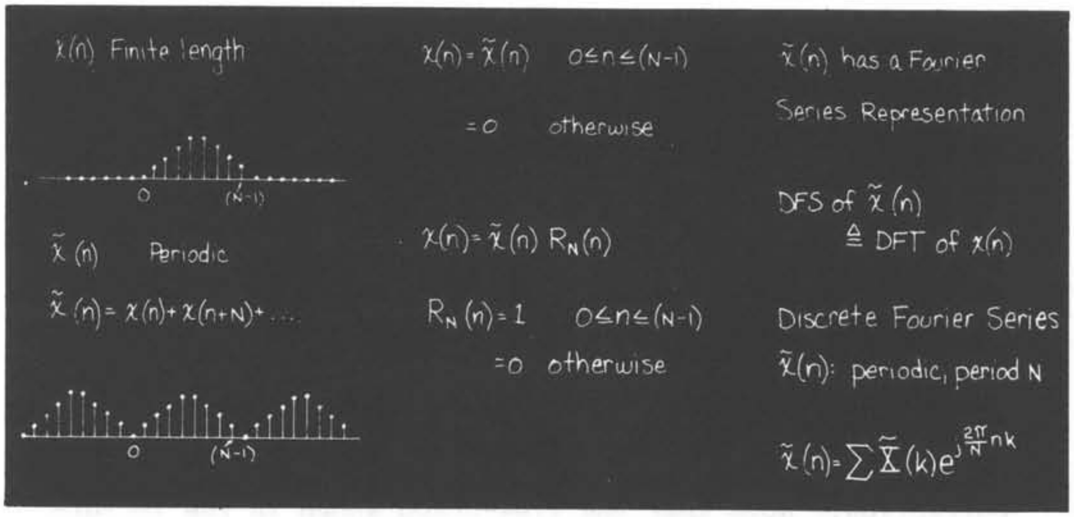
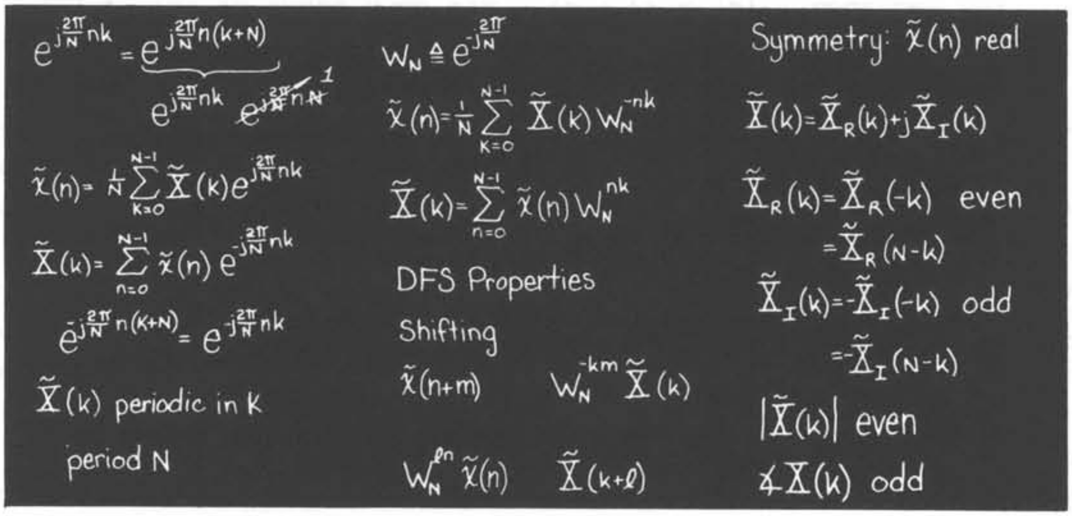
THE DISCRETE FOURIER SERIES

1. Lecture 8 - 43 minutes



a.



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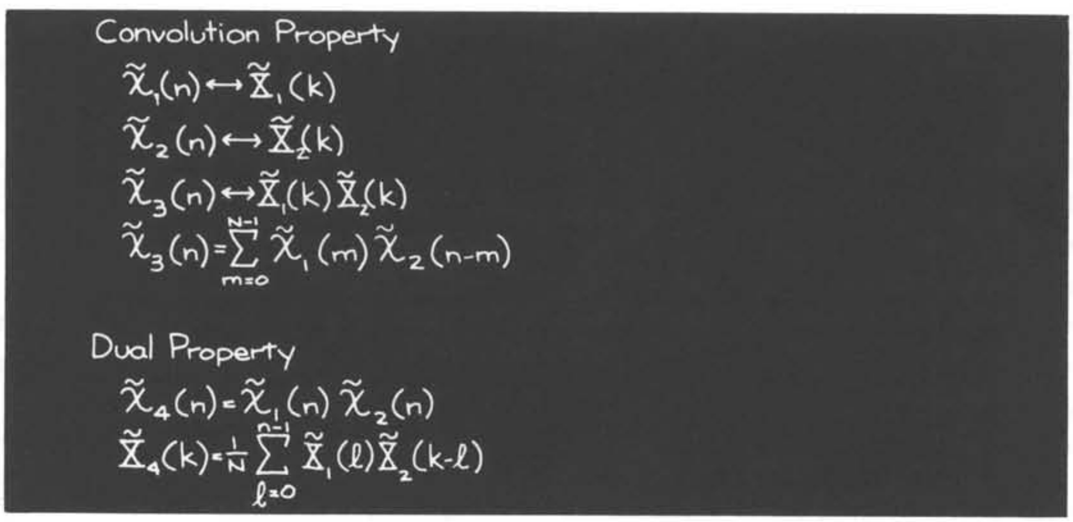

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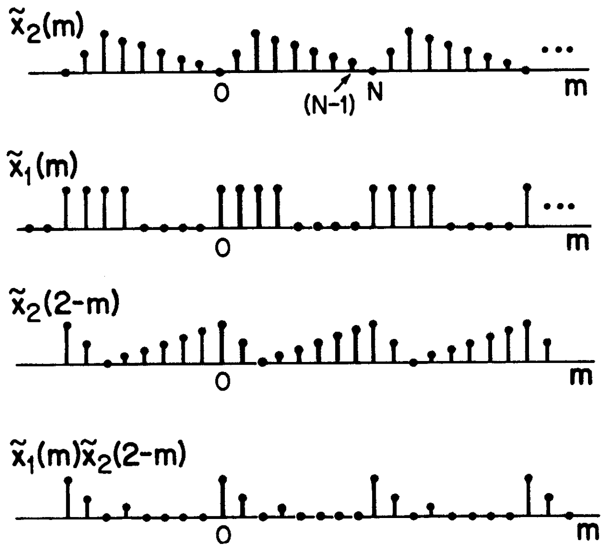

Illustration of the sequences involved in forming a periodic convolution.

d.

Correction: Note that the view-graph shown in the lecture illustrating periodic convolution does not correctly illustrate the sequence $\tilde{x}_1(m)\ \tilde{x}_2(2 - m)$. The copy above of this figure includes the correction

2. Comments

In the past several lectures we considered the Fourier transform and z-transform as tools for the representation and analysis of discrete-time signals and systems. In this lecture we introduce the Fourier series representation for periodic sequences. In the next lecture the Fourier series will be applied to developing a Fourier representation of finite length sequences, referred to as the Discrete Fourier Transform by utilizing the fact that finite length and periodic sequences are closely related. Because of the importance of this relationship in eventually interpreting properties of the Discrete Fourier Transform, this lecture begins with a discussion of the relationship betweeen finite length and periodic sequences. The remainder of the lecture concentrates on the Fourier series representation of periodic sequences.

In representing continuous-time signals through the Fourier series, the representation in general requires an infinite number of harmonically related complex exponentials (or sines and cosines). In contrast, in the discrete-time case there are only a finite number (N) of distinguishable harmonically related complex exponentials with fundamental period N. Thus there are only a finite number (N) of distinguishable Discrete Fourier Series coefficients. In developing the Fourier series representation it is convenient to interpret the Fourier series coefficients as a periodic sequence with period N. This interpretation is consistent with the property that there are only N distinguishable coefficients since the distinguishable values in a periodic sequence are represented by a single period.

The lecture concludes with a discussion of some of the properties of Fourier series coefficients. A property which will play an important role in lecture 10 is the convolution property, and in particular the definition of periodic convolution.

3. Reading

Text: Sections 8.0 (page 514) through 8.3.

4. Problems

Problem 8.1

(a) The sequence $\tilde{x}_1(n)$ of Figure P8.1-1 is periodic with period 4. Determine the Fourier series coefficients $\tilde{X}_1(k)$

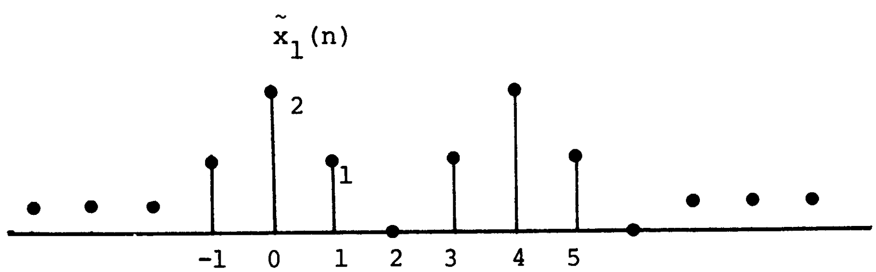

Figure P8.1-1

Problem 8.2

In the lecture it was stated that for a real periodic sequence $\tilde{x}(n)$ the Fourier series coefficients $\tilde{X}(k)$ are conjugate symmetric, i.e. $\tilde{X}(k) = \tilde{X}^*(-k)$. By using the Discrete Fourier Series analysis and synthesis pair, (eqs 8.11 and 8.12 in text) show that this symmetry property is true.

Problem 8.3

Another important symmetry property of the Fourier series coefficients states that if $\tilde{x}(n)$ is real and even, then $\tilde{X}(k)$ is real and even. By using the Discrete Fourier Series analysis and synthesis pair show that the symmetry property is true.

Problem 8.4

In figure P8.4-1 is shown a real periodic signal $\tilde{x}(n)$. Utilizing the properties discussed in section 8.2 and without explicitly evaluating the Fourier series coefficients, determine whether or not the following are true or not for the Fourier series coefficients $\tilde{X}(k)$.

(i) $\tilde{X}(k) = \tilde{X}(k + 10)$ for all k

(ii) $\tilde{X}(k) = \tilde{X}(-k)$ for all k

(iii) $\tilde{X}(0) = 0$

(iv) $\tilde{X}(k)\, e^{jk\frac{2\pi}{5}}$ is real for all k

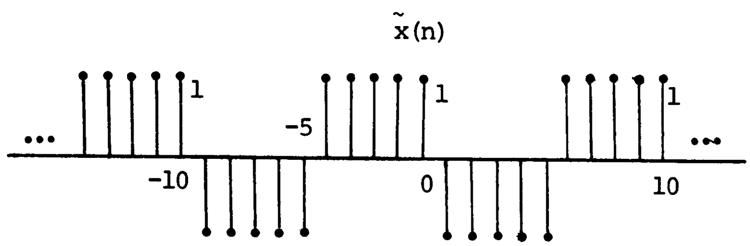

Figure P8.4-1

Problem 8.5

In figure P8.5-1 are shown two periodic sequences both with period 6. Determine and sketch $\tilde{x}_3(n)$ the result of a periodic convolution of these two sequences.

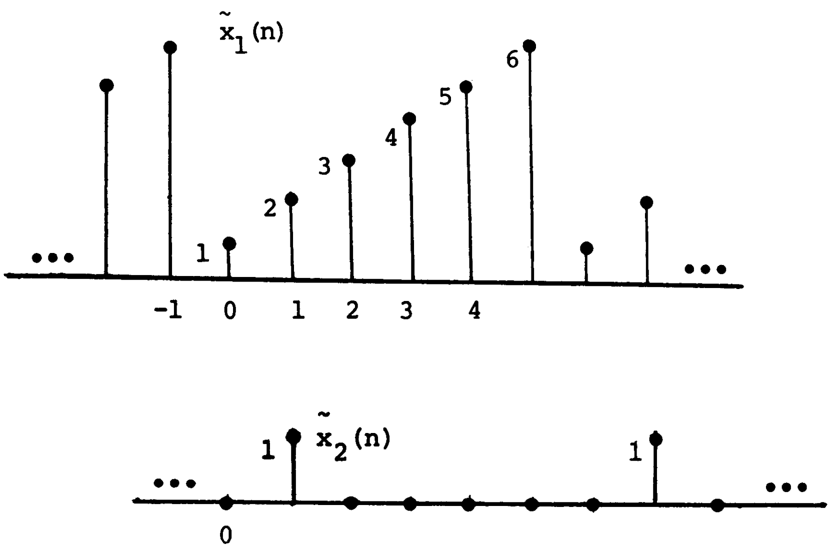

Figure P8.5-1

Problem 8.6*

$\tilde{x}(n)$ denotes a periodic sequence with period N and $\tilde{X}(k)$ denotes its discrete Fourier series coefficients. The sequence $\tilde{X}(k)$ is also a periodic sequence with period N. Determine, in terms of $\tilde{x}(n)$, the discrete Fourier series coefficients of $\tilde{X}(k)$.

Problem 8.7*

In Figure P8.7-1 are shown several periodic sequences x(n). These sequences can be expressed in a Fourier Series as

$$\tilde{x}(n) = \sum_{k=0}^{N-1} X(k)\ e^{j(2\pi/N)\ kn}$$

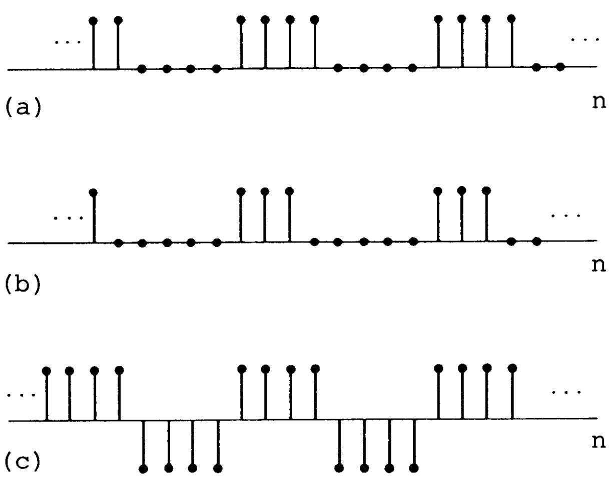

Figure P8.7-1

(a) For which sequences can the time origin be chosen such that all the X(k) are real?

(b) For which sequences can the time origin be chosen such that all the X(k) (except X(0)) are imaginary?

Problem 8.8*

If $\tilde{x}(n)$ is a periodic sequence with period N, it is also periodic with period 2N. Let $\tilde{X}_1(k)$ denote the DFS coefficients of $\tilde{x}(n)$ considered as a periodic sequence with period N and $\tilde{X}_2(k)$ denote the DFS coefficients of $\tilde{x}(n)$ considered as a periodic sequence with period 2N. $\tilde{X}_1(k)$ is of course, periodic with period N and $\tilde{X}_2(k)$ is periodic with period 2N. Determine $\tilde{X}_2(k)$ in terms of $\tilde{X}_1(k)$.