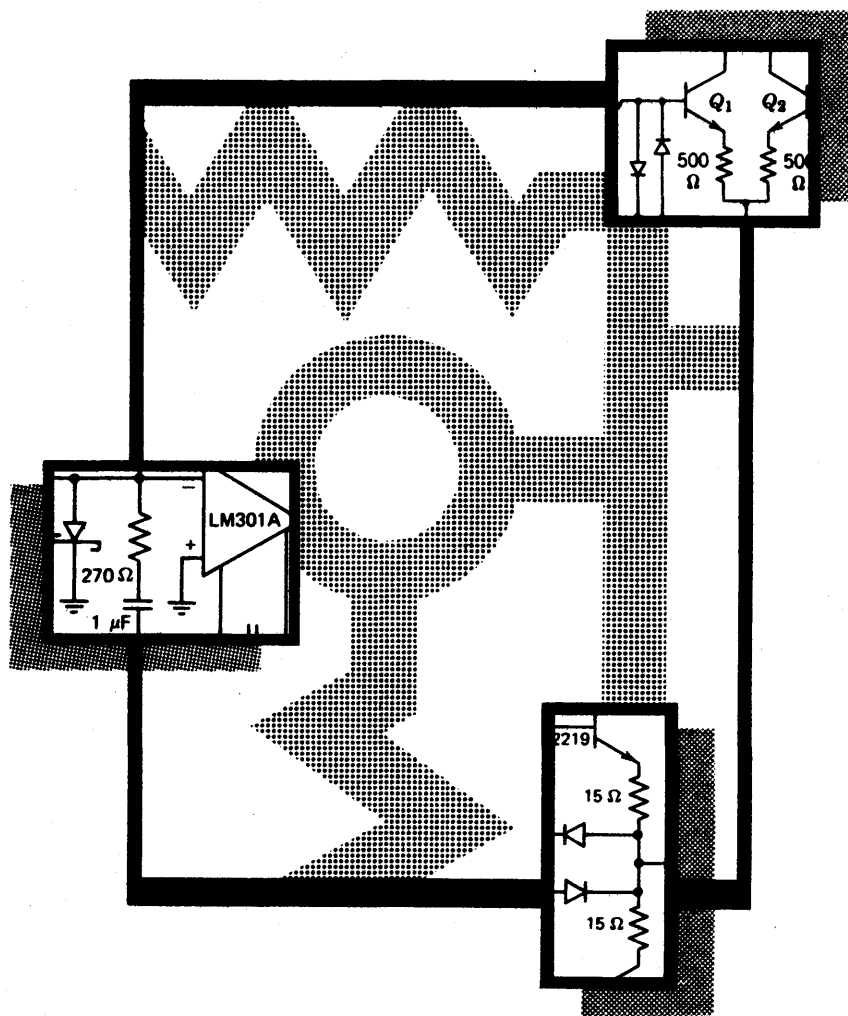


# Describing Functions

# 15



**Blackboard 15.1**

Describing Functions

Linear-  
 $V_i \sin \omega t \rightarrow [G(j\omega)] \rightarrow V_o \sin(\omega t + \theta)$   
 $\frac{V_o}{V_i} = |G(j\omega)|, \theta = \angle G(j\omega)$

Nonlinear-  
 $v_i = E \sin \omega t \rightarrow [NL] \rightarrow v_o$   
 $v_o = B_1 \sin \omega t + A_1 \cos \omega t + B_2 \sin 2\omega t + A_2 \cos 2\omega t + \dots$

$|G_D(E, \omega)| = \frac{\sqrt{A_1^2 + B_1^2}}{E}$   
 $\angle G_D(E, \omega) = \tan^{-1} \frac{A_1}{B_1}$

$G_D(E) = \frac{4}{\pi E} \angle 0^\circ$

$|G_D(E)|$  vs  $E \rightarrow$  (Graph showing inverse relationship)

$a(j\omega)G_D(E, \omega) = -1$

15-1

**Blackboard 15.2**

If NL is  $\omega$  ind.  
 $a(j\omega)G_D(E) = -1$   
 or  
 $a(j\omega) = -\frac{1}{G_D(E)}$

$\frac{1}{G_D(E)} = \frac{4E}{\pi} \angle 0^\circ$   
 $-\frac{1}{G_D(E)} = \frac{\pi E}{4} \angle -180^\circ$

$\frac{\pi E}{4} = 0.125, E = \frac{1}{2\pi}$   
 Stable Amplitude?

$\frac{1}{(s+1)^2} \rightarrow \frac{1}{2\pi} \sin \sqrt{3}t$

$|a(j\omega) + \frac{1}{G_D(E)}|$  vs  $\omega$  (Graph showing intersection)

$\angle a(j\omega) \rightarrow \angle \frac{1}{G_D(E)}$

$\omega = \sqrt{3}$  (Graph showing phase margin)

$|a(j\omega)G_D(E, \omega)|$  vs  $\omega$  (Graph showing stability margin)

15-2

**Blackboard 15.3**

Harmonic Distortion:  
 @ output of NL,  $\frac{3^{rd}}{1^{st}} = \frac{1}{3}$

$\frac{|a(j\sqrt{3})|}{|a(j3\sqrt{3})|} = 0.057$

distortion  $\approx 2\%$

15-3

---

Describing-function analysis offers a way to apply the powerful frequency-domain methods that are so useful in linear-systems analysis to nonlinear systems. The describing function indicates the gain-and-phase shift that a nonlinear element provides to an input sinusoid, considering only the fundamental component of the output.

### Comments

While describing-function analysis can be used to estimate the magnitudes of all signals in a nonlinear system that is driven with a sinusoid, the computational requirements for this type of detailed analysis are generally not justifiable. However, describing functions do provide a valuable way of estimating the amplitude, frequency, and harmonic distortion of certain kinds of oscillators.

---

Textbook: Sections 6.3 through 6.3.3.

### Reading

---

### Problems

---

**Problem 15.1 (P6.6)**

---

**Problem 15.2 (P6.7)**

MIT OpenCourseWare  
<http://ocw.mit.edu>

RES.6-010 Electronic Feedback Systems  
Spring 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.