE DIRECTION OF ALL BRANCHES
2. INTERCHANGE INPUT AND OUTPUT

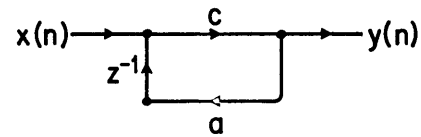
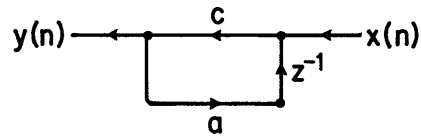
TRANSFER FUNCTION REMAINS THE SAME

f.

Example 1

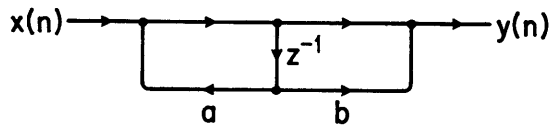


Example of Transposition theorem.

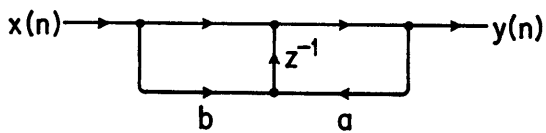
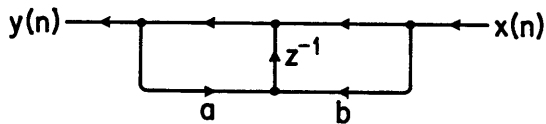


g.

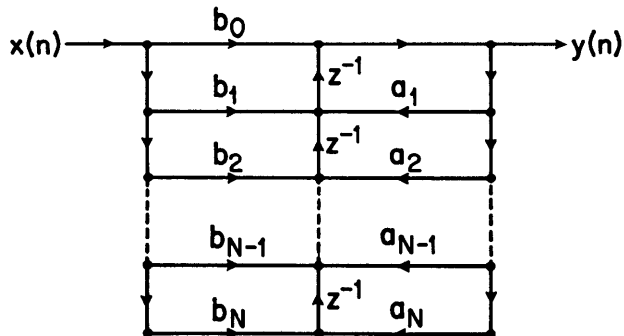
Example 2



Example of Transposition theorem.



h.



Transposed direct form II structure.

i.

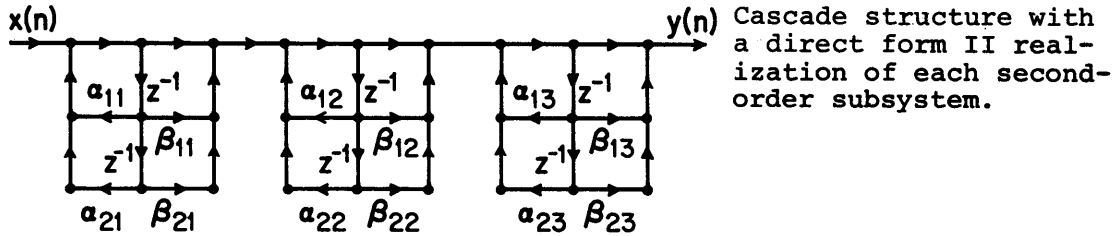
Cascade Structure

Factorization of the z-transform for the cascade structure.

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 - \sum_{k=1}^N a_k z^{-k}}$$

$$= A \prod \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}$$

j.



k.

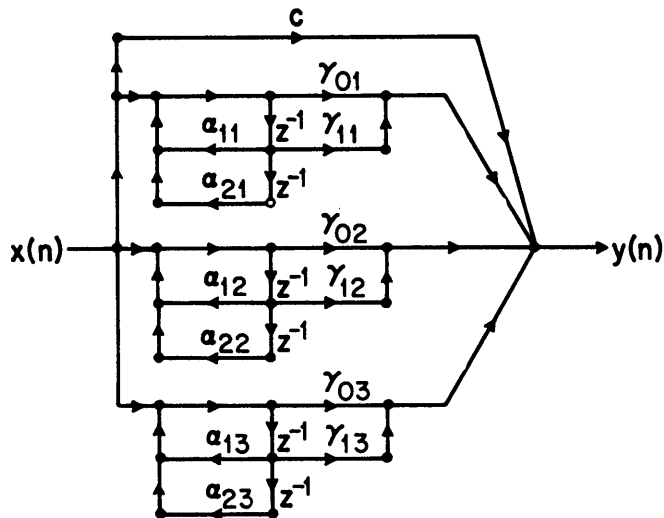
$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 - \sum_{k=1}^N a_k z^{-k}}$$

Partial Fraction expansion for parallel structure.

$$= \sum_{k=1}^{N_1} \frac{A_k}{1 - c_k z^{-1}} + \sum_{k=1}^{N_2} \frac{B_k (1 - e_k z^{-1})}{(1 - d_k z^{-1})(1 - d_k^* z^{-1})} + \sum_{k=0}^{M-N} c_k z^{-k}$$

$$= \sum_{k=1}^{\frac{N+1}{2}} \frac{(\gamma_{0k} + \gamma_{1k} z^{-1})}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}} + \sum_{k=0}^{M-N} c_k z^{-k}$$

l.



Parallel form realization with real and complex poles grouped in pairs.

m.

2. Comments

In the previous lecture we introduced the representation of linear digital networks. There are many network structures which can be used to implement a specified transfer function. While all of these structures are mathematically equivalent, they may have different implications for a hardware or software realization of a digital filter.

In this lecture we discuss some of the more common structures, specifically the direct-form, cascade form and parallel form. The notion of canonic structures, i.e. structures utilizing the minimum possible number of delays is also considered. Since delay branches in the flow graph represent memory registers in a hardware implementation it is generally desirable to minimize the number of delays in a structure. Implementation of a digital filter inevitably involves trade-offs, however, and the importance of minimizing delays may need to be balanced against other considerations.

We also present in this lecture the transposition theorem for signal-flow graphs. This theorem is useful in obtaining new structures. For example we present in this lecture the direct-form II structure and its transpose. Both of these structures are canonic in the number of delays. They differ in the order in which the poles and zeros are implemented, a consideration which is often important when implementing a digital filter with a small register length.

3. Reading

Text: Sections 6.3 (page 300) and 6.4.

4. Problems

Problem 12.1

Consider the discrete-time linear causal system defined by the difference equation

$$y(n] - \frac{3}{4}y[n - 1] + \frac{1}{8}y[n - 2] = x[n] + \frac{1}{3}x[n - 1]$$

Draw a signal flow graph to implement this system in each of the following forms:

- (a) Direct form I.
- (b) Direct form II.
- (c) Cascade.
- (d) Parallel.

For the cascade and parallel forms use only first-order sections.

Problem 12.2

In Figure P12.2-1 (a)-(c) several networks are shown. Determine the transpose of each and verify that in each case the original and transpose networks have the same transfer function.

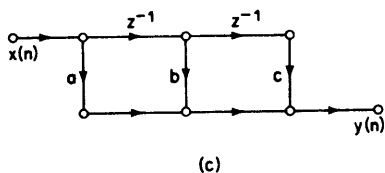
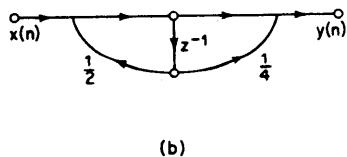
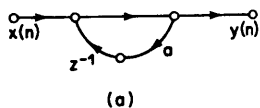


Figure P12.2-1

Problem 12.3

In Figure P12.3-1 (a)-(f) six digital networks are shown. Determine which one of the last five i.e., (b) through (f) has the same transfer function as (a). You should be able to eliminate some of the possibilities by inspection.

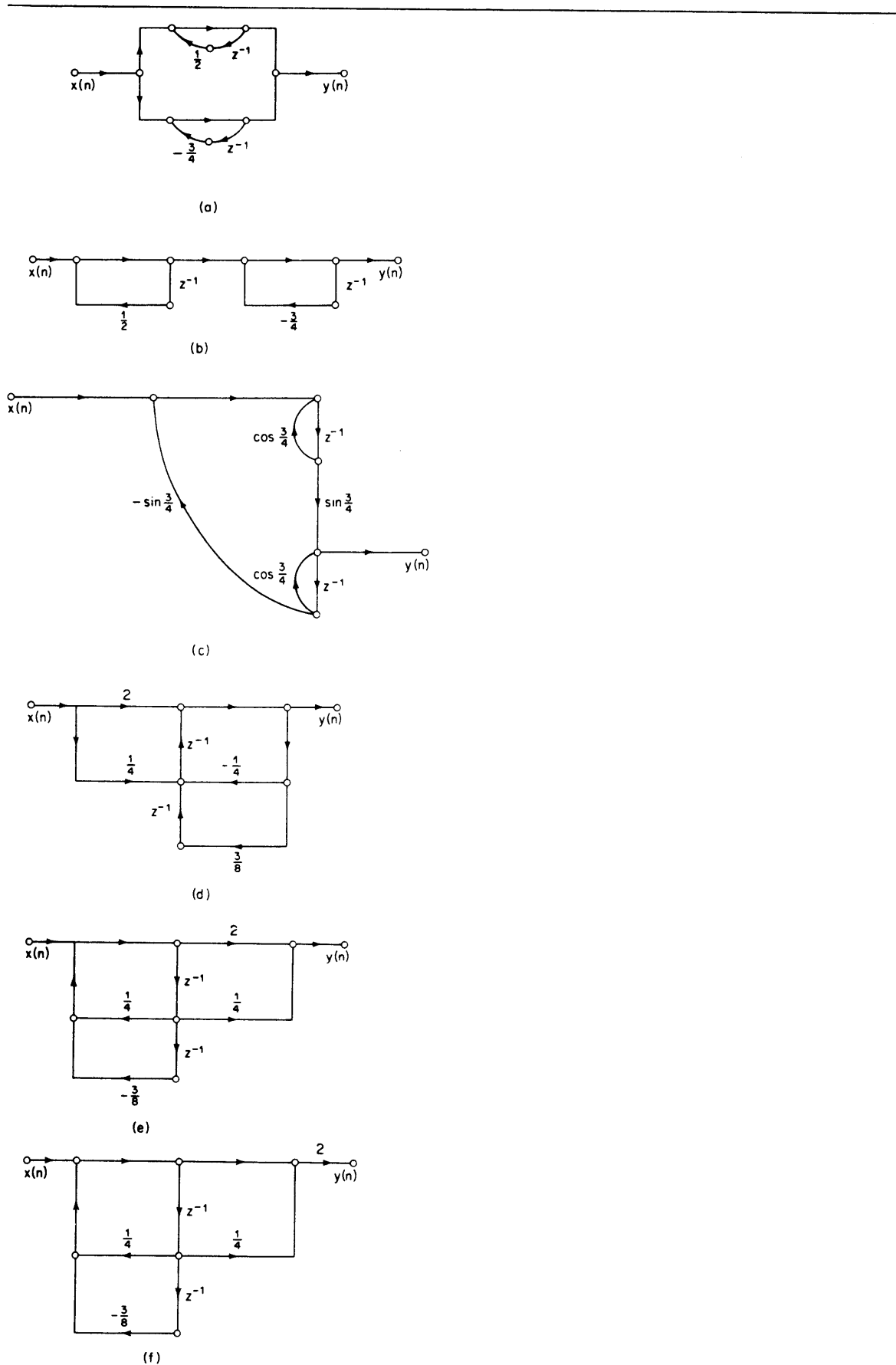


Figure P12.3-1

Problem 12.4*

The system with transfer function $H(z) = \frac{z^{-1} - a}{1 - az^{-1}}$ is an allpass system,

i.e. the frequency response has unity magnitude.

(a) Draw a network realization of this system in direct II form; and indicate in particular the number of delay branches required and the number of branches requiring multiplication by other than +1 or -1.

(b) An alternative implementation is suggested by noting that the difference equation of the allpass system can be expressed as

$$y(n) - ay(n - 1) = x(n - 1) - a x(n)$$

or equivalently,

$$y(n) = a [y(n - 1) - x(n)] + x(n - 1)$$

Draw a network realization of this equation requiring two delay branches but only one branch with a multiplication by other than +1 or -1.

The primary disadvantage to the network in (b) as compared to that in (a) is that two delay branches are required. In some applications, however, it is necessary to implement a cascade of allpass sections. For N allpass sections it is possible to utilize a realization of each in the form determined in part (b) but using only (N + 1) delay branches. This is accomplished essentially by sharing a delay between sections.

(c) Consider the allpass system with transfer functions

$$H(z) = \frac{z^{-1} - a}{1 - az^{-1}} \quad \frac{z^{-1} - b}{1 - bz^{-1}}$$

Draw a network realization of this system by "cascading" two networks of the form obtained in part (b) in such a way that only three delay branches are required.

Problem 12.5*

Speech production can be modeled as a linear system representing the vocal cavity, excited by puffs of air released through the vocal cords. In synthesizing speech on a digital computer, one approach is to represent the vocal cavity as a connection of cylindrical acoustic tubes with equal length but with different cross-sectional areas, as depicted in Figure 12.5-1. Let us assume that we want to simulate this system in terms of the volume velocity representing air flow. The input is coupled into the vocal tract through a small constriction, the vocal

cords. We will assume that the input is represented by a change in volume velocity at the left end, but that the boundary condition for traveling waves at the left end is that the net volume velocity must be zero. This is analogous to an electrical transmission line driven by a current source. The output is considered to be the volume velocity at the right end. We assume that each section is lossless.

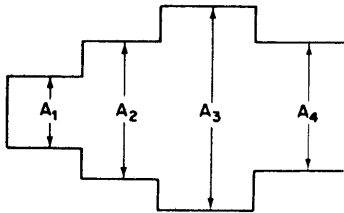


Figure P12.5-1.

At each interface between sections a forward-traveling wave is transmitted to the next section with one coefficient and reflected as a backward-traveling wave with a different coefficient. Similarly, a backward-traveling wave arriving at an interface is transmitted with one coefficient and reflected with a different coefficient. Specifically, if we consider a forward-traveling wave f_+ in a tube with cross-sectional area A_1 arriving at the interface with a tube of cross-sectional area A_2 , then the forward-traveling wave transmitted is $(1 + \alpha)f_+$ and the reflected wave is αf_+ where

$$\alpha = \frac{A_2 - A_1}{A_1 + A_2}$$

Consider the length of each section to be 3.4 cm with the velocity of sound in air 34,000 cm/s. Draw a digital network that will implement the four-section tube in Figure P12.5, with the output sampled at a 20-kHz rate.

In spite of the lengthy introduction, this is a reasonably straightforward problem. If you find it hard to think in terms of acoustic tubes, think in terms of transmission-line sections with different characteristic impedances. Just as with transmission lines, it is difficult to express the impulse response in closed form. Draw the network directly from physical considerations, in terms of forward- and backward-traveling pulses in each section.

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Resource: Digital Signal Processing
Prof. Alan V. Oppenheim

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