S O L U T I O N S

Compensation 8

Note: All references to Figures and Equations whose numbers are *not* preceded by an "S" refer to the textbook.

Solution 8.1 (P4.13)

As suggested in Lecture 8, to perform a Nyquist analysis, we first sketch the Bode plot. The transfer function of interest is the af product given by

$$a(s)f(s) = \frac{10^6(0.01s + 1)^2}{(s + 1)^3} \times f_o \qquad \textbf{(S8.1)}$$

Using the methods of Section 3.4 of the textbook, the Bode plot is sketched in Figure S8.1. From this Bode plot, a gain-phase Nyquist plot is generated in Figure S8.2. From this figure, it is apparent that for some range of intermediate values of f_o, the -1 points will be enclosed within the contour, and the system will be unstable. However, for small enough *or* large enough values of f_o, the system will be stable. For instance, from Figure S8.2, the system is stable if $f_o = 1$, and it is certainly stable for any $f_o > 1$.

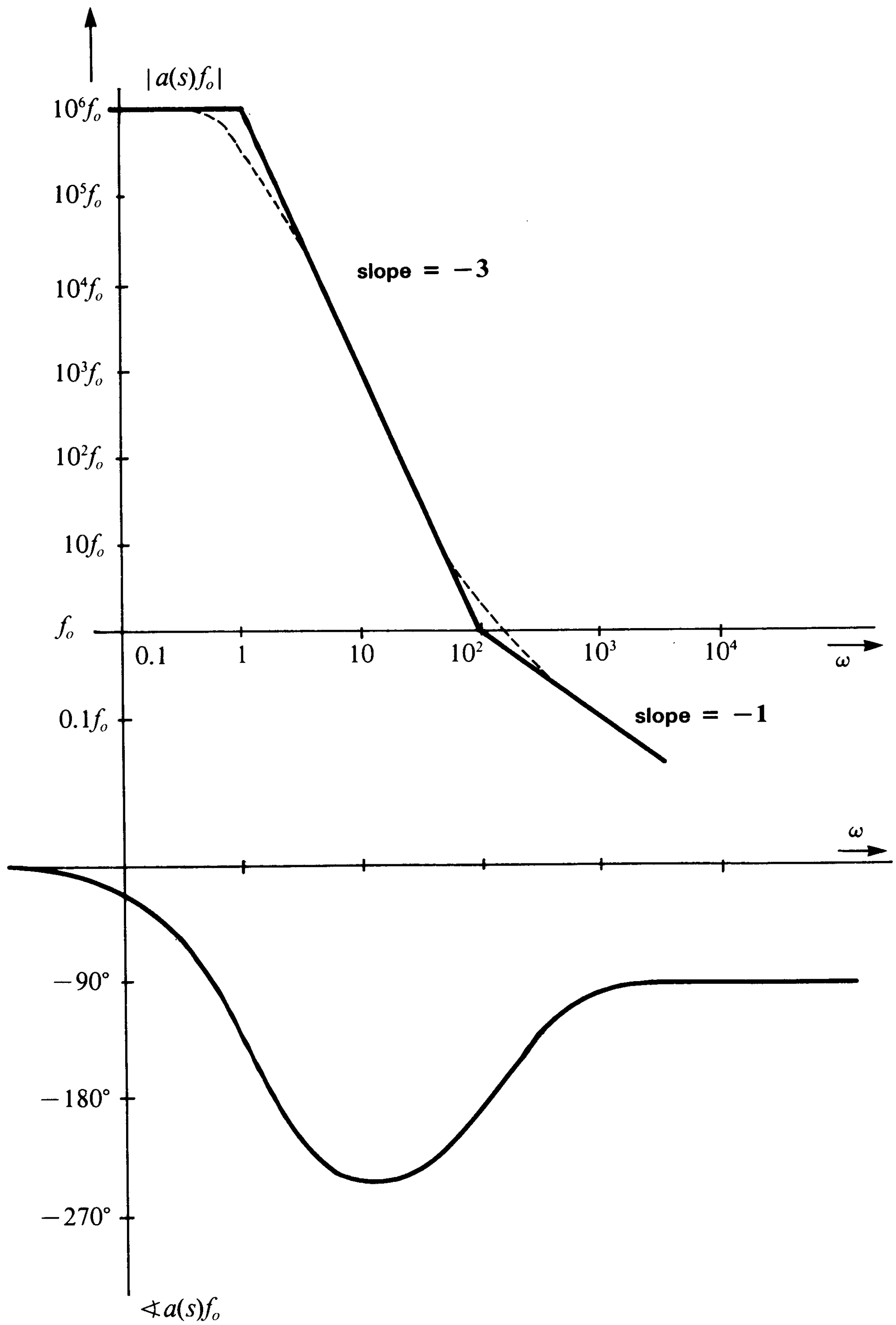
This same result can be obtained from a root-locus construction as shown in Figure S8.3. Because the two zeros are a factor of 100 farther from the origin than the three poles, the root-locus branches will initially follow asymptotes of $\pm 60°$ and $180°$ from the real axis, by Rules 7 and 5. The two branches that leave the real axis at $\pm 60°$ will enter the right half of the s plane at about $\omega = 1.7$ for large enough values of f_o. However, these two branches must rejoin the negative real axis at a point to the left of the two poles at $s = -100$, by Rules 2 and 3. Thus the branches cross back into the left half of the s plane and the system is stable for sufficiently large f_o. (Because only a qualitative analysis is required, the exact point at which the branches reenter the negative real axis will not be solved for.) Thus, both the Nyquist and root-locus analyses indicate that the system is stable for small values of f_o, unstable for intermediate values of f_o, and stable for large f_o.

Now, we can use a Routh analysis to determine the values of f_o that separate these regions of stability and instability. The characteristic equation is

$$1 + a(s)f_o = 1 + 10^6 f_o \frac{(0.01s + 1)^2}{(s + 1)^3} = 0 \qquad \textbf{(S8.2)}$$

After clearing fractions and collecting terms, we have

Figure S8.1 Bode plot for Problem 8.1 (P4.13).



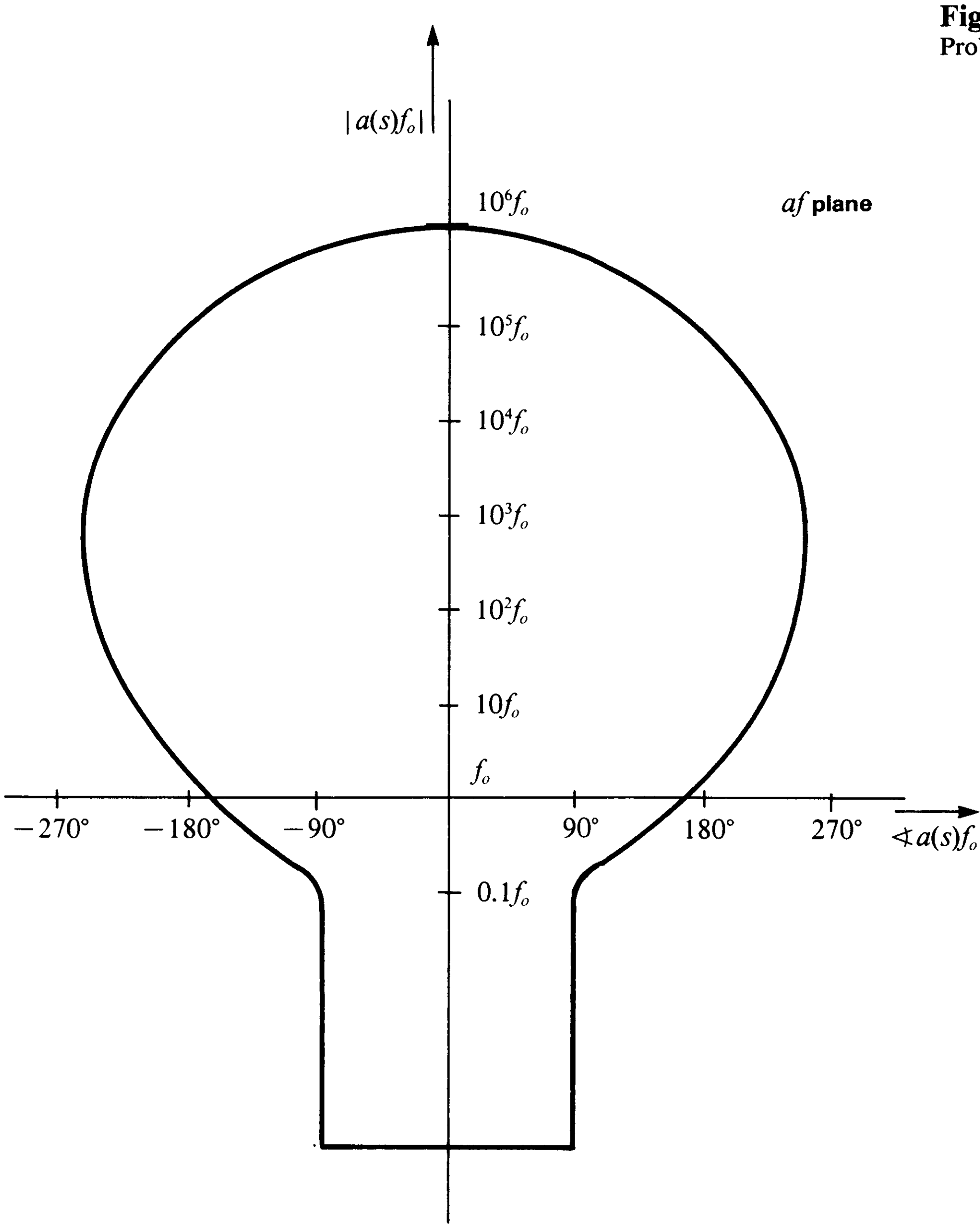
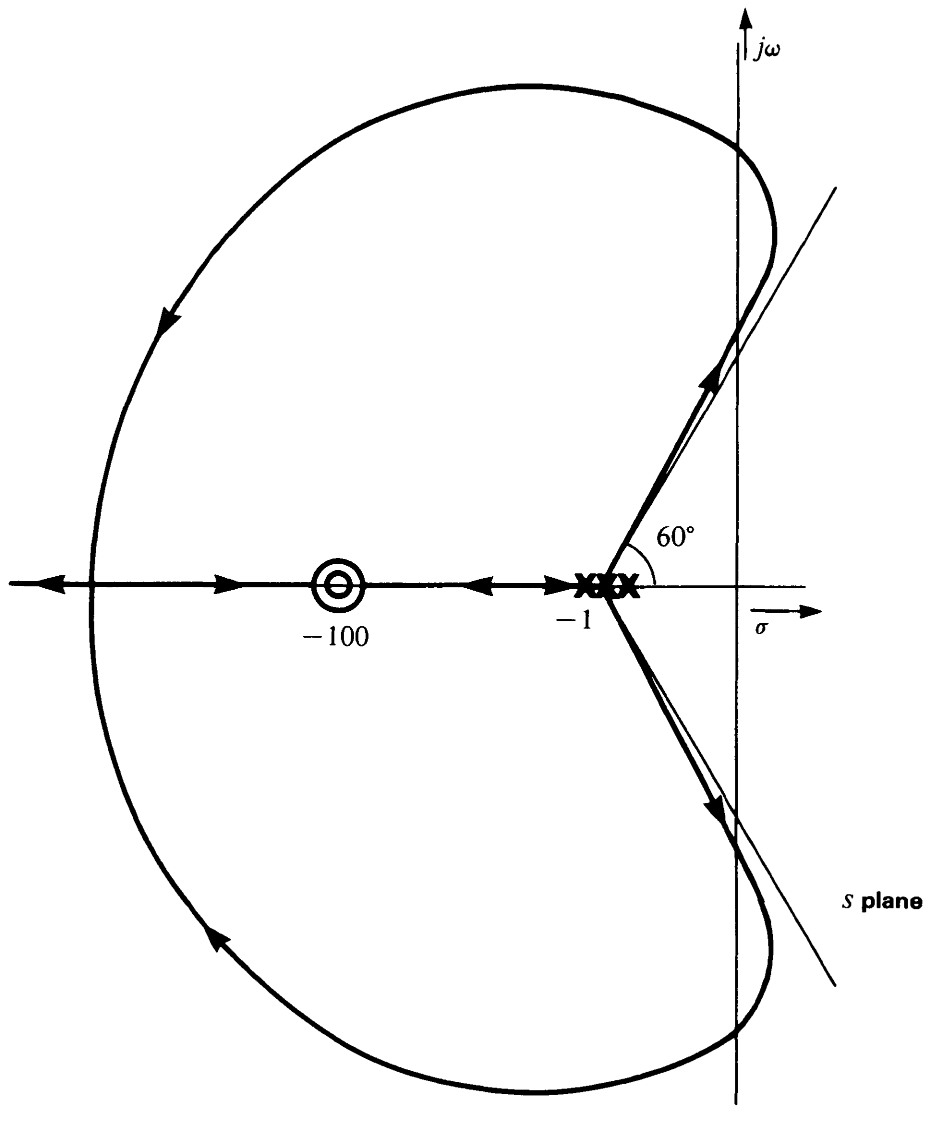


Figure S8.2 Nyquist plot for Problem 8.1 (P4.13).

Figure S8.3 Root locus for Problem 8.1 (P4.13).



$$s^3 + (3 + 10^2 f_o)s^2 + (3 + 2 \times 10^4 f_o)s + 1 + 10^6 f_o = 0 \qquad \textbf{(S8.3)}$$

From the polynomial, the Routh array is constructed as

$$\begin{array}{cc}
1 & 3 + 2 \times 10^4 f_o \\
3 + 10^2 f_o \; 1 & + 10^6 f_o \\
\dfrac{2 \times 10^6 f_o^2 - 0.94 \times 10^6 f_o + 8}{3 + 10^2 f_o} & 0 \\
1 + 10^6 f_o & 0
\end{array} \qquad \textbf{(S8.4)}$$

The third row becomes zero (indicating poles on the imaginary axis) when

$$2 \times 10^6 f_o^2 - 0.94 \times 10^6 f_o + 8 = 0 \qquad \textbf{(S8.5)}$$

which is solved by

$$\begin{aligned}
f_o &= \frac{0.94 \times 10^6 \pm \sqrt{(0.94 \times 10^6)^2 - 64 \times 10^6}}{4 \times 10^6} \\
&= 0.2350000 \pm 0.2349915 \qquad \textbf{(S8.6)} \\
&= 8.5 \times 10^{-6}, 0.47 \qquad \textbf{(S8.7)}
\end{aligned}$$

Note that high numerical precision is required to extract the root at $f_o = 8.5 \times 10^{-6}$. This problem could be avoided by framing the Routh calculation in terms of $a_o f_o$. However, a scientific calculator can carry out this calculation with sufficient accuracy. We carry this precision only where necessary, and round the answers of Equation S8.7 to two significant figures. The third row is negative for

$$8.5 \times 10^{-6} < f_o < 0.47 \qquad \textbf{(S8.8)}$$

Thus, the system is stable for

$$f_o < 8.5 \times 10^{-6}$$

and

$$f_o > 0.47 \qquad \textbf{(S8.9)}$$

which are the two borderline values the problem statement asks for.

Solution 8.2 (P5.1)

The circuit of Figure S8.4 provides an ideal gain of -10, and allows lowering of the loop-transmission magnitude.

The block diagram for this connection is as in Figure S8.5.

Figure S8.4 Connection with gain of -10, which allows lowering of the loop-transmission magnitude.

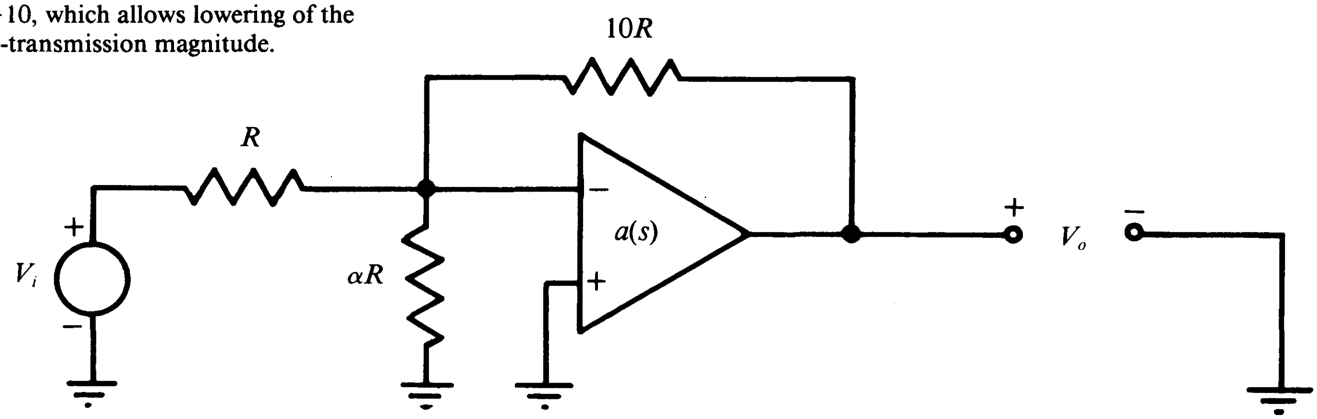
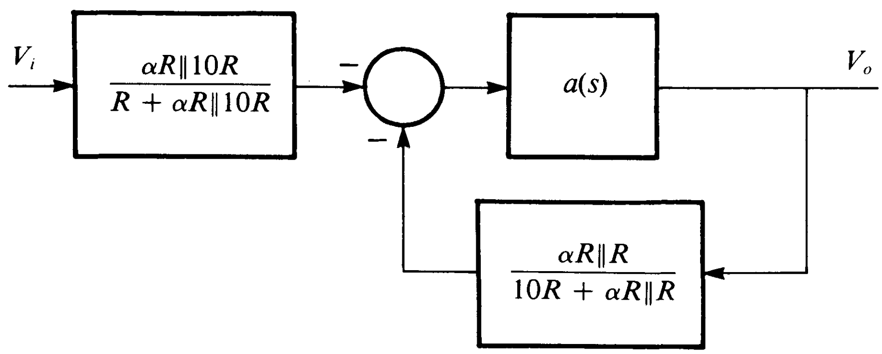


Figure S8.5 Block diagram for circuit of Figure S8.4.



The value of R cancels out of both blocks in which it appears. After algebraically reducing the expressions in these blocks, we have the diagram of Figure S8.6.

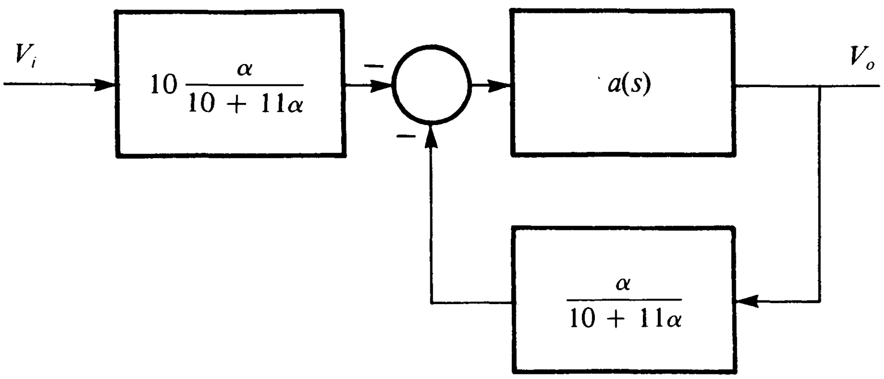


Figure S8.6 Reduced block diagram for Problem 8.2 (P5.1).

This can be further reduced as shown in Figure S8.7.

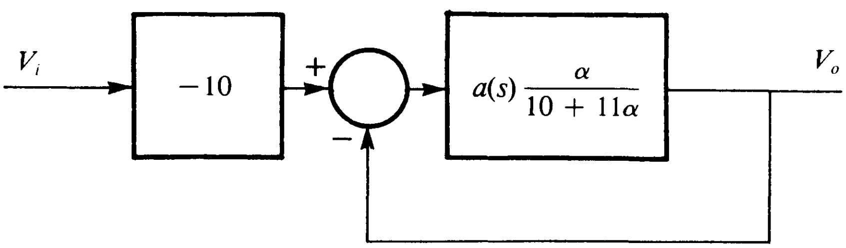


Figure S8.7 Reduced block diagram for Problem 8.2 (P5.1).

From the form of the block diagram, this system has an ideal gain of −10 as stated earlier. The negative of the loop transmission is

$$\begin{aligned} -\text{L.T.} &= a(s)\frac{\alpha}{10+11\alpha} \\ &= \frac{2\times 10^5}{(0.1s+1)(10^{-5}s+1)^2}\times\frac{\alpha}{10+11\alpha} \end{aligned} \qquad \textbf{(S8.10)}$$

From Figure 4.26*b*, the required damping ratio for $P_o = 1.1$ is approximately 0.6. From Figure 4.26*a*, this implies a phase margin of about 58°. That is, the loop-transmission phase must be −122° at the crossover frequency ω_c. The form of $a(s)$ allows us to readily solve for this frequency, because at frequencies where the two poles at $s = -10^5$ are contributing any significant phase shift, the pole at $s = -10$ is contributing −90° of phase. Thus, at ω_c, the phase due to the two poles must be −32°. Applying Equation 3.47 from the textbook, we can write

$$-32^\circ = -2\tan^{-1}10^{-5}\omega_c \qquad \textbf{(S8.11)}$$

which is solved for ω_c as

$$\omega_c = 10^5 \tan 16° = 2.87 \times 10^4 \text{ rad/sec} \qquad \textbf{(S8.12)}$$

Now, we need to pick α to set the loop-transmission magnitude equal to unity at this frequency. Applying Equation 3.46 to the three poles gives

$$1 = \frac{2 \times 10^5}{\sqrt{0.01\omega_c^2 + 1}\,(10^{-10}\omega_c^2 + 1)} \times \frac{\alpha}{10 + 11\alpha} \qquad \textbf{(S8.13)}$$

Substituting in $\omega_c = 2.87 \times 10^4$ gives

$$1 = 64.4\frac{\alpha}{10 + 11\alpha} \qquad \textbf{(S8.14)}$$

which is solved by

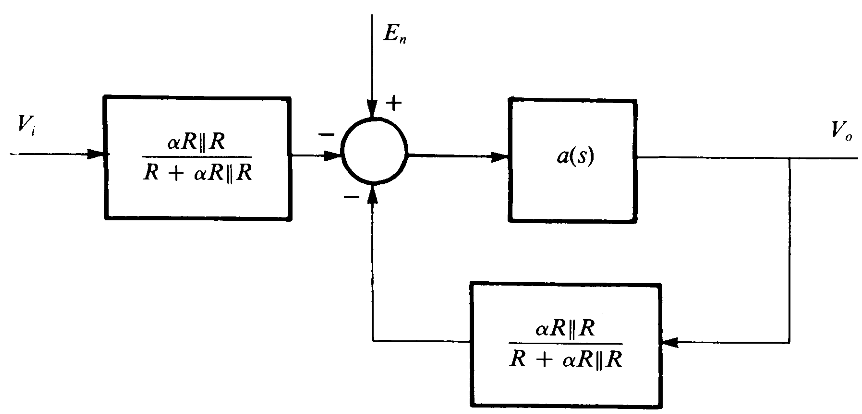
$$\alpha \simeq 0.19 \qquad \textbf{(S8.15)}$$

Thus, the value of the attenuation resistor is $0.19R$.

Solution 8.3 (P5.2)

This is the same topology as in Problem 8.2 (P5.1). The only difference is that the noise voltage E_n adds directly to the error signal, and the ideal gain is -1 rather than -10. The appropriate modifications to the block diagram of Figure S8.5 give the block diagram of Figure S8.8, which represents the connection of Figure 5.23a.

Figure S8.8 Block diagram for circuit of Figure 5.23a.



By a block-diagram manipulation, Figure S8.8 reduces to Figure S8.9.

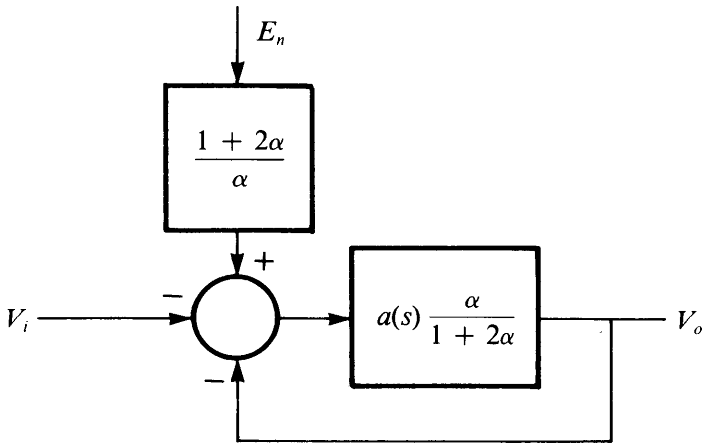


Figure S8.9 Reduced block diagram.

From Figure S8.9, at frequencies where $\left| a(s)\dfrac{\alpha}{1 + 2\alpha} \right| \gg 1$

$$\frac{V_o(s)}{E_n(s)} \simeq \frac{1 + 2\alpha}{\alpha} \tag{S8.16}$$

For $\alpha \gg 1$, this ratio is about 2. For $\alpha \ll 1$, the ratio $\dfrac{V_o(s)}{E_n(s)}$ becomes very large, verifying the assertion at the end of Section 5.2.1 that this type of attenuation increases voltage noise at the amplifier output.